MODIFICATION OF A HEURISTIC METHOD FOR THE CAPACITATED FACILITY LOCATION PROBLEM: A NOTE

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ABSTRACT

We give a modified version of a heuristic, available in the relevant literature, of the capacitated facility location problem. A numerical experiment is performed to compare the two heuristics. The study would help to design heuristics for different generalizations of the problem.

KEYWORDS

Capacitated Facility Location Problem, Heuristics, Performance

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1. INTRODUCTION

We may describe the capacitated facility location problem (CFLP) as follows. In an $m \times n$ CFLP we have m possible locations for setting up some facilities from where an item would be supplied to n customers or demand points. There is a fixed cost to locate a facility at the i-th location and it is given by f_i. Variable or operational cost of supplying one unit from a facility, set up at i-th location, to the j-th customer is c_{ij}. A facility at i-th location has a capacity of S_i and j-th customer has a demand of D_j. The problem is to find an optimal solution, specifying the number and locations of facilities and supply quantity from each facility to each customer, minimizing the total of fixed and variable costs. The CFLP may be written, with the preceding notation, and x_{ij} denoting the supply from the i-th location to the j-th customer, as the following mixed integer linear program (P1):

$$Minimize \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} f_i y_i$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} \le S_i y_i, \forall i = 1, \dots, m;$$

$$\sum_{i=1}^{m} x_{ij} = D_j, \forall j = 1, \dots, n;$$

$$y_i = 1 \text{ or } 0, \forall i = 1, \dots, m;$$

$$x_{ij} \ge 0, \forall i, \forall j.$$

The preceding mathematical programming formulation, or an equivalent one, has been often used in the literature (see, e.g., [2, 4]) to describe the CFLP. If all S_i are very large, larger than equal to the total demand $\sum_j D_j$, then we have an un-capacitated facility location problem (UFLP) or, as sometimes called, a simple facility / plant location problem. Both of CFLP and UFLP are NP-complete problems. There can be many variants and extensions of the CFLP. CFLP is a versatile optimization model, with a wide scope of application.

Heuristic methods are often the only possible solution approach for the problem, particularly for large instances. A type of heuristic methods considers improving the solution, beginning with an initial solution, with dropping open locations, adding a closed location or interchanging a closed location with an open one. Different variants of such add-drop-interchange (ADI) heuristics have been discussed by Kuehn & Hamburger [3], Jacobsen [2], Domschke & Drexl [1], Sinha [5] and others. Sinha [5] has given a heuristic method (subsequently called as Method 1), which in numerical experiments has shown noteworthy performance. However, some kind of critical instances where the heuristic methods may fail for the CFLP are described by Sinha [6].

In this article, we consider some modifications of Method 1, in order to increase solution efficiency. The modified version is also an ADI heuristic, but varies in some steps. Not possible theoretically, numerical experiments have been performed to compare the two methods.

This article is organized in the following manner. We describe the modified heuristic method (Method 2) in the next section. Results of numerical experiments on the method are given in Section 3. This is followed by concluding remarks.

2. THE MODIFIED HEURISTIC

We shall use, for convenience, the same notation and the manner of description as used for Method 1. We consider the transportation problem that is obtained if fixed costs are omitted and a subset of the locations is considered. Let the set of all possible locations be $I = \{1, 2, ..., m\}$ and the set of customers be as $J = \{1, 2, ..., n\}$. Denote the problem as (P2).

$$Minimize \sum_{i\in K} \sum_{j\in J} c_{ij}\, x_{ij}$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} \leq S_i, \forall i \in K$$

$$\sum_{i=1}^{m} x_{ij} = D_j, \forall j \in J$$

$$x_{ij} \geq 0, \forall i \in K, \forall j \in J,$$

with $K \subseteq I$ as a set of some possible locations. A feasible solution of (P2) is also feasible for (P1). In the heuristic method, which is described next, (P2) is solved to get an initial solution which is then improved in subsequent iterations. It is assumed that, $f_i \geq 0$, $S_i > 0$, $\forall\, i$; and, $\sum_i S_i > \sum_j D_j$.

<u>Method 2</u>

A near optimal solution is obtained by solving an instance in the following two phases.

Phase 1

Let T_i = minimum $\{S_i, \Sigma D_j\}$. An initial solution is obtained with solving problem (P2) with $K = I$ and costs modified as, $c'_{ij} = c_{ij} + f_i/T_i$, $\forall$ i, $\forall$ j. A CFLP solution, i.e., a solution of (P1), is obtained by considering the locations which have nonzero supply to at least one demand point, in the optimal solution obtained for (P2). K is updated accordingly to include only such locations.

Phase 2

The solution obtained in Phase 1 is tried to be improved through adding/ dropping/ interchanging locations. Let I_1 and I_2 be the sets of used and unused locations, corresponding to a CFLP solution, at the beginning of an iteration. The following steps are carried out in an iteration.

Step 1. (i) Try dropping the used locations in the following manner. For $\forall$ $k \in I_1$, the steps, as given next, are performed.

(a) Initialize as, $r_i = S_i - \sum_{j \in J} x_{ij}$, $\forall$ $i \in I_1 \setminus k$; $D_k = 0$, $p = 0$.
(b) If $p < n$, $p = p + 1$; $R = x_{kp}$ and go to (c). Else, stop.
(c) If $R > 0$, go to (d), else go to (b).

(d) Let $c_{i^*p} = \min\{c_{ip} \mid i \in I_1 \setminus k, r_i > 0\}$. If such i^* exists then, $z = \min\{R, r_{i^*}\}$, $D_k = D_k + z \times (c_{i^*p} - c_{kp})$, $R = R - z$, $r_{i^*} = r_{i^*} - z$ and go to (c). Else, location k cannot be dropped, stop setting D_k at a large value.
Let $C_{(\text{drop})} = \max\{f_k - D_k \mid k \in I_1\} = f_{k^*} - D_{k^*}$.

(ii) Try adding the unused locations in the following manner. For $\forall$ $l \in I_2$, the subsequent steps are carried out.
(a) Initialize as, $R = S_l$, $D_l = 0$, $A = \varnothing$.
(b) If $R = 0$, stop. Else go to (c).

(c) Let $c_{i^*j^*} = \max\{c_{ij} - c_{lj} \mid c_{ij} > c_{lj}, x_{ij} > 0, i \in I_1, j \in J, (i, j) \notin A\}$. If such i^* and j^* exist then, $z = \min\{x_{i^*j^*}, R\}$ and $D_l = D_l + z \times (c_{i^*j^*} - c_{lj})$, $A = A + (i^*, j^*)$, $R = R - z$, and go to (b). Else, stop.
Let $C_{(\text{add})} = \max\{D_l - f_l \mid l \in I_2\} = D_{l^*} - f_{l^*}$.

(iii) Try to interchange an unused location with a used location, with $\forall$ $k \in I_1$, $\forall$ $l \in I_2$. In interchanging, first, k is dropped, considering l is also used, as in (ii). Then, if all supply capacity of l has not been used, it is attempted to be added, as in (i), with the remaining capacity.

Let $C_{(\text{interchange})} = \max\{f_k - D_k - f_l + D_l \mid k \in I_1, l \in I_2\} = f_{k^*} - D_{k^*} - f_{l^*} + D_{l^*}$.

Step 2. Calculate $\overline{C} = \max\{C_{(\mathrm{add})}, C_{(\mathrm{drop})}, C_{(\mathrm{interchange})}\}$. If $\overline{C} > 0$, add, drop or interchange the corresponding locations k^*, l^* to update *K*, and solve (P2) with costs c_{ij}. Return to Step 1. Else, i.e., $\overline{C} \leq 0$, stop with the current solution, with used locations in *K*.

Specifically, the modifications made are as described. In Phase 1, in the cost transformation, we use T_i, instead of S_i. This is done to avoid the situation that the effect of fixed cost is unnecessarily reduced, when a supply location has very high supply capacity. In actual problems, although, such data may not be found much. Another modification is done in step (ii), (c). Instead of taking maximum of the costs (c_{ij}) of the arcs, we take the maximum of the difference of the costs ($c_{ij} - c_{lj}$). This would more correctly identify the improvement through an add; although, that does not ensure a better or equally efficient final solution.

3. NUMERICAL EXPERIMENT AND OBSERVATIONS

It is not possible to identify theoretically which method would work better when. We have conducted a numerical experiment, with random instances, to compare the two methods. Such instances have been generated in the following way.

i. Get *m*, *n*, CR as inputs (Capacity ratio (CR) = Total supply ($\sum_i S_i$) / Total demand ($\sum_j D_j$));

ii. $c_{ij} = u$, for $i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$;

iii. $S_i = 5 \times n \times \mathrm{CR} / m + 100 \times u,$ rounded to the nearest integer, for $i = 1, 2, \ldots, m$;

iv. Get total demand $A = \sum_i S_i / \mathrm{CR}$, rounded to the nearest integer;

v. $D_j = 5 + (A - 5n) \times (u_j / \sum_{l=1}^{n} u_l)$, rounded to the nearest integer, for $j = 1, 2, \ldots, n-1$; $D_n = A - \sum_{j=1}^{n-1} D_j$. If D_n is less than 5, it is increased to 5; S_1 is increased by the same quantity.

Fixed costs are generated in three ways, giving three types of random instances; Type 1: $f_i = 0.5 \times S_i \times u$; Type 2: $f_i = 5.0 + \sqrt{S_i} \times u$; Type 3: $f_i = 25$. In the preceding, each *u* is an independent random deviate in (0, 1).

Efficiency of a solution is calculated as,

Efficiency = (1 − (Cost of the solution – Cost of an optimal solution) / Cost of an optimal solution) × 100%.

We have first evaluated the methods with 15 × 100 instances of Type 1, with CR = 4. For these instances, optimal solutions have been obtained by considering exhaustively all combinations of open facility locations. In all the 30 randomly generated instances, both the heuristic methods have given the same solution. In 20 instances, the solution obtained is optimal. Average Efficiency is 99.4% for both the methods, minimum being 88.9%. This indicates satisfactory performance of both the methods.

The methods are also compared with the three types of randomly generated instances with size 100 × 250, with CR = 2, 4, 6 for each type. For a particular type and a particular CR, 15 instances are used. The types of such random instances are summarized in Table 1.

Table 1: Random Instances (Large) in the Numerical Experiment

Obs. No.	Instance Type	Fixed Cost (approximately) of Supply Locations	Instance Size ($m \times n$)	CR	Number of Instances Verified
1	Type 1	Proportional to the supply capacity	100 × 250	2	15
2				4	15
3				6	15
4	Type 2	Proportional to the square root of supply capacity	100 × 250	2	15
5				4	15
6				6	15
7	Type 3	Constant	100 × 250	2	15
8				4	15
9				6	15

As we have large sized instances here, the methods are compared only between those-selves, but not with an optimal solution. For Type 1, the methods have yielded the same solutions for all instances. For Type 2, differences have occurred only for two instances, the instances having CR = 6. For Type 3 (constant fixed cost), there are more differences. For the highest deviation, the difference reaches 10% (with respect to the better solution; Method 2 being better for the particular instance). But there is no indication of clear superiority of any of the method. For Type 3 instances, paired t test comparisons show that there is not enough evidence to reject the null hypothesis of equality of average solution values, at the level of significance 5%. This holds for all CR values. In Table 2, we give the experimental observations for the 100 × 250 instances. The numerical experiments have been done with Microsoft Excel, the heuristic methods being implemented in Visual Basic. A Pentium IV personal computer with 1.86 GHz processor, 1 GB RAM internal memory, with Windows XP Professional operating system has been used. The maximum time taken to solve the 100 × 250 instances has been 1092.8 second (s) for Method 1; whereas this has been 1054.5 s for Method 2 (which has occurred for the same instance) (it may be possible to improve the time requirement by better implementation of the methods). Time requirements are near and no method has less time requirement for all the instances uniformly.

4. CONCLUDING REMARKS

We have considered some modifications of an add-drop-interchange heuristic method, presented in the relevant literature earlier. Apparently, such modifications should further improve the performance of the method. Numerical experiments with randomly generated instances have been done to compare the two methods. Observations suggest that, both the methods have satisfactory performance and suitable for practical applications. There are sometimes differences, but there is no indication of clear superiority of any of the method. Any one of the methods may be used; or, to increase solution efficiency both the methods may be used, if possible.

We feel that, the numerical experiment involving the two heuristic methods has given some insights about the CFLP. Such insights and the methods may be of help to obtain more efficient methods for the many possible generalizations of the CFLP.

Table 2: Comparison of the Two Heuristic Methods

Obs. No	**Instance Type**	**CR**	**Number of Instances Method 1 Better**	**Number of Instances Method 2 Better**	**Average Ratio of Solution Costs (Method1/Method 2)**
1	Type 1	2	0	0	100%
2	,,	4	0	0	100%
3	,,	6	0	0	100%
4	Type 2	2	0	0	100%
5	,,	4	0	1	100.04%
6	,,	6	0	1	100.08%
7	Type 3	2	4	5	100.41%
8	,,	4	4	8	100.41%
9	,,	6	4	1	99.94%

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