INVENTORY MODEL FOR DETERIORATING ITEMS WITH QUADRATIC DEMAND, PARTIAL BACKLOGGING AND PARTIAL TRADE CREDIT

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SUMMARY

This article develops inventory model for deteriorating items with quadratic demand. Shortages are allowed. We study the partial delay in payments. The objective is to find the optimal cycle times that minimize the total cost. In this study first, we developed arithmetical model and procedure of finding the most favorable solution is developed. Also the solution process is developed in order to minimize the total cost. The total costs are calculated by various principles. A solution process is developed to find the optimal solution and statistical examples are presented to demonstrate the result of the proposal model. Compassion study of the most favorable solution with respect to the parameters of the system is carried out and recommendations for further research are provided.

KEYWORDS:

1. PROLOGUE

Inventory management is the administration of supply, storage and convenience of items in order to ensure a sufficient supply without excessive oversupply. It can also be referred as domestic control - an accounting method or system designed to promote efficiency or assure the implementation of a policy or safeguard assets or avoid fraud and error etc. The main target of inventory organization deals with minimization of the inventory carrying costs for which it is necessary to determine the optimal stock and optimal time of refill for the future. Researchers had developed inventory models by assuming quadratic demand for items like fruits, vegetables, materials, iron goods, clothes, etc., since the demand rate is flexible due to buyers and sellers.

Due to the customer's choice and the arrival of new products may decline the demand rate. Attempting the obvious fact of time-varying demand pattern in the deteriorating inventory models yields very much real time application. Inventory has two types based on time – varying demands namely discrete and continuous time. The continuous – time inventory models were developed the demand pattern by linearly increasing or decreasing.

throughout their life time owed to delay, injure, spoilage, consequence of the other reasons. Yet to be paid to this fact, calculating and maintaining the inventory of deteriorating items becomes a tough problem for the assessments.

If the suppliers offered the customers the permissible delay in payments if the order quantity is less than the predetermined quantity and such system is termed as partial trade credit. The partial trade credit is one of the central features in supply chain management. In real life situation this partial trade credit is more matched to the retailer.

The willingness of the customer to wait for shortages with the length of the waiting time. Here shortages are as backlogging; the impact on the cost from the delay of the products can be balanced out.

2. LITERATURE REVIEW

Abubakar Musa et.al [1] described several deteriorating items that do not start deteriorating immediately they were held in stock. And also reduction of these items as soon as they are stored would depend on demand, and when deterioration begins, depend on both demand and deterioration. Aggarwal [2] discussed an inventory model for a system with charge is time dependent. Amutha and Chandrasekaran [3] developed model with constant deterioration rate and time dependent holding cost. R. Begum et al. [5] developed the model with time dependent on quadratic demand rate.

K.Annadurai and R.Uthayakumar [4] developed the partial trade credit financing in a supply chain by EOQ – based model for decomposing items together with shortages and also they discussed about partial permissible delay in payments even if the order quantity is less than the pre-determined quantity.

Chaintanya Kumar Tripathy et at.[6] another model with quadratic demand when deterioration rate depends on two parameters and also discussed to be the subject to time depend Weibull deterioration, they include the shortages and replenishment is infinite. Deb et at. [7] studied an inventory for assuming the shortages and found how the profit was. Ghare [8] studied about the demand which is decaying exponential. Goyal and Giri [9] methodically review the recent trend in deteriorating inventory models. Inventory model for the most favorable pricing and ordering policies for the vendor with trade credit was formulated by Hwang and Shinn [11] they have suggested the condition that the supplier offers trade credit to the purchaser if the order quantity is greater than or equal to a pre determined quantity.

Mitra et al. [12] an inventory model for constant demand and the deteriorating items and also they included the variable holding cost considering the shortage costs. Mohan andVenkateswarlu [13] developed an EOQ model for deteriorating products by considering rate of demanded is quadratic with respect to time. Narayan Singh et al. developed a three echelon supply chain inventory model with permissible delay in payments. Soni H et al. [17] analyzed by agree to the shortages and the partial trade credit financing in a supply chain by EOQ-based model for decaying items. Liao [14] developed an EOQ model with non instantaneous acknowledgment and exponential deteriorating item under two-level trade credit.

Hardik N. Soni [10] recognized an inventory model for deteriorating items under stock- needy demand and two-level trade credit and obtained the optimal replenishment policy. Sarkar et.al [15] described with a most favorable inventory refill policy for a deteriorating item considering time-quadratic demand and time-dependent partial backlogging which depends on the length of the waiting time for the next replenishment over a finite time horizon and variable replenishment cycle. There is no inventory model, by EOQ – based model for deteriorating items with partial trade credit under quadratic demand with shortages. This paper is organized as follows. We described the required EOQ – based inventory model with partial trade credit and also formulated several derivations. In this paper we consider the demand as quadratic form. By this form we gauge the total costs. To highlights and visualize the results, examples are given. Then sensitivity psychotherapy is carried out and some executive insights are obtained.

Vandana and Sharma [18] developed an economic production quantity inventory model for non-instantaneous deteriorating item sunder trade credit policy. Vandana and Sharma [19] prepared an inventory model for non- instantaneous deteriorating items with quadratic demand rate and shortages under trade credit policy.

In this article, we suppose that the inventory model for deteriorating items with quadratic demand partial backlogging and partial trade credit. The inventory is assumed to deteriorate at a stable rate and shortage is allowed. In this study, first we developed a mathematical model and procedure for finding the optimal solutions and our approach is illustrated through a numerical example. We provide the entries and hypothesis for the proposed model in section 3. Arithmetic model is established in section 4. A section 5 describes the solution process. Statistical examples are provided to illustrate the proposed model in section 6.The compassion study of the most favorable solution with respect to parameters of the system is carried out in Section 7. Finally, we present the termination and future research in section 8.

3. ENTRIES AND HYPOTHESIS

The mathematical model in this paper is developed on the basis of the following entries and hypothesis

3.1. Entries

To expand the proposed model, we accept the following entries

A	Setup costs or ordering costs per order
D	Annual demand (i.e., $a + bt + ct^2$; $a \neq 0$; $b \neq 0$; $c \geq 0$)
Q	order quantity
T	Inventory cycle length
t_1	Length of time (with no shortages)
b_1	Backlogging parameter
a_1	The deterioration rate
w	The measure at which the full delay payment acceptable per order
d	the little bit of the delay payments acceptable by the provider per order

C_0	Unit purchase cost of the item
C_1	Shortage cost for backlogged item
C_2	Unit cost of lost sales
M	delay period offered by the seller for settling the accounts to the vendor
I_e	Annual interest which can be earned
I_p	Annual interest payable
P	Defective rate in an order lot
I_{max}	$I_1(0) = I_{max}$
$TC_1(t_1, T_1)$	if $M \leq t_1$
$TC_2(t_1, T_2)$	if $t_w \leq t_1 \leq M$
$TC_3(t_1, T_3)$	if $0 < t_1 < t_w$

3.2. Hypothesis

The following hypotheses are used in the development of the model:

1. The rate of demand is assumed to be in quadratic form.
2. Shortages are allowed and backlogged
3. The longer waiting time is ,the smaller the backlogging rate will be .Let

   $B(t) = \dfrac{1}{1 + b_1(T - t)}$ where t is the waiting time up to the next refill with $0 \leq b_1 \leq 1$

4. The sellers may offer the partial delay in payments even if the order quantity is less than the pre-determined quantity.
5. The total annual cost $TC(t_1, T)$ = Ordering cost + Holding cost + Opportunity cost + Shortages cost due to backlogging + Deterioration cost +Cost of interest charges for the unsold items after the permissible delay in payments – Cost of interest earned from the last sales during the permissible period.

4. ARITHMETICAL MODEL

An inventory model, a arithmetical model to find the most favorable refill in the cycle time and to minimize the total annual cost is developed.
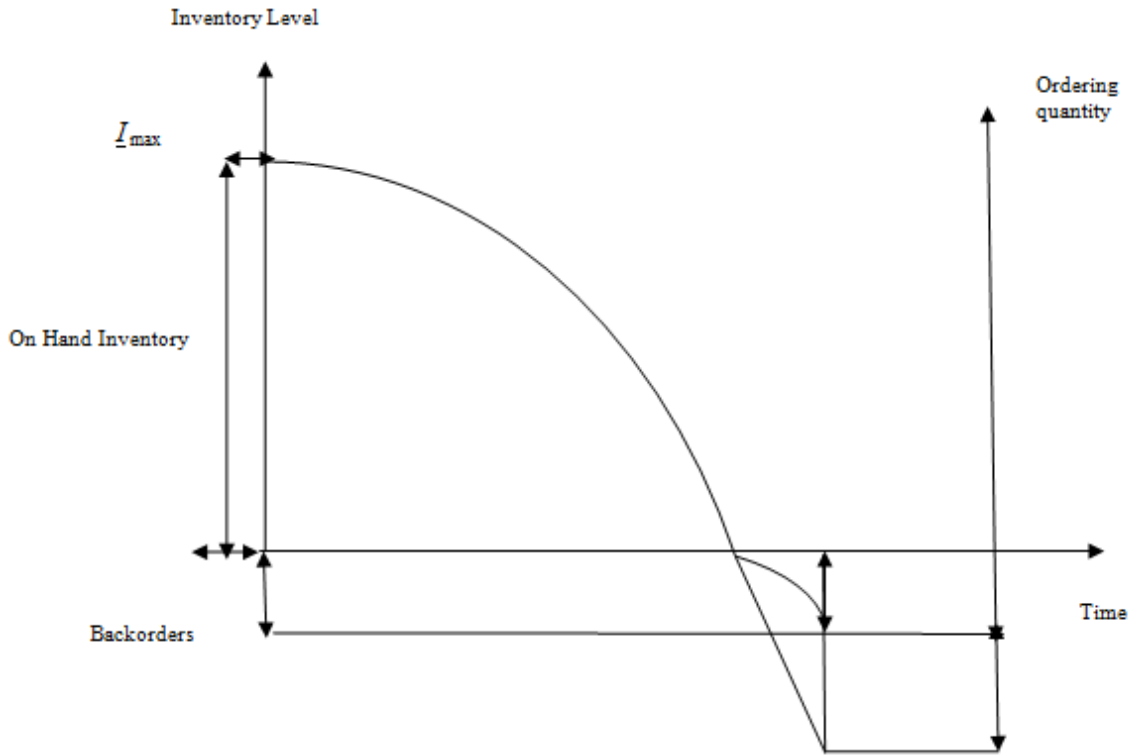
Figure 1 Graphical image of inventory system with shortages

Let $I_1(t)$ be the inventory at time $t_1\,(0\leq t\leq t_1)$.Hence the differential equation representing the inventory status is given by,

$$\frac{dI_1(t)}{dt}+a_1 I_1(t)=-(a+bt+ct^2) \tag{1}$$

With frontier condition $I_1(0)=I_{max}$

The solution of equation (1) is given by

$$I_1(t)= -\frac{bt}{a_1}-\frac{ct^2}{a_1}+\frac{2ct}{a_1^2}-\frac{a}{a_1}+\frac{b}{a_1^2}-\frac{2c}{a_1^3} \quad \text{if } 0\leq t\leq t_1 \tag{2}$$

So, the maximum inventory altitude for each cycle can be obtained as

$$I_{max} = I_1(0) = \left(\frac{1}{a_1} \left[\frac{b}{a_1} - a - \frac{2c}{a_1^2} \right] \right)$$

$$I_1(t) = \left(\frac{2ct}{a_1^2} + \frac{b}{a_1^2} - \frac{bt}{a_1} - \frac{ct^2}{a_1} - \frac{a}{a_1} - \frac{2c}{a_1^3} \right) \tag{3}$$

Shortage interval the demand at t is partially backlogged at the fraction $B(t) = \dfrac{1}{1 + b_1(T - t)}$.

Thus the differential equation leading the amount of demand backlogged is as below.

$$\frac{dI_2(t)}{dt} = \frac{-(a + bt + ct^2)}{1 + b_1(T - t)}; \ t_1 \le t \le T \tag{4}$$

With frontier condition $I_2(t_1) = 0$

The solution of equation (4) is given by

$$I_2(t) = a(t_1 - t) + ab_1 T(t_1 - t) + \frac{ab_1}{2}(t_1^2 - t^2)$$

$$+ \frac{b}{2}(t_1^2 - t^2) - \frac{bb_1 T}{2}(t_1^2 - t^2) + \frac{bb_1}{2}(t_1^3 - t^3) + \frac{c}{3}(t_1^3 - t^3) - \frac{cb_1 T}{3}(t_1^3 - t^3)$$

if $t_1 \le t \le T$ $\tag{5}$

Thus we have,

$$I(t) = \begin{cases} -\left(\dfrac{bt}{a_1} + \dfrac{ct^2}{a_1} - \dfrac{2ct}{a_1^2} + \dfrac{a}{a_1} - \dfrac{b}{a_1^2} + \dfrac{2c}{a_1^3} \right) & \text{if } 0 \le t \le t_1 \\[2mm] a(t_1 - t) + ab_1 T(t_1 - t) + \dfrac{ab_1}{2}(t_1^2 - t^2) \\[2mm] + \dfrac{b}{2}(t_1^2 - t^2) - \dfrac{bb_1 T}{2}(t_1^2 - t^2) + \dfrac{bb_1}{2}(t_1^3 - t^3) \\[2mm] + \dfrac{c}{3}(t_1^3 - t^3) - \dfrac{cb_1 T}{3}(t_1^3 - t^3) & \text{if } t_1 \le t \le T \end{cases} \tag{6}$$

By letting t= T in equation (5), we can obtain the maximum amount of demand backlogged per cycle as

$$\Lambda = -I_2(T) = -\begin{pmatrix} a(t_1 - T) + ab_1 T(t_1 - T) + \dfrac{ab_1}{2}(t_1^2 - T^2) \\[2mm] + \dfrac{b}{2}(t_1^2 - T^2) - \dfrac{bb_1 T}{2}(t_1^2 - T^2) + \dfrac{bb_1}{2}(t_1^3 - T^3) \\[2mm] + \dfrac{c}{3}(t_1^3 - T^3) - \dfrac{cb_1 T}{3}(t_1^3 - T^3) \end{pmatrix} \tag{7}$$

Hence the order quantity per cycle is given by

$$Q = I_{max} + \Lambda = \left(\dfrac{1}{a_1}\left[\dfrac{b}{a_1} - a - \dfrac{2c}{a_1^2} \right] \right) - \begin{pmatrix} a(t_1 - T) + ab_1 T(t_1 - T) + \dfrac{ab_1}{2}(t_1^2 - T^2) \\[2mm] + \dfrac{b}{2}(t_1^2 - T^2) - \dfrac{bb_1 T}{2}(t_1^2 - T^2) + \dfrac{bb_1}{2}(t_1^3 - T^3) \\[2mm] + \dfrac{c}{3}(t_1^3 - T^3) - \dfrac{cb_1 T}{3}(t_1^3 - T^3) \end{pmatrix} \tag{8}$$

We can obtain the time interval that w units are exhausted to zero due to the both demand and the deterioration as $t_w = \dfrac{1}{a_1}\log\left(\dfrac{a_1 w}{a + bt + ct^2} + 1 \right).$ $\tag{I}$

If the fully delayed payment is permitted then the inequality $Q \geq w$ iff $t_1 \geq t_w$. If $Q < w$ then the vendor must take a loan with the interest charged I_p. To pay the supplier the partial payment $(1-d)C_0 Q$ when the order is filled at time zero. From the constant pays revenue the retailer will be able to pay off the loan $(1-d)C_0 Q$ at time $\dfrac{(1-d)C_0}{P}\left(\dfrac{e^{a_1 t_1} - 1}{a_1} \right).$ $\tag{II}$

Now, the total annual cost of the inventory model for the vendor is computed using the following various components.

1. Ordering cost = $\dfrac{A}{T}$

2. Holding cost = $-\left[\dfrac{h}{Ta_1}\left(at_1 + \dfrac{bt_1^2}{2} + \dfrac{ct_1^3}{3} - \dfrac{bt_1}{a_1} - \dfrac{ct_1^2}{a_1} + \dfrac{2ct_1}{a_1^2} \right) \right]$

3. Deterioration cost = $-\left[\dfrac{C_0}{T}\left(at_1 + \dfrac{bt_1^2}{2} + \dfrac{ct_1^3}{3} - \dfrac{bt_1}{a_1} - \dfrac{ct_1^2}{a_1} + \dfrac{2ct_1}{a_1^2} \right) \right]$

4. Shortage cost due to backlogging $= -\dfrac{C_1}{T}\left[a\left(\left(\dfrac{-aT^2}{2}\right) + \dfrac{ab_1T^3}{2} - \dfrac{ab_1T^3}{6} - \dfrac{bT^3}{6} + \dfrac{bb_1T^4}{6} \right) \right.$

$$\left. -\left(\dfrac{-at_1^2}{2} + \dfrac{ab_1Tt_1^2}{2} - \dfrac{ab_1t_1^3}{6} - \dfrac{bt_1^3}{6} + \dfrac{bt_1^3b_1}{6} \right) \right]$$

5. Opportunity cost due to lost sales $= \dfrac{C_2}{T}\left[a\left(\dfrac{b_1T^2}{2} - b_1Tt_1 - \dfrac{b_1t_1^2}{2} \right) \right.$

$$\left. +b\left(\dfrac{-5b_1T^3}{6} - \dfrac{b_1Tt_1^2}{2} - \dfrac{b_1t_1^2}{3} \right) - \dfrac{cb_1t_1^3T}{3} \right]$$

Regarding the interest charged and interest earned, based on the values M, t_w, t_0 we have the following three possible.

$$(i)\, t_0 > M \geq t_w \ , \ (ii)\ t_0 \geq t_w > M \ \text{and (iii) } t_w > t_0 > M$$

Case (i)

In this case there are three sub - cases.

Sub-case (i): M $\leq t_1$

Imbursement of the goods is matured and the vendor starts paying the capital opportunity cost for the items in store with rate I_p. Hence, the annual interest payable is

$$= \dfrac{I_p C_0}{T a_1} \int\limits_M^{t_1} (a + bt + ct^2)(e^{a_1(t_1-t)} - 1)dt$$

$$= -\dfrac{I_p C_0}{T a_1^2}\left[\left(a + bt_1 + \dfrac{b}{a_1} + ct_1^2 + \dfrac{2ct_1}{a_1} + \dfrac{2c}{a_1^2} \right) \right.$$

$$\left. -\left(ae^{a_1(t_1-M)} + bMe^{a_1(t_1-t)} + be^{a_1(t_1-t)} + cM^2e^{a_1(t_1-t)} + \dfrac{2cMe^{a_1(t_1-t)}}{a_1} + \dfrac{2ce^{a_1(t_1-t)}}{a_1^2} \right) \right]$$

We assume that during the time the amount is not settled, the interest earned from the permissible period is I_e per year.

Hence, $= \dfrac{I_e P}{T} \int\limits_0^M (a + bt + ct^2)tdt$

$$= \frac{I_e P}{T} \int_0^M (at + bt^2 + ct^3)dt = \frac{I_e P}{T} \left(\frac{aM^2}{2} + \frac{bM^3}{3} \frac{cM^4}{4} \right)$$

Sub-case (ii): $t_w \leq t_1 \leq M$

In this case, there is no interest payable is charged by the retailer, but the interest earned is received by them. Therefore the interest payable is zero in this case and the annual interest earned is I_e per year.

Hence,

$$= \frac{I_e P}{T} \left[\int_0^{t_1} (a + bt + ct^2) t \, dt + \int_0^{t_1} (M - t_1)(a + bt + ct^2) dt \right]$$

$$= \frac{I_e P}{T} \left[\left(a \left(Mt_1 - \frac{t_1^2}{2} \right) + b \left(\frac{Mt_1^2}{2} - \frac{t_1^3}{6} \right) + \frac{cMt_1^3}{3} \right) \right]$$

Sub-case (iii): $0 < t_1 < t_w$

If $t_1 < t_w$, then the retailer must borrow the partial payment $(1-d)C_0Q$ at time zero and pay the suppliers and then pay off the loan from the sales revenue at time $\frac{(1-d)C_0}{P} \left(\frac{e^{a_1 t_1} - 1}{a_1} \right)$. From the time zero to $\frac{(1-d)C_0}{P} \left(\frac{e^{a_1 t_1} - 1}{a_1} \right)$ the annual interest charged

and payment is, the annual interest payable $= \frac{I_e C_0^2 (a + bt + ct^2)(1-d)^2}{2PT} \left(\frac{e^{a_1 t_1} - 1}{a_1} \right)^2$

And the annual interest earned $= \frac{I_e P}{2T} \left[(a + bt + ct^2) \left(t_1 - \frac{(1-d)C_0}{P} \left(\frac{e^{a_1 t_1} - 1}{a_1} \right) \right) \right]^2$

$$+ \frac{I_e P(M - t_1)}{T} \left[(a + bt + ct^2) \left(t_1 - \frac{(1-d)C_0}{P} \left(\frac{e^{a_1 t_1} - 1}{a_1} \right) \right) \right]$$

Thus, the total cost for the retailer can be expressed as,

$$TC(t_1,T) = \begin{cases} TC_1(t_1,T_1) & if\ M \le t_1 \\ TC_2(t_1,T_2) & if\ t_w \le t_1 \le M \\ TC_3(t_1,T_3) & if\ 0 < t_1 < t_w \end{cases} \tag{9}$$

Now we calculate the values of TC_1, TC_2, TC_3.

Hence,

$$\begin{aligned}
TC_1(t_1,T) =\ & \frac{A}{T_1} - \left(\frac{h+C_0 a_1}{T_1 a_1}\right)\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} - \frac{bt_1}{a_1} - \frac{ct_1^2}{a_1} + \frac{2ct_1}{a_1^2}\right) \\
& - \frac{C_1}{T}\left[\left(\left(\frac{-aT^2}{2}\right) + \frac{ab_1 T^3}{2} - \frac{ab_1 T^3}{6} - \frac{bT^3}{6} + \frac{bb_1 T^4}{6}\right)\right. \\
& - \left(\frac{-at_1^2}{2} + \frac{ab_1 T t_1^2}{2} - \frac{ab_1 t_1^3}{6} - \frac{bt_1^3}{6} + \frac{bt_1^3 b_1}{6}\right)\bigg] + \frac{C_2 b_1}{T_1}\left[a\left(\frac{T_1^2}{2} - T_1 t_1 - \frac{t_1^2}{2}\right)\right. \\
& + b\left(\frac{-5T_1^3}{6} - \frac{T_1 t_1^2}{2} - \frac{t_1^3}{3}\right) - \frac{ct_1^3 T_1}{3}\bigg] - \frac{I_p C_0}{T_1 a_1^2}\left[\left(a + bt_1 + \frac{b}{a_1} + ct_1^2 + \frac{2ct_1}{a_1} + \frac{2c}{a_1^2}\right)\right. \\
& - \left(ae^{a_1(t_1-M)} + bMe^{a_1(t_1-t)} + be^{a_1(t_1-t)} + cM^2 e^{a_1(t_1-t)} + \frac{2cMe^{a_1(t_1-t)}}{a_1} + \frac{2ce^{a_1(t_1-t)}}{a_1^2}\right)\bigg] \\
& - \frac{I_e P}{T_1}\left(\frac{aM^2}{2} + \frac{bM^3}{3}\frac{cM^4}{4}\right)
\end{aligned} \tag{10}$$

$$\begin{aligned}
TC_2(t_1,T_2) =\ & \frac{A}{T_2} - \left(\frac{h+C_0 a_1}{T_2 a_1}\right)\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} - \frac{bt_1}{a_1} - \frac{ct_1^2}{a_1} + \frac{2ct_1}{a_1^2}\right) \\
& - \frac{C_1}{T}\left[\left(\left(\frac{-aT^2}{2}\right) + \frac{ab_1 T^3}{2} - \frac{ab_1 T^3}{6} - \frac{bT^3}{6} + \frac{bb_1 T^4}{6}\right)\right. \\
& - \left(\frac{-at_1^2}{2} + \frac{ab_1 T t_1^2}{2} - \frac{ab_1 t_1^3}{6} - \frac{bt_1^3}{6} + \frac{bt_1^3 b_1}{6}\right)\bigg] \\
& + \frac{C_2 b_1}{T_2}\left[a\left(\frac{T_2^2}{2} - T_2 t_1 - \frac{t_1^2}{2}\right) + b\left(\frac{-5T_2^3}{6} - \frac{T_2 t_1^2}{2} - \frac{t_1^3}{3}\right) - \frac{ct_1^3 T_2}{3}\right] \\
& - \frac{I_e P}{T_2}\left[\left(a\left(Mt_1 - \frac{t_1^2}{2}\right) + b\left(\frac{Mt_1^2}{2} - \frac{t_1^3}{6}\right) + \frac{cMt_1^3}{3}\right)\right]
\end{aligned} \tag{11}$$

$$TC_3(t_1, T_3) = \frac{A}{T_3} - \left(\frac{h + C_0 a_1}{T_3 a_1}\right)\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} - \frac{bt_1}{a_1} - \frac{ct_1^2}{a_1} + \frac{2ct_1}{a_1^2}\right)$$

$$- \frac{C_1}{T}\left[\left(\left(\frac{-aT^2}{2}\right) + \frac{ab_1 T^3}{2} - \frac{ab_1 T^3}{6} - \frac{bT^3}{6} + \frac{bb_1 T^4}{6}\right)\right.$$

$$- \left(\frac{-at_1^2}{2} + \frac{ab_1 T t_1^2}{2} - \frac{ab_1 t_1^3}{6} - \frac{bt_1^3}{6} + \frac{bt_1^3 b_1}{6}\right)\right] + \frac{C_2 b_1}{T_3}\left[a\left(\frac{T_3^2}{2} - T_3 t_1 - \frac{t_1^2}{2}\right)\right.$$

$$\left.+ b\left(\frac{-5T_3^3}{6} - \frac{T_3 t_1^2}{2} - \frac{t_1^3}{3}\right) - \frac{ct_1^3 T_3}{3}\right] + \frac{I_e C_0^2 (a + bt + ct^2)(1-d)^2}{2PT_3}\left(\frac{e^{a_1 t_1} - 1}{a_1}\right)^2$$

$$- \frac{I_e P}{2T_3}\left[(a + bt + ct^2)\left(t_1 - \frac{(1-d)C_0}{P}\left(\frac{e^{a_1 t_1} - 1}{a_1}\right)\right)\right]^2$$

$$- \frac{I_e P(M - t_1)}{T_3}\left[(a + bt + ct^2)\left(t_1 - \frac{(1-d)C_0}{P}\left(\frac{e^{a_1 t_1} - 1}{a_1}\right)\right)\right] \qquad (12)$$

6. SOLUTION PROCESS

To minimize the total cost, we determine the optimal storage cost and the optimal replenishment time.

Case 1: $t_w \le t_1 \le M$

Sub-case (i)

Equation (10) is minimum when $\dfrac{\partial TC_1(t_1, T_1)}{\partial t_1} = 0$ and $\dfrac{\partial TC_1(t_1, T_1)}{\partial T_1} = 0$ which yields

$$\frac{\partial TC_1(t_1, T_1)}{\partial t_1} \Rightarrow -\left(\frac{h + C_0 a_1}{T_1 a_1}\right)\left(a + bt_1 + ct_1^2 - \frac{b}{a_1} - \frac{2ct_1}{a_1} + \frac{2c}{a_1^2}\right)$$

$$- \frac{C_1}{T_1}\left[a\left(t_1 + b_1 T t_1 - \frac{b_1 t_1^2}{2}\right) - \frac{bt_1^2}{2} + \frac{bt_1^2 b_1}{2}\right]$$

$$+ \frac{C_2 b_1}{T_1}\left[a(-T_1 - t_1) + b(-T_1 t_1 - t_1^2) - c(t_1^2 T_1)\right]$$

$$- \frac{I_p C_0}{T_1 a_1^2}\left[\left(b + 2ct_1 + \frac{2c}{a_1}\right) - \left(aa_1 e^{a_1(t_1 - M)} + bM a_1 e^{a_1(t_1 - M)}\right.\right.$$

$$\left.\left.+ ba_1 e^{a_1(t_1 - M)} + cM^2 a_1 e^{a_1(t_1 - M)} + \frac{2cM a_1 e^{a_1(t_1 - M)}}{a_1} + \frac{2ca_1 e^{a_1(t_1 - M)}}{a_1^2}\right)\right] = 0 \quad (13)$$

$$\frac{\partial TC_1(t_1,T_1)}{\partial T_1} \Rightarrow -\frac{A}{T_1^2} + \left(\frac{h+C_0a_1}{T_1^2 a_1}\right)\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} - \frac{bt_1}{a_1} - \frac{ct_1^2}{a_1} + \frac{2ct_1}{a_1^2}\right)$$

$$+ \frac{C_1 aT^2}{2} + \frac{2C_1 aT^3 b_1}{6} - \frac{C_1 bT^3}{6} + \frac{C_1 at_1^2}{2} + \frac{C_1 ab_1 t_1^3}{6}$$

$$+ \frac{C_1 bt_1^3}{6} - \frac{C_1 bt_1^3 b_1}{6} - \frac{3aC_1 T^2 b_1}{2} + \frac{C_1 ab_1 T^2}{2} - \frac{C_1 bt_1^3 b_1}{6}$$

$$- \frac{C_2 b_1}{T_1^2}\left[a\left(\frac{T_1^2}{2} - T_1 t_1 - \frac{t_1^2}{2}\right) + b\left(\frac{-5T_1^3}{6} - \frac{T_1 t_1^2}{2} - \frac{t_1^3}{3}\right) - \frac{ct_1^3 T_1}{3}\right]$$

$$+ \frac{C_2 b_1}{T_1}\left[a(T_1 - t_1) + b\left(\frac{-5T_1^2}{2} - \frac{t_1^2}{2}\right) - \frac{ct_1^3}{3}\right]$$

$$+ \frac{I_p C_0}{T_1^2 a_1^2}\left[\left(a + bt_1 + \frac{b}{a_1} + ct_1^2 + \frac{2ct_1}{a_1} + \frac{2c}{a_1^2}\right)\right.$$

$$-\left(ae^{a_1(t_1-M)} + bMe^{a_1(t_1-M)} + be^{a_1(t_1-M)}\right.$$

$$+ cM^2 e^{a_1(t_1-M)} + \frac{2cMe^{a_1(t_1-M)}}{a_1} + \frac{2ce^{a_1(t_1-M)}}{a_1^2}\Bigg)\Bigg]$$

$$+ \frac{I_e P}{T_1^2}\left[\frac{aM^2}{2} + \frac{bM^3}{3}\right] = 0 \tag{14}$$

From the above equation, now we calculate the value of T_1^2,

$$T_1^2 = \left[2\left(A - \left(\frac{h+C_0 a_1}{a_1}\right)\left(at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} - \frac{bt_1}{a_1} - \frac{ct_1^2}{a_1} + \frac{2ct_1}{a_1^2}\right)\right.\right.$$

$$+ \frac{C_1 at_1^2}{2} + \frac{C_1 ab_1 t_1^3}{6} + \frac{C_1 bt_1^3}{6} - \frac{C_1 bt_1^3 b_1}{6}$$

$$+ \frac{C_2 b_1 at_1^2}{2} + \frac{C_2 bb_1 t_1^3}{3} - \frac{I_o C_0}{a_1^2}\left[\left(a + bt + ct_1^2 + \frac{b}{a_1} + \frac{2ct_1}{a_1} + \frac{2c}{a_1^3}\right)\right.$$

$$-\left(ae^{a_1(t_1-M)} + bMe^{a_1(t_1-M)} + be^{a_1(t_1-M)} + cM^2 e^{a_1(t_1-M)} + \frac{2ce^{a_1(t_1-M)}}{a_1} + \frac{2ce^{a_1(t_1-M)}}{a_1^2}\right)\Bigg]$$

$$-I_e P\left(\frac{aM^2}{2} + \frac{bM^3}{3}\right)\Bigg] \div \left[C_1 a - 3C_1 ab_1 - C_1 b + C_2 b_1 a\right] \tag{15}$$

From equation (15) we have to calculate the value of T_1

$$T_1 = Sqrt\left(\left[2\left(A-\left(\frac{h+C_0a_1}{a_1}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)\right.\right.\right.$$

$$+\frac{C_1at_1^2}{2}+\frac{C_1ab_1t_1^3}{6}+\frac{C_1bt_1^3}{6}-\frac{C_1bt_1^3b_1}{6}+\frac{C_2b_1at_1^2}{2}+\frac{C_2bb_1t_1^3}{3}$$

$$-\frac{I_oC_0}{a_1^2}\left[\left[\left(a+bt+ct_1^2+\frac{b}{a_1}+\frac{2ct_1}{a_1}+\frac{2c}{a_1^3}\right)\right.\right.$$

$$-\left(ae^{a_1(t_1-M)}+bMe^{a_1(t_1-M)}+be^{a_1(t_1-M)}+cM^2e^{a_1(t_1-M)}+\frac{2ce^{a_1(t_1-M)}}{a_1}+\frac{2ce^{a_1(t_1-M)}}{a_1^2}\right)\right]$$

$$\left.\left.-I_eP\left(\frac{aM^2}{2}+\frac{bM^3}{3}\right)\right]\right]\div\left[C_1a-3C_1ab_1-C_1b+C_2b_1a\right] \tag{16}$$

Sub-case (ii)

Equation (11) is minimum when $\dfrac{\partial TC_2(t_1,T_2)}{\partial t_1}=0$ and $\dfrac{\partial TC_2(t_1,T_2)}{\partial T_2}=0$ which yields

$$\frac{\partial TC_2(t_1,T_2)}{\partial t_1}\Rightarrow -\left(\frac{h+C_0a_1}{T_2a_1}\right)\left(a+bt_1+ct_1^2-\frac{b}{a_1}-\frac{2ct_1}{a_1}+\frac{2c}{a_1^2}\right)$$

$$-\frac{C_1}{T_1}\left[a\left(t_1+b_1Tt_1-\frac{b_1t_1^2}{2}\right)-\frac{bt_1^2}{2}+\frac{bt_1^2b_1}{2}\right] \qquad\qquad +\frac{C_2b_1}{T_2}\left[a(-T_2-t_1)+b(-T_2t_1-t_1^2)\right.$$

$$-c(t_1^2T_2)]$$

$$-\frac{I_eP}{T_2}\left[a(M-t_1)+b\left(Mt_1-\frac{t_1^2}{2}\right)+cMt_1^2\right]\quad=\quad 0 \tag{17}$$

$$\frac{\partial TC_2(t_1,T_2)}{\partial T_2}\Rightarrow -\frac{A}{T_2^2}+\left(\frac{h+C_0a_1}{T_2^2}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)$$

$$+\frac{C_1aT^2}{2}+\frac{2C_1aT^3b_1}{6}-\frac{C_1bT^3}{6}+\frac{C_1at_1^2}{2}+\frac{C_1ab_1t_1^3}{6}$$

$$+\frac{C_1bt_1^3}{6}-\frac{C_1bt_1^3b_1}{6}-\frac{3aC_1T^2b_1}{2}+\frac{C_1ab_1T^2}{2}-\frac{C_1bt_1^3b_1}{6}$$

$$+\frac{C_2}{T_2^2}\left[a\left(\frac{b_1T_2^2}{2}-b_1T_2t_1-\frac{b_1t_1^2}{2}\right)+b\left(\frac{-5b_1T_2^3}{6}-\frac{b_1T_2t_1^2}{2}-\frac{b_1t_1^3}{3}\right)+\frac{cb_1t_1^3T_2}{3}\right]$$

$$+\frac{C_2}{T_2}\left[a(b_1T_2-b_1t_1)+b\left(\frac{-5b_1T_2^2}{2}-\frac{b_1t_1^2}{2}\right)-\frac{cb_1t_1^3}{3}\right]$$

$$+\frac{I_eP}{T_2^2}\left[a\left(Mt_1-\frac{t_1^2}{2}\right)+b\left(\frac{Mt_1^2}{2}-\frac{t_1^3}{6}\right)+\frac{cMt_1^3}{3}\right]=0 \qquad (18)$$

From the above equation we calculate the value of T_2^2

$$T_2^2=\left[2\left(A-\left(\frac{h+C_0a_1}{a_1}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)+\frac{C_1at_1^2}{2}+\frac{C_1ab_1t_1^3}{6}+\frac{C_1bt_1^3b_1}{6}\right.\right.$$

$$\left.+\frac{C_2ab_1t_1^2}{2}+I_eP\left[a\left(Mt_1-\frac{t_1^2}{2}\right)+b\left(\frac{Mt_1^2}{2}-\frac{t_1^3}{6}\right)+\frac{cMt_1^3}{3}\right]\div\left(-3C_1a-C_2ab_1t_1\right) \qquad (19)$$

From equation (19) we get value of T_2,

$$T_2=Sqrt\left[2\left(A-\left(\frac{h+C_0a_1}{a_1}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)\right.\right.$$

$$+\frac{C_1at_1^2}{2}+\frac{C_1ab_1t_1^3}{6}+\frac{C_1bt_1^3b_1}{6}$$

$$\left.+\frac{C_2ab_1t_1^2}{2}+I_eP\left[a\left(Mt_1-\frac{t_1^2}{2}\right)+b\left(\frac{Mt_1^2}{2}-\frac{t_1^3}{6}\right)+\frac{cMt_1^3}{3}\right]\div\left(-3C_1a-C_2ab_1t_1\right) \qquad (20)$$

Sub-case (iii)

Equation (12) is minimum when $\dfrac{\partial TC_3(t_1,T_3)}{\partial t_1}=0$ and $\dfrac{\partial TC_3(t_1,T_3)}{\partial T_3}=0$ which yield

$$\frac{\partial TC_3(t_1,T_3)}{\partial t_1}\Rightarrow-\left(\frac{h+C_0a_1}{T_3a_1}\right)\left(a+bt_1+ct_1^2-\frac{b}{a_1}-\frac{2ct_1}{a_1}+\frac{2c}{a_1^2}\right)$$

$$-\frac{C_1}{T_1}\left[a\left(t_1+b_1Tt_1-\frac{b_1t_1^2}{2}\right)-\frac{bt_1^2}{2}+\frac{bt_1^2b_1}{2}\right]+\frac{C_2b_1}{T_3}\left[a(-T_3-t_1)+b\left(-T_3t_1-t_1^2\right)-c\left(t_1^2T_3\right)\right]$$

$$+\frac{I_eP}{2T_3}\left(a+bt+ct^2\right)$$

$$\left[2t_1-2\left(\frac{(1-d)C_0}{Pa_1}\left(\frac{t_1e^{a_1t_1}}{a_1}\right)+2\left(\frac{(1-d)C_0}{Pa_1}\left(\frac{e^{a_1t_1}}{a_1^2}\right)\right)+\frac{2(1-d)C_0}{Pa_1}\right]=0 \quad (21)$$

$$\frac{\partial TC_2(t_1,T_3)}{\partial T_3}\Rightarrow-\frac{A}{T_3^2}+\left(\frac{h+C_0a_1}{T_3^2}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)$$

$$+\left(\frac{C_1}{T_3^2}\right)\left[a\left(\frac{T_3^2}{2}+\frac{ab_1T_3^2}{2}-\frac{ab_1T_3^3}{6}\right)-\frac{bT_3^3}{6}\right)$$

$$-\left(-\frac{at_1^2}{2}+\frac{ab_1t_1^2T_3}{2}-\frac{ab_1t_1^3}{6}-\frac{bt_1^3b_1}{6}\right)\right]-\frac{C_1}{T_3}\left[a\left(-T_3+\frac{3ab_1T^2}{2}-\frac{3ab_1T^2}{6}-\frac{3bT^2}{6}-\frac{ab_1t_1^2}{2}\right)\right.$$

$$+\frac{C_2}{T_3^2}\left[a\left(\frac{b_1T_3^2}{2}-b_1T_3t_1-\frac{b_1t_1^2}{2}\right)+b\left(\frac{-5b_1T_3^3}{6}-\frac{b_1T_3t_1^2}{2}-\frac{b_1t_1^3}{3}\right)+\frac{cb_1t_1^3T_3}{3}\right]+\frac{C_2}{T_3}\left[a\left(b_1T_3-b_1t_1\right)\right.$$

$$+b\left(\frac{-5b_1T_3^2}{2}-\frac{b_1t_1^2}{2}\right)-\frac{cb_1t_1^3}{3}\right]-\frac{I_pC_0^2\left(a+bt+ct^2\right)\left(1-d\right)^2}{2PT_3^2}\left(\frac{e^{a_1t_1}-1}{a_1}\right)^2+\frac{I_eP}{2T^2}\left[\left(a+bt+ct^2\right)\right.$$

$$\left(t_1-\frac{\left(1-d\right)C_0}{P}\left(\frac{e^{a_1t_1}-1}{a_1}\right)\right)^2+\frac{I_eP\left(M-t_1\right)}{T^2}\left[\left(a+bt+ct^2\right)\right.$$

$$\left(t_1-\frac{\left(1-d\right)C_0}{P}\left(\frac{e^{a_1t_1}-1}{a_1}\right)\right)=0 \tag{22}$$

From the above equation we calculate the value of T_3^2 ,

$$T_3^2=\left[2\left(-A+\left(\frac{h+C_0a_1}{a_1}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)\right.\right.$$

$$+\frac{C_1at_1^2}{2}+\frac{C_1ab_1t_1^3}{6}+\frac{C_1bt_1^3}{6}-\frac{C_1bb_1t_1^3}{6}+\frac{C_1ab_1t_1^2}{2}+\frac{C_2ab_1t_1^2}{2}+\frac{C_2bb_1t_1^3}{3}$$

$$-\frac{I_pC_0^2\left(a+bt+ct^2\right)}{2P}\left(1-d\right)^2\left(\frac{e^{a_1t_1}-1}{a_1}\right)^2-\frac{I_eP}{2}\left[\left(a+bt+ct^2\right)\left(t_1-\frac{\left(1-d\right)C_0}{P}\left(\frac{e^{a_1t_1}-1}{a_1}\right)\right)\right]^2$$

$$-I_eP(M-t_1)\left(a+bt+ct^2\right)\left(t_1-\frac{\left(1-d\right)C_0}{P}\left(\frac{e^{a_1t_1}-1}{a_1}\right)\right)\right]$$

$$\div\left(-C_1a-C_2ab_1+5C_2bb_1\right) \tag{23}$$

From the equation (23) we calculate T_3 ,

$$T_3=\text{sqrt}\left[2\left(-A+\left(\frac{h+C_0a_1}{a_1}\right)\left(at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}-\frac{bt_1}{a_1}-\frac{ct_1^2}{a_1}+\frac{2ct_1}{a_1^2}\right)\right.\right.$$

$$+\frac{C_1at_1^2}{2}+\frac{C_1ab_1t_1^3}{6}+\frac{C_1bt_1^3}{6}-\frac{C_1bb_1t_1^3}{6}+\frac{C_1ab_1t_1^2}{2}+\frac{C_2ab_1t_1^2}{2}+\frac{C_2bb_1t_1^3}{3}$$

$$-\frac{I_p C_0^2 (a+bt+ct^2)}{2P}(1-d)^2\left(\frac{e^{a_1 t_1}-1}{a_1}\right)^2 - \frac{I_e P}{2}\left[(a+bt+ct^2)\left(t_1 - \frac{(1-d)C_0}{P}\left(\frac{e^{a_1 t_1}-1}{a_1}\right)\right)\right]^2$$

$$-I_e P(M-t_1)\left(a+bt+ct^2\right)\left(t_1 - \frac{(1-d)C_0}{P}\left(\frac{e^{a_1 t_1}-1}{a_1}\right)\right)\right] \div \left(-C_1 a - C_2 a b_1 + 5C_2 b b_1\right) \quad (24)$$

6. STATISTICAL EXAMPLES

In order to demonstrate the solution method, let us consider an inventory system with the following data: Let A= $500 per order, h = $15 per unit per order, M = 2/12, C1= $ 20 per unit,C2 = $30 per unit, P = $85 per unit, I_p = 0.15 per dollar per year, I_e = 0.12 per dollar per year, a_1=0.08, b_1=0.50, d=0.8, w=200, C_0=25, a=55, b=65, c=75,and t=2 years

Table 1 Illustration of the solution procedure for the numerical example

a	b	c	T_1	T_2	T_3	TC_1	TC_2	TC_3
5	10	20	1.3576	1.4882	3.5176	77.5000	80.022	183.680
t_w			1.6667	1.8524	2.8286			
15	20	25	1.0803	1.3457	3.4020	201.2000	108.37	354.22
t_w			1.0195	1.2407	2.1294			
15	25	35	0.3054	2.7949	3.3652	272	532.85	518.70
t_w			0.0668	0. 0574	0.1936			
55	65	75	1.8637	1.9394	4.4971	100.23	125.56	700.99
t_w			0.3959	0.4078	0.9591			

7. COMPASSION STUDY

Table 2 Computational outcome with respect to different values of h, A, M

Changing parameters	Change in parameters	T_1	T_2	T_3	TC_1	TC_2	TC_3
	20	2.6601	2.566	0.8790	258.55	722.37	711.13
h	25	3.3463	2.7724	0.9790	100.270	80.486	87.201
	30	3.6205	2.9700	0.7050	102.93	87.884	152.19
	300	2.6601	2.566	0.8790	258.55	722.37	711.13
A	350	3.3453	2.7734	0.9798	100.30	80.490	87.221
	400	3.6225	2.9715	0.7055	102.97	87.897	152.25
	1/12	2.6601	2.566	0.8790	258.55	722.37	711.13
M	2/12	3.3443	2.7730	0.9791	100.10	80.483	87.211
	3/12	3.6200	2.9710	0.7047	102.80	87.880	152.20

Table 3 Computational results with respect to different values of b_1, a_1, I_e, I_p

Changing parameters	Values of parameters	T_1	T_2	T_3	TC_1	TC_2	TC_3
b_1	0.30	3.1218	1.1682	1.6770	177.06	344.43	173.33
	0.35	1.0466	1.5802	1.2390	384.40	204.09	97.663
	0.40	3.5340	3.5661	1.3610	891.76	744.99	131.43
a_1	0.02	3.1218	1.1682	1.6770	177.06	344.43	173.33
	0.04	1.0456	1.5812	1.2395	384.44	204.15	976.67
	0.08	3.5345	3.5663	1.3615	891.80	745.00	131.50
I_e	0.15	3.1218	1.1682	1.6770	177.06	344.43	173.33
	0.10	1.0462	1.5800	1.2387	384.35	204.00	976.53
	0.25	3.5341	3.5650	1.3605	891.70	744.93	131.47
I_p	0.20	3.1218	1.1682	1.6770	177.06	344.43	173.33
	0.25	1.0469	1.5805	1.2388	384.33	203.99	976.59
	0.30	3.5346	3.5666	1.3613	891.71	744.87	131.23

8. TERMINATION

In this paper, we have examined the partial trade credit financing in a supply chain by EOQ-based model for decomposing items together with shortages. We also assumed that the demand is quadratic. A mathematical is developed for the optimal solution with the shortage point, ordering quantity in order to minimize the total cost. A Statistical example is presented to demonstrate the solution process and the compassion study for various values of the parameters also offered. The proposed model can be further extended in it several ways. For example, it may be extended to multiple items with partial backlogging and also developing the model to changing the demand as cubic.

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