A single stage single constraints linear fractional programming problem: An approach

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ABSTRACT

In the present paper we present a new method for solving a class of single stage single constraints linear fractional programming (LFP) problem. The proposed method is based on transformation the objective value and the constraints also. After reducing the fractional program in to equivalent linear program with the help of transformation technique, after that we apply Simplex method to find objective value. Numerical examples are constructed to show the applicability of the above technique

KEYWORDS

Linear fractional programming problem(LFPP); Linear program; Simplex method

Mathematical Subject Classi_cation (2010): 90C05; 90C32; 46N10

1. INTRODUCTION

Linear fractional programming problem (LFPP) deals with problems in which objective function is a ratio of two linear functions. Maximizing the efficiency of an economicsystem leads to optimization problems whose objective function is a ratio. Linear fractional problems may be found in different fields such as data development analysis, taxprogramming, risk and portfolio theory, logistic and location theory [7, 6, 5, 4, 3]. Also,linear fractional programming is used to achieve the highest ratio of outcome to cost, theratio representing the highest efficiency.

Charnes and Cooper [8] proposed several methods for solving linear fractional program by transforming it to an equivalent linear program. Bitran and Novaes [18] considered updated objective functions method to solve linear fractional program by solving a sequence of linear programs whereas Dinkelbach [9] used parametric approach to solve a linear fractional programming problems. Later on, several authors extended the approach by Dinkelbach [9] to solve fractional programming problems, *e.g.*, generalized fractional programming problems [10, 11] and the minimum spanning tree with sum of ratios problems [12]. Almogy and Levin [13] extended the parametric approach of Dinkelbach [9] to solve sum of ratios problems. Falk and Palocsay [16] showed that the approach in [13] does not always lead to appropriate solutions and they extended the parametric approach of Dinkelbach to solve sum of ratios, product of ratios and product of linear functions in [14]. Tammer*et al.* [15] considered Dinkelbach approach to solve multiobjective linear fractional programming problems by estimating the parameters. However, their approach does not necessarily guarantee an efficient solution. Gomes *et al.* [17] focused on multiobjective linear programming problem having weights established some optimality conditions.

During recent years, complexity of problems arising in different fields prompted researchers to develop efficient algorithms to solve linear fractional programs. Valipour*et al.* [19] suggested an iterative parametric approach for solving multiobjective linear fractional programming (MOLFP) problems. Cambini*et al.* [20] reviewed methods for solving biobjective linear fractional programming. Recently, Tantawy [21] suggested an iterative method based on conjugate gradient projection method for solving linear fractional programming problem.

In the present paper, we first convert linear fractional programming problem (LFPP) to linear program one with the help of transformation technique. We also present an example to clarify the proposed method. In section 2 some notations and definitions is given while in section 3 we give the main result with suitable numerical example and a concluding remark is given in section 4.

2. DEFINITIONS AND METHODOLOGY

A linear fractional programming problem occurs when a function is minimize or maximize and the objective function is ratio (numerator and denominator) and the constraints are linear type function. Consider a linear fractional programming problem:

$$Max/Min(Z) = \frac{c^T x+\alpha}{d^T x+\beta}$$

$$s.t. \qquad Ax \leq b$$

$$x \geq 0$$

This problem can be applied to find an optimal productivity solution, minimizing or maximizing the ratio between storage cost and production cost under the storage constraints, *Z*=the total ratio cost between storage cost and production cost,

c^T = storage cost per unit,

d^T =production cost per unit,

α and β are constant.

2.1 Methodology

Transformation of Objective:

$$Max(Z) = \frac{c^T x+\alpha}{d^T x+\beta}$$

Multiplying β on both numerator and denominator we have,

$$Max(Z) = \frac{c^T x\beta + \alpha.\beta}{\beta(d^T x + \beta)}$$
$$= \frac{c^T x\beta + d^T x\alpha - d^T x\alpha + \alpha\beta}{\beta(d^T x + \beta)}$$
$$= (c^T - d^T.\frac{\alpha}{\beta}).\frac{x}{d^T x + \beta} + \frac{\alpha}{\beta}$$
$$Max(Z) = P^T y + \frac{\alpha}{\beta}$$

where $P^T = (c^T - d^T.\frac{\alpha}{\beta})$,$y = \frac{x}{d^T x + \beta}$,$g = \frac{\alpha}{\beta}$

$$Max(Z) = P^T y + g$$

Transformation of constraints:

$$(Ax - b) \leq 0,$$
$$= \quad \frac{\beta(Ax - b)}{\beta(d^T x + \beta)} \leq 0,$$
$$= \quad \frac{A\ \ x\beta - b\ \ \beta}{\beta\ \ (d^T x + \beta)} \leq 0,$$
$$= \quad \frac{A\ \ x\ \ \beta + b\ \ d^T x - b\ \ d^T x - b\ \ \beta}{\beta(d^T x + \beta)} \leq 0,$$
$$= \quad (A + d^T.\frac{b}{\beta}).\frac{x}{d^T x + \beta} \leq 0,$$
$$= \quad Gy \leq h.$$

where $G = (A + d^T.\frac{b}{\beta})$,$h = \frac{b}{\beta}$

Now consider the linear programming problem from the above transformation of objecttive and transformation of constraints we have,

$$MaxF(x) = P^T y + g$$
$$s.t \quad Gy \leq h$$
$$y \geq 0.$$

where $g = \frac{\alpha}{\beta}.y$is the variable and F(x) is the optimal value.

3. Example

Consider the following LFP problem

$$MaxF(x) = \frac{x_1+3}{x_2+1}$$

$$s.t. \quad -x_1 + x_2 \leq 1$$

$$2x_1 \leq 3$$

$$3x_1 + 2x_2 \leq 7$$

$$x_1, x_2 \geq 0$$

First we solve the LFP problem by using our proposed technique to its correspondinglinear programming problem,

$$MaxF(x) = y_1 - 2y_2 + 3$$

$$s.t. \quad -y_1 + 2y_2 \leq 1$$

$$2y_1 \leq 3$$

$$3y_1 + 9y_2 \leq 7$$

$$y_1, y_2 \geq 0$$

Now this problem is our linear programming problem and solved by regular simplex method. We find the variable is $\frac{3}{2}$, 0 and the optimal solution for the relaxed linear programming with optimal value $F(x) = 9/2$.This result is same as the result of [22]. The method is very useful because of his calculations involved are very simple and take least time as compare as other method for solving linear fractional programming problem. We also solved this problem by LINGO software and find objective solutions is same as our proposed method result.

4. CONCLUSIONS

In this paper, we present a transformation method for solving linear fractional programming problem when the objective function is ratio function and the set of constraints isin the form of linear inequality. Our proposed method based upon transformation technique. Our new method can be applied to any linear fractional programming problem,since it is a special thing of the mathematical program.

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