AN APPLICATION TO THE TRAVELLING SALESMAN PROBLEM

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ABSTRACT

ABC Appliances (Pvt) Ltd.,one of the leading companies in Sri Lanka has supplied and installed a very large number of air condition units all over the country. The company is currently provides a comprehensive after sales service for its customers. At present the service department is interesting in reducing the cost involving in regular after sale servicers. In this research we proposed a Travelling Salesman Problem (TSP) approach tominimize the cost involving in service tours. We used nearest neighbourhood search algorithm to obtain the solutions to the TSP. Computational examples show that the new service routes obtained using this algorithm will reduce the travelling cost significantly in comparison to existing routs.

KEYWORDS

Travelling Salesman Problem, Greedy Algorithm, NP Hard, Heuristic, Meta Heuristic, Nearest Neighbour

1. INTRODUCTION

ABC Appliances Company (pvt) Ltd., one of the leading companies in Sri Lanka, was established in November 1991 for the purpose of marketing a range of brand products; air conditioners, water pumps, ceiling fans, table and pedestal fans, four-wheel tractors and generators. The company is currently distributing these products island wide through a strong dealer network. The ABC Appliances Company, not only distributing air conditioners and accessories but also provides a comprehensive after sales service for its customers. The company maintains a service department consists of a service manager, three engineers, thirty technicians, five motor vehicles and three motor bicycles to provide after sales for the customers. The main responsibilities of the department are to service the air conditioners purchased from the company and deploy mobile teams to attend break-downs.

At present the service department is interesting in reducing the cost involving in regular after sale servicers for air conditioners. The service department has to visit its customers once a month to service the air conditioners. These customers are located island wide and therefore, travelling cost contributes a reasonable amount for the total cost on top of service cost. Thus the goal of this work is to propose scientific approach to minimize the travelling cost.

In this research we used the concept of Travelling Salesman Problem (TSP)([18],[19],[20])to determine the regular service routs that minimizes the travelling costs. In the classical TSP problem, the objective is to find the best possible way of visiting all the cities exactly once and returning to the starting point for a given set of cities and the cost of travel (or distance) between each possible pairs. Likewise, the serve department of ABC Company starts their service journeys from the head office and return back to the head office after visiting the locations that

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need to be serviced. Thus applying TSP approach to determine optimal service routs for ABC Company is relevant.

The TSP problem is a NP hard combinatorial optimization problem (COP) ([1],[9],[8]). Combinatorial optimization is a branch of discrete optimization and the COPs have a finite set of possible solutions. The best way to solve a COP is to enumerate all the feasible solutions in the search space using algorithms such as the branch and bound algorithm and the branch and cut algorithm ([10], [12],[16], [17]). However, enumerating all the feasible solutions is not always possible, thus solving this problem optimally in real time is impractical, especially as the search space complexity grows, the execution time can increase exponentially, making the search for a solution is not feasible ([1],[3],[9]). In this case the COP is defined as NP-hard (non-deterministic polynomial-time) COP ([6], [10],[15]). Because of the difficulties faced with enumerating all feasible solutions using these methods, alternative methods such as approximation algorithms, heuristic algorithms, and metaheuristic algorithm have been proposed. Therefore, in this research project we used a heuristic algorithm proposed for the TSP based on nearest neighbourhood search [7] to obtain the best route for the ABC Company.

The data collected from ABC Company is used in this algorithm to find the best routs for the company. Computational examples show that the new service routes obtained using this algorithm will reducethe travelling cost significantly in comparison to existing routs.

2. PROBLEM STATEMENT

ABC Appliances (Pvt) Ltd. has supplied and installed a very large number of air condition units all over the country. According to the service agreements, the company has to attend to urgent break- down calls promptly and to carry out regular service visits to each and every unit already installed. At present, the service department is following a schedule of places to be visited, which is prepared by its Management Department. According to the present schedule, the entire country is divided into six regions for the convenience of carrying out regular service tours. The six regions are given below. Each region is corresponding to a Tour/Route. Therefore, currently there are six different service tours/ routs in place.

Region 1(Tour No 1)

Colombo, Nittabuwa, Peradeniya, kegalle, Gampola, Mawanella, Nawalapitiya, Kothmale, Akurana, Kandy, Kundasale, Digana, Mahiyangana, Matale, Pussellawa, Dehiyattakandiya, Colombo.

Region 2 (Tour No 2)

Colombo, Anuradapura, Kaduruwella, Manampitiya,Valachchanai, Batticalo, Samanthurai, Ampara, Colombo.

Region 3 (Tour No 3)

Colombo, Dhargatown, Ahungalla, Hikkaduwa, Galle, Beruwela, Katukurunda, Matara Colombo.

Region 4 (Tour No 4)

Colombo, Kochchikade, Wennappuwa, Lunuwila, Marawila, Chilaw, Puttalam, Wariyapola, Hettipola, Kuliyapitiya, Narammala, Colombo.

Region 5 (Tour No 5)

Colombo, Awissawella, Eheliyagoda, Rathnapura, Kalawana, Rathnapura, Pelmadulla, Embilipitiya, Middeniya, Hungama, Tangalle, Hungama, Tissamaharamaya, Katharagama, Moneragala, Badulla, Welimada, Bandarawella, Balangoda, Pelmadulla, Colombo.

Region 6 (Tour No 6)

Colombo, Kurunegala, Uhumiya, Melsiripura, Galewela, Dambulla, Madawachchiya, Vavniya, Kebithigollewa, Tricomalee, Habarana, Colombo.

Apparently the Management Department has not taken into account the important factors such as optimal route in preparing the schedule. Hence, the Service Department is in the process of analysing the present schedule for the purpose of amending them suitably to minimize the time and cost involved in attending to regular services of air conditioner units. As there is a stiff competition for supply and installation of air conditioners it has become very essential to prepare a well-planned schedule so that the service department will be able to carry out regular service tours efficiently.

3. METHODOLOGY

First, we represent the cities and the roads connecting them in a network, which is called a road network. This will enable us to distinguish the paths currently used by the service department and the other alternative paths, which can be considered to reduce the travelling distance. A network consists of a set of points called nodes, with certain pairs of nodes being joined by lines called branches and a flow of some type on its branches ([2],[4]). In this research we consider a road network, where nodes are considered as cities, branches as roads linking these cities and flow as distance between any two cities. The problem that we deal here can be represented in six different road networks. The Table1 represents the distances between the cites for Tour No.1 and Figure 1 shows the road associated with tour No. 1. Remaining tour informationare given in the Appendix.

Origin	Destination	Distance (km)
Akurana	Kandy	11
Ambepussa	Kegalle	17
Colombo	Nittambuwa	40
Gampola	Nawalapitiya	13
Gampola	Pussaellawa	22
Kandy	Kurunegala	42
Kandy	Peradeniya	7
Kegalle	Mawanella	13
Kundasale	Digane	12
Kundasale	Kandy	5
Mahiyangana	Dehiattakandiya	35
Mahiyangana	Digane	49
Matale	Akurana	15
Mawanella	Peradeniya	19
Nittambuwa	Ambepussa	20
Peradeniya	Gampola	18

Table 1. Distance Table for Tour No 1

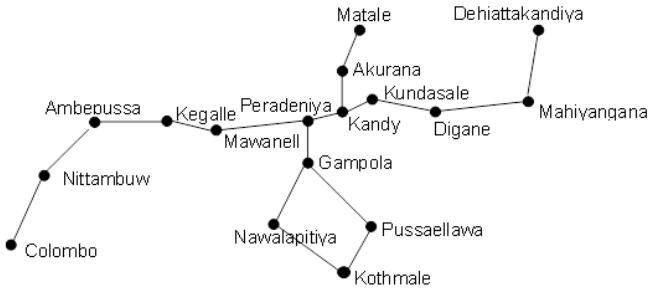

Figure 1. Network Diagram for Tour No 1

Since the service department is located at Colombo, the service tours should start from Colombo and back to Colombo after visiting the intermediate cities. As our objective is to minimize the travelling cost, which depends on the travelling distance, our approach should be to visit each city exactly once and come back to the starting point, which is Colombo. As mentioned in the Introduction, this problem can be formulated as TSP which is one the hard NP COP. Thus, we emphasize the mathematical formulation of TSP in Section 3.1 to obtain the best routs for ABC Company.

3.1. MATHEMATICAL FORMULATION OF THE TRAVELING SALESMAN PROBLEM

There are several mathematical formulations of the traveling salesman problem, and the one we have chosen to present in this report is due to Miller, Tuker, and Zemlin (1960)[14]. We begin the formulation process by first numbering the cities from 1 through n, with city 1 being designated as the home city with index set $I = \{i: i = 1, \dots, n\}$. Let $x \in R^n$ be the decision variable defined as follows:

$$x_{i,j} = \begin{cases} 1, & \text{if city } j \text{ is visited immidiately following city } i \\ 0, & \text{otherwise} \end{cases}$$

Let $c_{i,j}$ be the traveling distance between city i and city j. The combinatorial optimization model of the TSP can be formulated as follows:

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} C_{i,j} x_{i,j}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} x_{i,j} = 1, \quad for\ i \in I \qquad \rightarrow (1)$$

$$\sum x_{i,j} = 1, \quad for\ j \in I \qquad \rightarrow (2)$$

$$t_i - t_j + n x_{i,j} \leq n - 1, \quad for\ i, j \in I \quad \rightarrow (3)$$

where t_i, t_j are arbitrary real numbers.

Constraints (1) ensure that on the travelling salesman tour, each city is followed by exactly one city. Similarly, constraints (2) specify that a unique city is visited immediately before city. The remaining constraints (3) are referred to as subtour elimination constraints. Their purpose is to

ensure that a single tour result rather than a number of disjoint sub-tours. Because city 1 is the home city, the constraints operate by ensuring that every tour contains city 1.

3.2 HEURISTIC ALGORITHM FOR THE TRAVELLING SALESMAN PROBLEM

In this section we present the algorithm that we used to obtain the best solution of the TSP. In this algorithm, we assume that the given road network is a complete road network. The concept of the nearest neighbour search is used in this algorithm. The key of this algorithm is to always visit the nearest city by minimizing the traveling cost $c_{i,j}$ for $i, j \in I$and return to the starting city when all the other cities are visited.The pseudocode of the algorithm is given in Figure 2. In this algorithm the symbol $\bar{I}$ denote index set of the currently visited cities. The algorithm starts with a homecity k. Starting from the home city k the algorithm finds the nearest unvisited city by minimizing the traveling cost $c_{k,i}$where in $i \in I \backslash \bar{I}$ and visit that city. This algorithm is run until all cities are visited and return to the home city k. The algorithm returns the tour start with home city k and the corresponding cost.

Algorithm: Nearest Neighbour

Start with a sub-tour with home city k

1. Input: Number of cities (n), C (cost matrix), k(home city)
2. Initialization:$\bar{I} = k$ where :$\bar{I}$ denotes the currently visited cities and cost=0
   % need to find a tour covering all cities

while $\bar{I} \neq I$ *do*

- Consider the branch(k, i) for all $i \in I$
- % select the nearest unvisited neighbor $i$of k according to the minimum traveling cost
- ***if*** i *not* $\in$ $\bar{I}$**then**
- update $\bar{I} = k \cup \bar{I}$ and *cost=cost+ minimum cost* % minimum cost is the cost to visit the nearest neighbor
- set $i = k$
- ***End if***

end while
cost= cost +C(k, last city) % last city is the city visited at the end
Return: tour starting with city k

Figure 2

4. COMPUTATIONAL RESULTS

After analysing the road network and the current travel schedule it was revealed that the existing six routes can be reduced to three routes. Therefore, the new travel schedule is prepared by considering only three routes. Applying the Nearest neighbourhood search algorithm to these three routes we have arrived at a best solution for each region as shown below:

Proposed Service Tours

Region 1 (Tour 1)

Colombo, Nittabuwa, Kegalle, Mawanella, Peradeniya, Kandy, Akurana, Matale, Digana, Mahiyangana, Dehiyattakandiya, Kaduruwela, Valachchenai, Batticalo, Amparai, Monaragala, Badulla, Bandarawela, Welimada, Pussellawe, Gampola, Nawalapitiya, Awssawella, Colombo.

Region 2 (Tour 2)

Colombo, Awissawella, Eheliyagoda, Rathnapura, Kalawana, Pelmadulla, Balangoda, Wellawaya, Buttala, Kataragama, Thissamaharamaya, Hambantota, Hungama, Embilipitiya, Middeniya, Tangalle, Matara, Aluthgama, Galle, Hikkaduwa, Ahungalle, Dargatown, Beruwela, Katukurunda, Colombo.

Region 3 (Tour 3)

Colombo, Katunayake, Nigambo, Kochchikade, Wennappuwa, Lunuwila, Marawila, Chilaw, Puttalam, Anuradapura, Madawachchiya, Vayuniya, Tricomalee, Habarana, Dambulla, Wariyapola, Hettipola, Kuliyapitiya, Narammala, Kurunegala, Colombo.

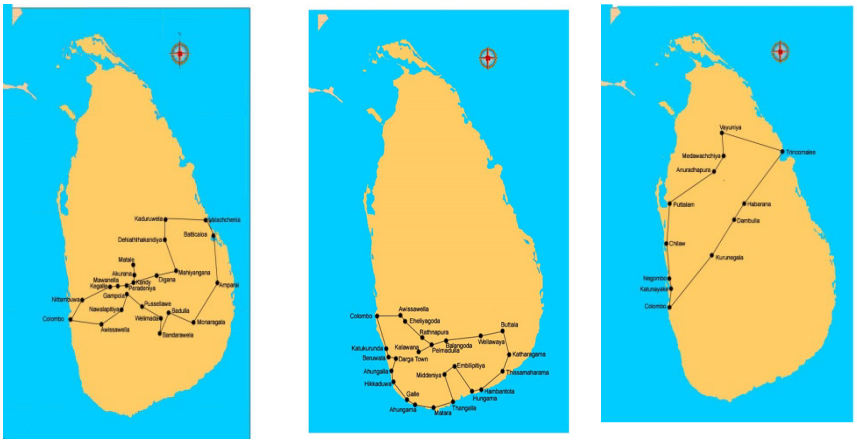

Figure 3-Proposed Service Routs

The solution given above is by far the best solution with respect to the distance. In fact there are other factors, which contribute to optimal route such as travelling time. But in this report we restrict our problem only to distance. In the discussion we compare the travelling cost for the current schedule with the proposed schedule. It is our understanding that this travelling cost can even reduce further if all the factors contribute to the traveling cost can be considered in finding the optimal route.

5. CONCLUSION AND DISCUSSION

Travelling cost for the current schedule and the proposed schedule are compared below:

Current schedule		**Proposed schedule**	
Tour No	**Travelling (km)**	**Tour No**	**Travelling (km)**
1	519	1	677
2	782	2	856
3	246	3	544
4	420		
5	790		
6	605		
Total	**3362**	**Total**	**2077**

Table 2.Comparison- Current schedule vs Proposed schedule

Table 2 exhibits the traveling distance for the current schedule and for the proposed schedule respectively.Company has incurred a total traveling distance of 3362 as per month by implementing the travel schedule prepared by the company's management division. This distance can be reduced to 2077 by implementing the proposed routs by applying TSP approach. This shows that the company can reduce the travelling distance by about 38%. This is by any mean a very significant reduction. Also, as a result the company can provide an efficient and a better service to the clients. The following two factors were not being considered in the proposed method:

1. Number of air conditioner units to be serviced in a city was not considered.

2. Average time to service air condition units was not considered.

(This may vary according to the size of the unit and the time delays due to various reasons beyond the control of the technicians.)

However, this proposed method can be further developed by taking corrective measures to minimize the effects of the above-mentioned shortcomings.

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APPENDIX

Table 4 -Distance Table for Tour No. 02

Origin	Destination	Distance (km)
Akurana	Kandy	11
Alawwa	Polgahawela	8
Ambepussa	Alawwa	9
Ambepussa	Kegalle	17
Ampara	Mahaoya	60
Anuradapura	Nikaweratiya	74
Anuradapura	Rambewa	15
Batticalo	Samanturei	50
Chenkaladi	Batticalo	13
Chenkaladi	Mahaoya	40
Habarana	Polonnaruwa	41
Kaduruwela	Manampitiya	10
Kandy	Kurunegala	42
Kandy	Peradeniya	7
Kegalle	Mawanella	13
Kegalle	Polgahawela	16
Kundasale	Digane	12
Kundasale	Kandy	5
Mahaoya	Meegaswatte	35
Mahiyangana	Digane	49
Manampitiya	Thirikkanmadu	45
Maradankadawala	Anuradapura	33
Maradankadawala	Ganewalpola	8
Mawanella	Peradeniya	19
Meegaswatte	Mahiyangana	30
Nittambuwa	Ambepussa	20
Polgahawela	Kurunegala	20
Polonnaruwa	Kaduruwela	4
Samanturei	Ampara	20
Thirikkanmadu	Valachchena	5
Valachchena	Chenkaladi	18
Vavuniya	Madewachchiya	26

Table 5 -Distance Table for Tour No. 03

Origin	Destination	Distance (km)
Ahungalla	Aluthgama	19
Aluthgama	Beruwela	5
Aluthgama	Dharga Town	2
Dharga Town	Neluwa	45
Galle	Hikkaduwa	20
Hikkaduwa	Ahungalla	22
Katukurunda	Beruwela	11
Katukurunda	Colombo	44
Neluwa	Hikkaduwa	51
Galle	Matara	35
Neluwa	Morawaka	22
Matara	Morawaka	40

Table 6 -Distance Table for Tour No. 04

Origin	Destination	Distance (km)
Alawwa	Polgahawela	8
Ambepussa	Alawwa	9
Colombo	Katunayake	25
Hettipola	Chilaw	30
Hettipola	Kuliyapitiya	15
Kochchikade	Nigambo	7
Kurunegala	Uhumiya	7
Lunuwila	Kochchikade	8
Madampe	Chilaw	12
Madampe	Kuliyapitiya	25
Marawila	Madampe	7
Marawila	Wennappuwa	7
Narammala	Alawwa	16
Nigambo	Katunayake	8
Nittambuwa	Ambepussa	20
Polgahawela	Kurunegala	20
Puththalam	Chilaw	55
Puththalam	Nikaweratiya	59
Uhumiya	Narammala	12
Wariyapola	Hettipola	20
Wariyapola	Kurunegala	15
Wariyapola	Nikaweratiya	6
Wennappuwa	Kochchikade	8
Wennappuwa	Lunuwila	3

Table 7 -Distance Table for Tour No. 05

Origin	Destination	Distance (km)
Awissawella	Eheliyagoda	14
Badulla	Bandarawela	26
Badulla	Passara	19
Badulla	Welimada	30
Balangoda	Beragala	30
Beragala	Wellawaya	23
Buttala	Kataragama	40
Buttala	Wellawaya	20
Colombo	Awissawella	58
Eheliyagoda	Rathnapura	29
Ginigathhena	Awissawella	40
Ginigathhena	NuwaraEliya	47
Hambanthota	Nonagama	15
Hungama	Middeniya	30
Hungama	Thangalla	15
Kalawana	Rathnapura	30
Kataragama	Thissamaharamaya	20
Kumblwela	Badulla	23
Kumblwela	Bandarawela	7
Madhampe	Embilipitiya	40
Madhampe	Pelmadulla	15
Mahiyangana	Badulla	57
Mathugama	Kalawana	36
Middeniya	Embilipitiya	17
Monaragala	Buttala	20
Nawalapitiya	Ginigathhena	10
Nonagama	Embilipitiya	33
Nonagama	Hungama	10
NuwaraEliya	Welimada	18
Pannegamuwa	Wellawaya	55
Pannegamuwa	Wirawila	5
Passara	Monaragala	58
Pelmadulla	Balangoda	24
Rathnapura	Pelmadulla	19
Thissamaharamaya	Pannegamuwa	4
Thissamaharamaya	Wirawila	7
Welimada	Bandarawela	24
Wellawaya	Kumblwela	21
Wirawila	Hambanthota	20

Table 8 -Distance Table for Tour No. 06

Origin	Destination	Distance (km)
Alawwa	Polgahawela	8
Ambepussa	Alawwa	9
Anuradapura	Rambewa	15
Dambulla	Galewela	15
Dambulla	Kekirawa	20
Galewela	Melsiripura	18
Habarana	Dambulla	24
Horuwpathana	Kabithigollewa	25
Horuwpathana	Mihinthale	41
Horuwpathana	Trincomalee	50
Kekirawa	Ganewalpola	7
Kurunegala	Uhumiya	7
Madewachchiya	Kabithigollewa	23
Maradankadawala	Anuradapura	33
Maradankadawala	Ganewalpola	8
Maradankadawala	Kekirawa	10
Maradankadawala	Mihinthale	38
Melsiripura	Kurunegala	25
Mihinthale	Anuradapura	10
Mihinthale	Rambewa	10
Polgahawela	Kurunegala	20
Rambewa	Madewachchiya	15
Trincomalee	Habarana	85
Uhumiya	Narammala	12
Vavuniya	Kabithigollewa	22
Vavuniya	Madewachchiya	26