SINGLE VENDOR-BUYER INTEGRATED INVENTORY SYSTEM FOR IMPERFECT PRODUCTION WITH VARIABLE LEAD TIME

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ABSTRACT

In this paper, the authors considered the single vendor single buyer integrated production inventory problem with stochastic demand for an imperfect production process. They relax the assumption that, the lead time is varying linearly with the lot size. That is, the lead time is composed of a lot size dependent run time and constant delay times such as moving, waiting and setup times. A solution procedure is mentioned for solving the proposed model and numerical examples are used to illustrate the benefit of integration. A sensitivity analysis is also performed to explore the effect of key parameter D (demand). A simple procedure is suggested to obtain an approximate solution of the proposed model. Examples are used to illustrate the model and explore the effect of important parameters on the production schedule and total expected cost.

KEY WORDS

Inventory production, Variable lead time, Economic order quantity, vendor-buyer, defective items and Inspection.

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INTRODUCTION

The single-vendor single-buyer cooperative production inventory model received a lot of interest in recent years by several researchers. The reason is this joint productive model has the skill to offer customers shorter waiting time and minimize the inventory cost. And also single-vendor single-buyer cooperative production helps to resolve the problem areas along the process enabling in trade to take vital action and further reduce price to get better the final value. The model is facing the customer is how much to order in each purchase order. On the other hand, the model is facing the seller is to make a decision the fiscal production batch size and the most economical number of shipments in which the whole order quantity to consumer will be supplied. Therefore, an integrated inventory rule is useful to decide the economic order quantity and shipment policy.

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LITERATURE REVIEW

Banerjee [1] developed a joint economic-lot-size model for a special case where a seller produces to order for a consumer on a lot-for-lot basis. By modifying Banerjee's lot-for-lot assumption, Goyal [5] extend a more universal joint economic lot size problem that provided a lower-joint total relevant cost. Hoque and Goyal [7] developed an optimal solution procedure for the single-vendor single-buyer production–inventory structure with uneven and equal sized shipments from the seller to the consumer and under the capacity restriction of the transport tackle. This supposition is questioned by Kim and Benton [10] and considered the effect of lot size on lead time and safety stock. Kim and Benton [10] established a linear association between lead time and lot size based on explanation of Karmarkar [9]. They included this lead time lot size relation into the classical probabilistic continuous review (Q, r) model.

In many literature, the single vendor-buyer integrated inventory model, that is frequently assumed that the shortages are not allowed and demand is deterministic. Ben-Daya and Hariga [2] extended this by taking the annual customer demand to be stochastic with variable lead time and thereby allowing shortages. Vandana, B.K.Sharma [13] allowing shortages and considered an economic ordered quantity model for retailers partial permissible delay in payment linked to order quantity. Most of studies deal with the perfect production process. Although production process is often considered to be ideal, but in reality, it is not possible that a manufacture progression is 100% defect-free. Huang [8] considered the imperfect production process. In that the defective percentage is considered among each lot size. In Vandana, B.K.Sharma., [14] an EPQ model with non-instantaneous deteriorating items are considered. In an integrated model, if the production is unsatisfactory then the seller who has to give warranty cost for faulty items, it is favorable to him, in particular, and to the supply chain as a whole, to spend amount (invest) in reducing the number of faulty items produced as in [12].

And finally Dey.O and Giri.B.C [4] developed the optimal vendor investment for reducing defect rate in vendor-buyer integrated system for imperfect production process. In that the production is imperfect in stochastic demand. This paper deals the similar case of Dey.O and Giri.B.C [4] but the lead time is linear in Q and without any investment. Therefore we considered the stochastic demand, an imperfect production process and finite screening period in this paper.

MODEL

In this paper, we assume that the buyer is using a continuous review (Q, r) inventory policy. The batch quantity and reorder level are often determined under the assumption of a constant lead time. However, from a real life point of view, lead time should be considered as a function of the production batch size. In this section, the classical (Q, r) continuous review inventory policy with deterministic variable lead time is considered for the buyer. In particular, we assume that the lead time is proportional to the lot size produced by the vendor in addition to a fixed delay due to transportation, nonproductive time, etc., that is $L(Q) = pQ + b$.

To extend the proposed model, the following notations are used:

D	Expected demand rate (units/unit time),

P	Production rate $(1/p)$,
A	Buyer's ordering cost,
F	Fixed transportation cost per shipment,
h_v	Vendor's holding cost per unit,
h_{b1}	Buyer's holding cost for defective item per unit per time,
h_{b2}	Buyer's holding cost for non-defective item per unit per time,
s	Buyer's screening cost per unit per time,
x	Buyer's screening rate per unit,
y	Percentage of defective items in each batch size Q,
π	Buyer's shortage cost per item per time,
w	Vendor's unit warranty cost for defective items,
$L = L(Q)$	Lead time, is directly proportional to the order quantity, *(i.e.)* $L(Q) = pQ + b$ where b is fixed delay time due to transportation, production time of the products,
$S(r, L(Q))$	Expected shortage quantity per shipment,
Q	Batch size (Decision variable),
n	Number of shipments (Decision variable),
r	Reorder level (Decision variable)

We develop the model with the following assumptions

- A single-vendor single-buyer integrated inventory model for a single item.
- Demand per unit time is normally distributed with mean D and standard deviation σ.
- The buyer places an order for nQ items. The vendor produces these items and gives to the buyer as n equal shipments.
- The buyer follows the classical (Q, r) continuous review with variable lead time $L(Q) = pQ + b$.
- The demand in lead time is normally distributed with mean $DL(Q)$ and standard deviation $\sigma\sqrt{L(Q)}$.

- The reorder level r =Expected demand during lead time + safety stock $= DL(Q) + k\sigma\sqrt{L(Q)}$ where, $k\sigma\sqrt{L(Q)}$ is safety stock and k is the safety factor.
- Shortages are allowed and completely backlogged.
- y is a percentage of defective items in each batch size Q.
- The non-defective production rate is greater than the demand rate. *(i.e.)* $P(1-y) > D.$
- Unit screening rate is greater than the demand rate. *(i.e.)* $x > D.$
- The seller offers the warranty cost for the defective items to the consumer.

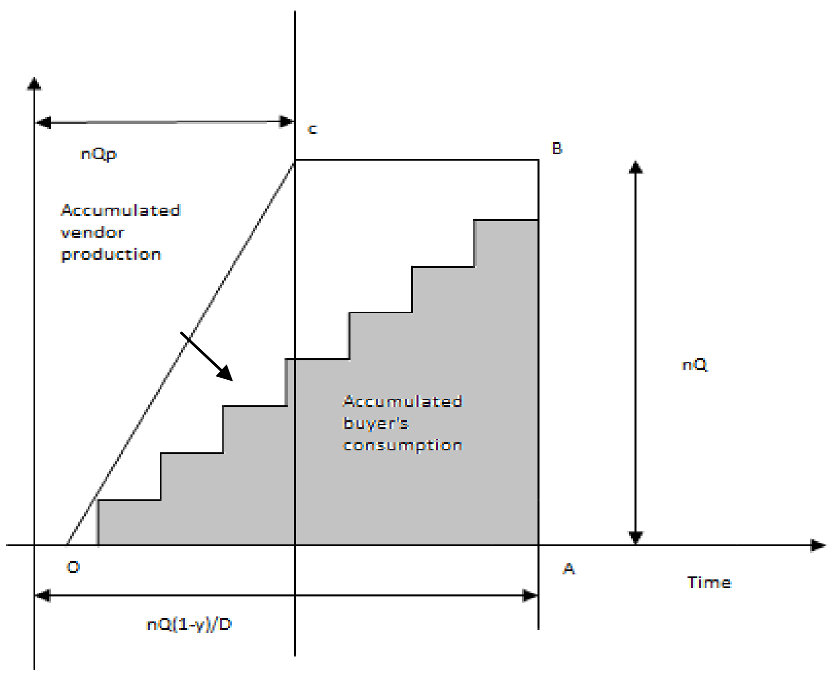


Fig : 1 Vendor buyer holding area.

The buyer places an order of size Q for non defective items to the vendor. In order to reduce the production cost, the vendor produces nQ items and transfers Q batches of nQ items each at regular intervals of $Q(1-y)/D$ units of time on average. The length of each ordering cycle is therefore $Q(1-y)/D$ and the length of the complete production cycle is $nQ(1-y)/D$.

Inventory profile for the vendor buyer is depicted in Fig 1

The problem is to find the number of shipments *n*, the shipment size Q, and the reorder point r, that minimize the total cost.

We assume the demand is normally distributed with mean D and standard deviation σ. Therefore the demand during lead time also normally distributed with mean $DL(Q)$ and standard deviation $\sigma\sqrt{L(Q)}$.

Expected annual cost for the buyer is

$$ETCB(Q,k,n,L(Q)) = \frac{D(A+nF)}{nQ(1-y)} + h_{b1}\left(Qy - \frac{DQy}{2x(1-y)}\right) + h_{b2}\left(k\sigma\sqrt{L(Q)} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)}\right)$$

$$+\pi\frac{D}{Q(1-y)}S(r,L(Q)) + s\frac{D}{1-y} \qquad (1)$$

where $S(r,L(Q)) = \int_r^\infty (x-r)f\left(x, DL(Q), \sigma\sqrt{L(Q)}\right)dx$, x is the demand during lead time with probability density function $f(x)$.

$$S(r,L(Q)) = \int_r^\infty (x-r)f(x, DL(Q), \sigma\sqrt{L(Q)}\,dx$$

$$= \sigma\sqrt{L(Q)}\int_r^\infty \left(\frac{x-DL(Q)}{\sigma\sqrt{L(Q)}} - \frac{r-DL(Q)}{\sigma\sqrt{L(Q)}}\right)\frac{1}{\sigma\sqrt{L(Q)}\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-DL(Q)}{\sigma\sqrt{L(Q)}}\right)^2}dx$$

$$= \sigma\sqrt{L(Q)}\,\psi(k) \qquad (2)$$

where $\psi(k) = \int_k^\infty (z-k)\phi(z)dz$, $\phi(z)$ is a standard normal probability density function, $z = \frac{x-DL(Q)}{\sigma\sqrt{L(Q)}}$, and $k = \frac{r-DL(Q)}{\sigma\sqrt{L(Q)}}$.

$\psi(k)$ may also written as

$$\psi(k) = \int_k^\infty (z-k)\phi(z)dz,$$

$$= \frac{1}{\sqrt{2\pi}} \int_k^{\infty} z e^{-\frac{z^2}{2}} dz - k\left(1 - \int_0^k \phi(z) dz\right)$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{k^2}{2}} - k(1 - F(k))$$

$$= \phi(k) - k\overline{F}(k) \qquad (3)$$

where $\overline{F}(k)$ is the complement of the cumulative distribution function. *(i.e,)* $\overline{F}(k) = 1 - F(k)$

Now the expected annual cost for vendor is

$$ETCV(Q,n) = \frac{BD}{nQ(1-y)} + h_v \frac{Q}{2}\left[n\left(1 - \frac{Dp}{1-y}\right) - 1 + \frac{2Dp}{1-y}\right] + w\frac{Dy}{1-y}.$$

Therefore the integrated annual expected cost is

$$ETC(Q,k,n,L(Q)) = \frac{D(A+B+nF)}{nQ(1-y)} + h_{b1}\left(Qy - \frac{DQy}{2x(1-y)}\right) + h_{b2}\left(k\sigma\sqrt{L(Q)} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)}\right)$$

$$+\pi\frac{D\sigma\psi(k)\sqrt{L(Q)}}{Q(1-y)} + s\frac{D}{1-y} + \frac{BD}{nQ(1-y)} + h_v\frac{Q}{2}\left[n\left(1 - \frac{Dp}{1-y}\right) - 1 + \frac{2Dp}{1-y}\right] + w\frac{Dy}{1-y} \qquad (4)$$

Solution procedure

The total cost function *ETC* is convex in *n.* Since it is easy to see that

$$\frac{\partial}{\partial n} ETC = -\frac{D(A+B)}{n^2 Q(1-y)} + h_v \frac{Q}{2}\left(1 - \frac{Dp}{1-y}\right)$$

$$\frac{\partial^2}{\partial n^2} ETC = \frac{D(A+B)}{n^3 Q(1-y)} > 0 \qquad \forall n \geq 1. \qquad (5)$$

For fixed value of *n, ETC* can also be shown to be convex in *k.*

$$\frac{\partial}{\partial k} ETC = h_{b2}\sigma\sqrt{L(Q)} + \frac{\pi D\sigma\sqrt{L(Q)}}{Q(1-y)} \frac{\partial}{\partial k}(\psi(k))$$

$$= h_{b2}\sigma\sqrt{L(Q)} - \frac{\pi D\sigma\sqrt{L(Q)}}{Q(1-y)} \overline{F}(k) \qquad (6)$$

$$\frac{\partial^2}{\partial k^2} ETC = -\frac{\pi D\sigma\sqrt{L(Q)}}{Q(1-y)}\left(1-\frac{1}{\sqrt{2\pi}}\int_0^k e^{-\frac{z^2}{2}}dz\right) \tag{7}$$

$$= \frac{\pi D\sigma\sqrt{L(Q)}}{Q(1-y)}\phi(z) > 0 \qquad\qquad \forall k \geq 0$$

Put

$$\frac{\partial}{\partial k} ETC = 0$$

We get

$$\overline{F}(k) = h_{b2}\frac{Q(1-y)}{\pi D} \tag{8}$$

Next, equating zero the first derivative of ETC with respect to Q.

$$\frac{\partial}{\partial Q} ETC = \frac{-G(n)D}{(1-y)Q^2} + h_{b1}\left(y - \frac{Dy}{2x(1-y)}\right) + \frac{h_v}{2}\left[n\left(1-\frac{Dp}{1-y}\right) - 1 + \frac{2Dp}{1-y}\right] + h_{b2}\left(\frac{k\sigma p}{2\sqrt{pQ+b}} + \frac{1-y}{2} + \frac{Dy}{2x(1-y)}\right)$$

$$+\frac{\pi D\sigma\psi(k)}{1-y}\left[\frac{\frac{pQ}{2\sqrt{pQ+b}} - \sqrt{pQ+b}}{Q^2}\right] = 0$$

$$\frac{-G(n)D}{Q^2} + H(n) + h_{b2}\frac{k\sigma p}{2\sqrt{L(Q)}}(1-y) + \frac{\pi D\sigma\psi(k)(pQ - 2L(Q))}{2Q^2\sqrt{L(Q)}} = 0$$

where,

$$H(n,y) = h_{b1}\left(y(1-y) - \frac{Dy}{2x}\right) + \frac{h_v}{2}(-(n-2)Dp + (n-1)(1-y)) + h_{b2}\left(\frac{(1-y)^2}{2} + \frac{Dy}{2x}\right)$$

$$G(n) = (A + B + nF)/n$$

$$H(n,y) + \frac{h_{b2}\sigma p(1-y)}{2\sqrt{L(Q)}}\left(k + \frac{\psi(k)}{\overline{F}(k)}\right) = \frac{D}{Q^2}\left(\pi\sigma\psi(k)\sqrt{L(Q)} + G(n)\right)$$

$$Q = \sqrt{\frac{D\pi\sigma\psi(k)\sqrt{L(Q)} + DG(n)}{H(n,y) + \frac{h_{b2}p\sigma(1-y)}{2\sqrt{L(Q)}}\left(k + \frac{\psi(k)}{\overline{F}(k)}\right)}} \tag{9}$$

ALGORITHM

Step 1. Set $ETC^{*} = \infty$ and $n = 1$.

Step 2. Compute $Q = \left[\sqrt{\frac{DG(n)}{H(n)}} \right]$ where $[x]$ the nearest integer of x .

Step 3. Find k from (8) and Compute $\psi(k)$ using (3)

Step 4. Compute Q' using (9) and, Set Q' by $[Q']$

Step 5. If $|Q' - Q| = 0$, compute $ETC(Q, n)$ and go to Step 6. If $|Q' - Q| > 0$, set Q by Q' and go to step 3.

Step 6. If $ETC^{*} \geq ETC(Q, n)$, then set $ETC*$ by $ETC(Q, n)$, $Q*$ by Q, $r*$ by r, and n by $n+1$ and go to step 2. Otherwise set $n*$ by $n-1$ and stop.

NUMERICAL EXAMPLE

For numerical studies, we consider the following data set:

D =1000; P =3200; A =50; F =35; h_v =4; h_{b1} =6; h_{b2} =10; s =0.25; x =2152; w =20; y =0.33;
π =100; b =0.01.

From table 1 we have the optimum number of shipments $n^{*} = 7$.
The optimum order quantity $Q^{*} = 106$.

The optimum reorder level $r^{*} = 45$.

The following table shows that optimal number of shipments and optimal batch quantity.

n	*Q*	*r*	*ETC*
1	327	115	14,073
2	223	83	13,298
3	176	68	13,024
4	149	59	12,881
5	130	53	12,815
6	116	49	12,784
7	**106**	**45**	**12,776**
8	98	43	12,782
9	91	40	12,794

Table:1

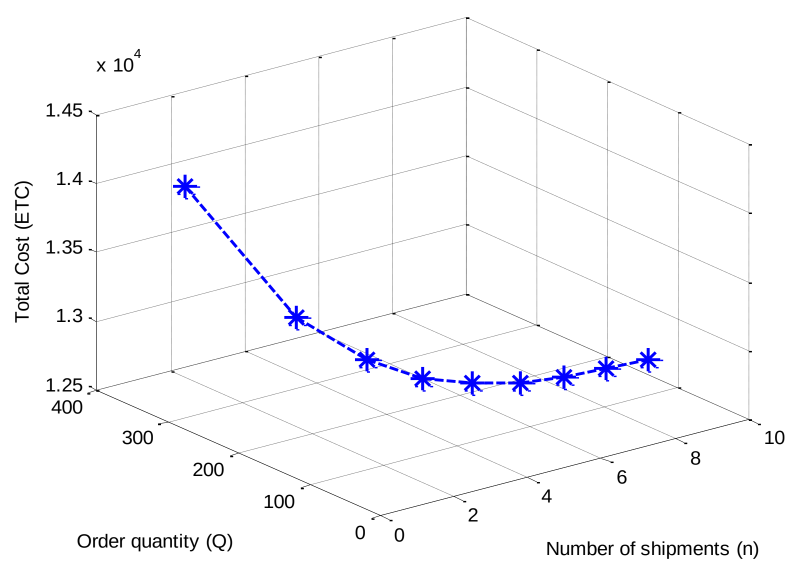
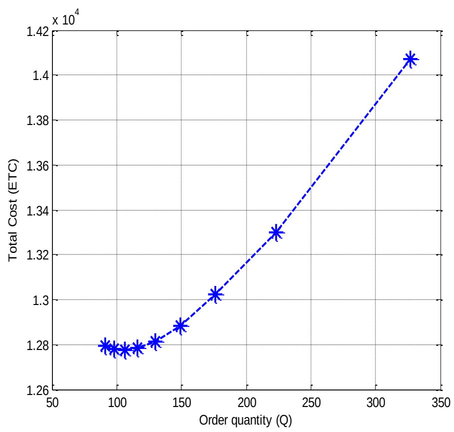


Fig 2 Graphical representation of optimal solutions for ETC with respect to Q and n.



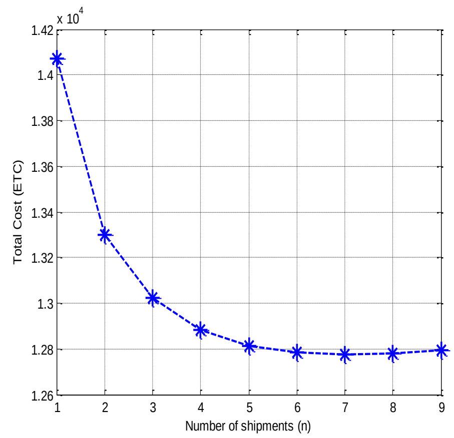


Fig 3 Fig 4

Graphical representation ETC with respect to Q Graphical representation ETC with respect to n.

SENSITIVE ANALYSIS

The sensitive analysis of demand D for D+50%, D+25%, D-50%, D+25% is performed Tables 2-5 in order to various demand affect the optimal solutions of the proposed model. Also the effect of demands depicted in Fig 5-8.

Effect of *D* when it is 50% extra

Demand	n	Q	r	ETC	k
	1	382	198	20,281	2.13
	2	269	145	19,147	2.26
	3	219	121	18,702	2.34
	4	189	107	18,471	2.39
1,500	5	168	97	18,334	2.43
(i.e.,	6	153	90	18,250	2.48
D+50%)	7	142	84	18,201	2.5
	8	132	80	18,170	2.52
	9	124	76	18,153	2.54
	10	**118**	**73**	**18,148**	**2.56**
	11	112	70	18,148	2.57
	12	107	68	18,153	2.6
	13	103	66	18,165	2.61

Table 2

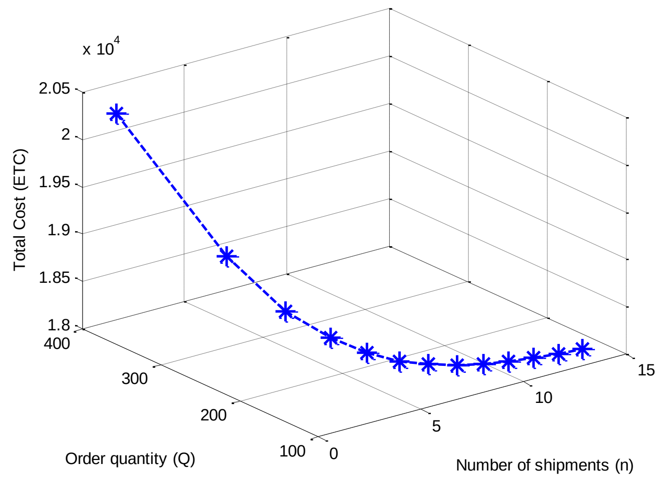


Fig 5 Graphical representation of optimal solutions for ETC with respect to Q and n when D=1500

Effect of *D* when it is 25% extra

Demand	*n*	*Q*	*r*	*ETC*	*k*
	1	357	156	17,191	2.07
	2	247	112	16,235	2.22
	3	198	93	15,874	2.3
1,250	4	169	81	15,675	2.36
(i.e.,	5	149	74	15,597	2.41
D+25%)	6	135	68	15,545	2.44
	7	124	64	15,518	2.48
	8	115	60	15,507	2.5
	9	**107**	**57**	**15,506**	**2.53**
	10	101	55	15,515	2.55
	11	96	53	15,530	2.57

Table 3

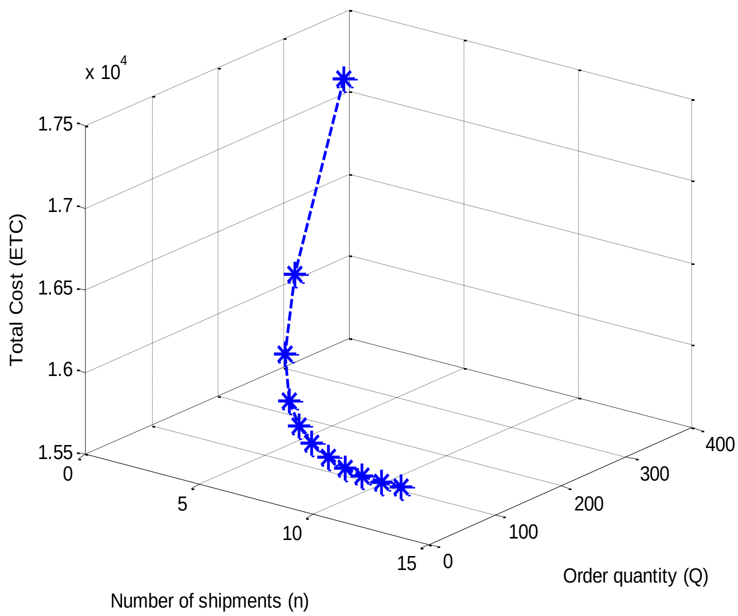


Fig 6 Graphical representation of optimal solutions for ETC with respect to Q and n when D=1250

Effect of *D* when it is 25% less

Demand	*n*	*Q*	*r*	*ETC*	*k*
750 (i.e., *D*-25%)	1	291	79	10,917	1.94
	2	194	56	10,309	2.11
	3	152	46	10,101	2.21
	4	126	40	10,009	2.28
	5	110	36	9,972	2.33
	6	**98**	**33**	**9,962**	**2.38**
	7	88	31	9,967	2.42
	8	81	29	9,981	2.44

Table 4

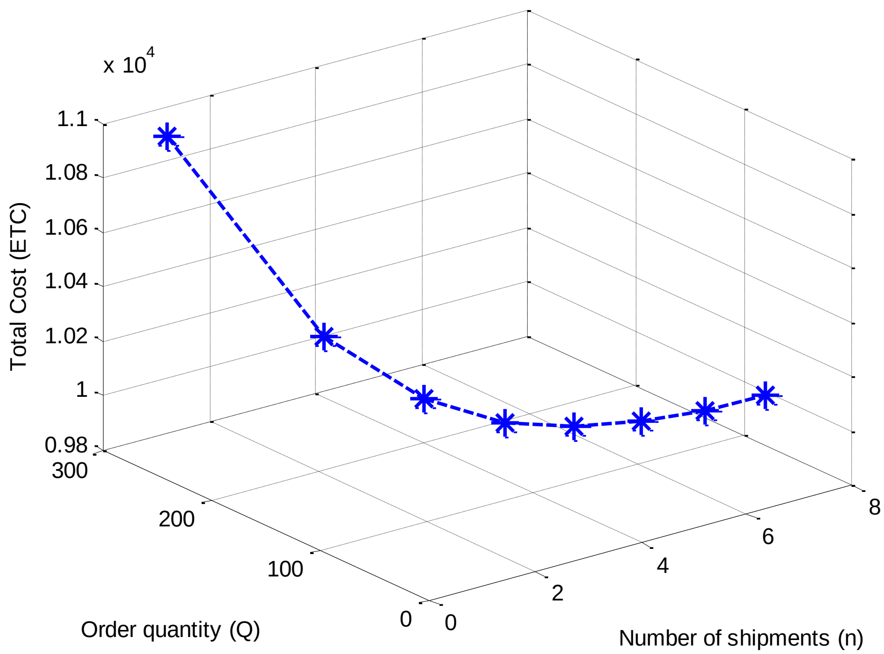


Fig 7 Graphical representation of optimal solutions for ETC with respect to Q and n when D=750

Effect of *D* when it is 50% less

Demand	*n*	*Q*	*r*	*ETC*	*k*
500 (i.e., *D*-50%)	1	245	46	7,696	1.84
	2	160	32	7,255	2.03
	3	123	27	7,113	2.13
	4	102	23	7,059	2.21
	5	**87**	**21**	**7,041**	**2.27**
	6	77	19	7,043	2.31
	7	70	18	7,059	2.35

Table 5

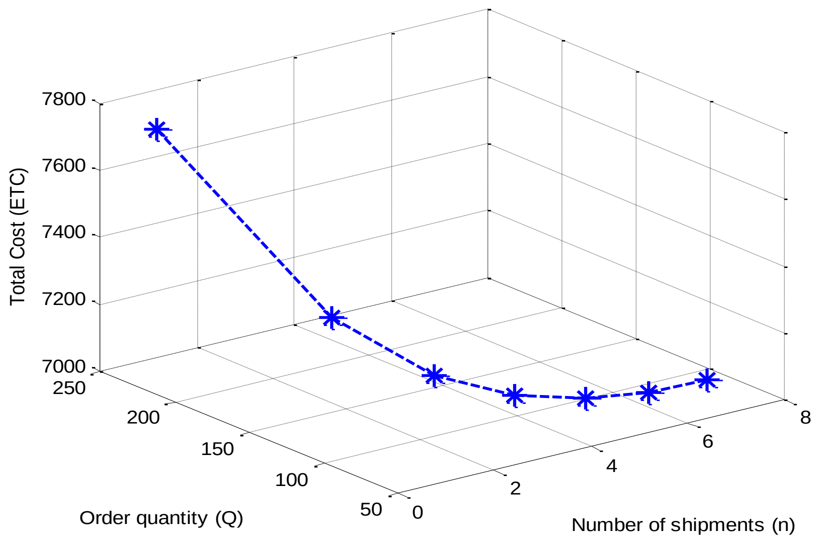


Fig 8 Graphical representation of optimal solutions for ETC with respect to Q and n when D=500

CONCLUSION

In this paper, we considered the single vendor single buyer integrated production inventory problem. Here, we assume that demand is stochastic; the lead time is variable and depends on batch size and other delays, such as transportation time. A simple procedure is suggested to obtain an approximate solution of the proposed model. Examples are used to illustrate the proposed model.

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