Fuzzy Inventory Model of Deteriorating Items under Power Dependent Demand and Inventory Level Dependent Holding Cost Function

Dr. Biswaranjan Mandal
Associate Professor of Mathematics, Acharya Jagadish Chandra Bose College,
Kolkata, West Bengal, India

Abstract:
The present paper deals with the development of a fuzzy inventory model of deteriorating items under power demand rate and inventory level dependent holding cost function. The deterioration rate, demand rate, holding cost and unit cost are considered as trapezoidal fuzzy numbers. Both the crisp model and fuzzy model are developed in this paper. The graded mean integration method(GM) and signed distance method(SD) are used to defuzzify the total cost of the present model. Both the models are illustrated by suitable numerical examples and a sensitivity analysis for the optimal solution towards changes in the system parameters are discussed. Lastly a graphical presentation is furnished to compare the total costs under the above two mentioned methods in the fuzzy model.

Key Words: Inventory, deterioration, fuzzy, trapezoidal fuzzy numbers and defuzzify.
Subject classification: AMS Classification No. 90B05

1. Introduction:

In our daily life, deterioration is a common issue. The deterioration or decay is very low for the items like steel, hardware, glassware where as food items, drugs, radioactive substances have sufficient deterioration in nature, and so it is harmful for use. We are studying many research papers where deterioration of items is assumed. In this field, many researchers like Ghare & Schrader [1], Covert & Philip [2], Ghimai et al[3], Hwang et al[4], Rajoria [5] , Dr Biswaranjan Mandal[6] are mentioned a few.

In inventory system, it is very common observation that many customers are influenced for purchasing goods when a large pile of goods are stuck in shelf in a super market. So consumption rate may vary with the stock level. In view of this, researchers like Levin [7], Datta & Pal[8], Baker et al [9], Tripathi et al [10] are noteworthy.

One of the weakness of the inventory management theory which has a great application in business sector is the unrealistic or uncertain assumptions of the different parameters. Bellan & Zadeh [11] first developed the fuzzy set theory in decision making process. After that, many researchers like Yao & Lee [12], Yao & Chiang [13] introduced fuzzy articles. More and more progresses are observed regarding development of inventory models from crisp parameters to fuzzy parameters. Vujosevic [14] and Hsieh et al [15] discussed the inventory models considering fuzzy sense where ordering cost and holding cost are fuzzy in nature called trapezoidal numbers. Many inventory models with major parameters as fuzzy are discussed by Chen [16], Park [17], Kao & Hsu [18], Sujit [19], Zadeh[20] etc for defuzzifying the total inventory cost.

In view of the above sort of situations and facts, we have studied an order level inventory system having both crisp and fuzzy models. The constant deterioration rate and power demand rate with inventory level dependent holding cost is assumed in the present model. Shortages are not allowed. The certain and uncertain nature of variables are considered in the proposed model. The trapezoidal type fuzzy number is used for representing fuzzy numbers. Both graded mean integration method (GM) and signed distance method (SD) are used for defuzzification in the present model. The solution procedure is illustrated considering suitable examples for both the models. Sensitivity analysis of the optimal solution with respect to the changes in the different parameters is discussed and lastly a graphical presentation is furnished to compare the total costs under the above two mentioned methods in the fuzzy model.

2. Definitions and Preliminaries :

The following definitions are needed during development of the fuzzy inventory model.

a) A fuzzy set X on the given universal set is a set of order pairs and defined by

$$\tilde{A} = \{(x, \lambda_{\tilde{A}}(x)) : x \in X\}$$, where $\lambda_{\tilde{A}} : X \to [0,1]$ is called membership function.

b) A fuzzy number $\tilde{A}$ is a fuzzy set on the real number R, if its membership function $\lambda_{\tilde{A}}$ has the following properties

$\lambda_{\tilde{A}}(x)$ is upper semi continuous.

$\lambda_{\tilde{A}}(x)$ =0, outside some interval $[a_1, a_4]$

Then $\exists$ real numbers a_2 and a_3, $a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$ and decreasing on $[a_3, a_4]$ and $\lambda_{\tilde{A}}(x)$ =1 for each $x \in [a_1, a_2]$.

c) A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is represented with membership function $\lambda_{\tilde{A}}$ as

$$\lambda_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ 1, a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, a_3 \le x \le a_4 \\ 0, otherwise \end{cases}$$

d) Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, the arithmetical operations are defined as:

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$$

$$\tilde{A} \Theta \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

$$\tilde{A}\phi\tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$$

$$\alpha \otimes \tilde{A} = \{ \begin{matrix} (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4), \alpha \geq 0 \\ (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1), \alpha < 0 \end{matrix}$$

e) Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, then the Graded Mean Integration Method of $\tilde{A}$ is defined as

$$P(\tilde{A}) = \frac{\frac{1}{2}\int_0^1 \alpha[A_L(\alpha) + A_G(\alpha)]d\alpha}{\int_0^1 \alpha d\alpha} = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

f) Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a fuzzy set defined on R, then the Signed Distance Method of $\tilde{A}$ is defined as

$$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_G(\alpha)]d\alpha = \frac{a_1 + a_2 + a_3 + a_4}{4}$$

3. Assumptions and Nomenclatures :

3.1 Assumptions:

The following fundamental assumptions used in the present paper are given as follows:

(i). Replenishment rate is infinite but size is finite.
(ii). Lead time is zero.
(iii). There is no repair or replacement of the deteriorated items occurring during the cycle.
(iv). Deterioration occur when the item is effectively in stock.
(v) The demand rate is power demand pattern
(vi) Holding cost follows as inventory level dependent.
(vii) Shortages are not allowed.

3.2 Nomenclatures:

The following fundamental nomenclatures are used in the present paper:

(i) i(t) : On hand inventory at time t.
(ii) T : The length of order cycle.
(iii) Q : The total amount of on-hand inventory.
(iv) A : The ordering cost per order.
(v) h : The holding cost per unit time.
(vi) c : The deterioration cost per unit item.

(vii) D(t) : The demand rate is inventory dependent defined as
D(t) = αi^{β}, $\alpha > 0$ and $0 < \beta < 1$ where α is annual demand parameter and β is demand elasticity parameter.

(viii) θ : The deterioration rate of an item, $0 < \theta < 1$.

(vii) C_D : Cost due to deterioration over the cycle period,

(ix) C_H : Holding cost over the cycle period

Where $C_H = \int_0^T h.i(t)dt$

(x) TC : Total average inventory cost per unit time.

(xi) $\tilde{\theta}$: The fuzzy deterioration parameter.

(xii) $\tilde{h}$: The fuzzy holding cost parameter.

(xiii) $\tilde{\alpha}$: The fuzzy demand parameter.

(xiv) $\tilde{\beta}$: The fuzzy demand elasticity parameter.

(xv) $\tilde{c}$: The fuzzy deterioration parameter.

(xvi) $\tilde{TC}$: The fuzzy total cost of the system per unit time.

4. Model development:

Let Q be the total amount of inventory purchased or produced at the beginning of each period t=0 .During [0,T], the stock will be gradually depleted due to the effect of deterioration and market demand, and ultimately falls to zero at t =T.
The instantaneous state of the inventory level i(t) at time t governed by the following equations

4.1 Crisp Model :

$$\frac{di(t)}{dt} + \theta i(t) = -\alpha\{i(t)\}^{\beta}, 0 \le t \le T \tag{4.1.1}$$

The boundary conditions are $i(0) = Q$ and $i(T) = 0$ (4.1.2)

The solution of the equation (4.1.1) is given by the following

$$i(t) = [\frac{\alpha}{\theta}\{e^{\theta(1-\beta)(T-t)} - 1\}]^{\frac{1}{1-\beta}}, 0 \le t \le T \tag{4.1.3}$$

Since $i(0) = Q$, we get from the equation (4.1.3) the following

$$Q = \frac{\alpha}{\theta}\{e^{\theta(1-\beta)T} - 1\}^{\frac{1}{1-\beta}} \tag{4.1.4}$$

The cost due to deterioration of units in the period [0,T] is given by

$$C_D = c\theta\int_0^T i(t)dt = c\theta\int_0^T [\frac{\alpha}{\theta}\{e^{\theta(1-\beta)(T-t)} - 1\}]^{\frac{1}{1-\beta}} dt \tag{4.1.5}$$

Since $0<\theta<1$ and $0<1-\beta<1$, neglecting higher powers of $\theta(1-\beta)(<<1)$ we get

$$C_D = c\theta\frac{1-\beta}{2-\beta}\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}}T^{\frac{2-\beta}{1-\beta}}, \qquad (4.1.6)$$

The inventory holding cost during the interval [0,T] is given by

$$C_H = h\int_0^T i(t)dt = h\int_0^T \frac{\alpha}{\theta}\{e^{\theta(1-\beta)(T-t)}-1\}]^{\frac{1}{1-\beta}}dt$$

$$= h\frac{1-\beta}{2-\beta}\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}}T^{\frac{2-\beta}{1-\beta}}, \text{ (neglecting higher powers of } \theta(1-\beta)(<<1) \qquad (4.1.7)$$

Therefore the average total cost per unit time is given by

$$\text{TC(T)} = \frac{A}{T}+\frac{1}{T}C_D+\frac{1}{T}C_H$$

Substituting the values of C_D and C_H from the expressions (4.1.6) and (4.1.7), we get the following expression

$$\text{TC(T)} = \frac{A}{T} + (c\theta+h)\frac{1-\beta}{2-\beta}\{\alpha(1-\beta)\}^{\frac{1}{1-\beta}}T^{\frac{1}{1-\beta}} \qquad (4.1.8)$$

The necessary condition for the minimization of the average cost $TC(T)$ is

$$\frac{dTC(T)}{dT}=0$$

After little calculation, the optimal replenishment T is expressed as following

$$\text{T}=\{\frac{A(2-\beta)}{c\theta+h}\}^{\frac{1-\beta}{2-\beta}}\{\alpha(1-\beta)\}^{\frac{1}{\beta-2}} \qquad (4.1.9)$$

Which gives the optimal cycle time T=T^* for the crisp model.

For minimum, the sufficient condition $\frac{\text{d}^2\text{TC(T)}}{\text{dT}^2}$ >0 would be satisfied.

The optimal values Q^* of Q and TC^* of TC are obtained by putting the optimal value $T=T^*$ from the expressions (4.1.4) and (4.1.8).

4.2 Fuzzy Model :

Consider

$$\tilde{\alpha} = (\alpha_1,\alpha_2,\alpha_3,\alpha_4),\quad \tilde{\beta} = (\beta_1,\beta_2,\beta_3,\beta_4),\quad \tilde{\theta} = (\theta_1,\theta_2,\theta_3,\theta_4)$$

$$\tilde{h} = (h_1,h_2,h_3,h_4),\ \tilde{c} = (c_1,c_2,c_3,c_4)$$

are as trapezoidal fuzzy numbers. The total cost of the system per unit time in fuzzy sense is given by the following

$$\tilde{TC}(T) = \frac{A}{T} + (\tilde{c}\tilde{\theta}+\tilde{h})\frac{1-\tilde{\beta}}{2-\tilde{\beta}}\{\tilde{\alpha}(1-\tilde{\beta})\}^{\frac{1}{1-\tilde{\beta}}}T^{\frac{1}{1-\tilde{\beta}}} \qquad (4.2.1)$$

We defuzzify the fuzzy total cost $\tilde{TC}(T)$ using the following two methods:

1) Graded Mean Integration Method (GM)
2) Signed Distance Method (SD).

4.2.1 ***Graded Mean Integration Method (GM)*** :

By Graded Mean Integration Method, the total cost is given by

$$\tilde{TC}_{GM}(T) = \frac{1}{6}[\tilde{TC}_{GM1}(T) + 2\tilde{TC}_{GM2}(T) + 2\tilde{TC}_{GM3}(T) + \tilde{TC}_{GM4}(T)] \qquad (4.2.1.1)$$

where $\tilde{TC}_{GM1}(T) = \frac{A}{T} + (c_1\theta_1 + h_1)\frac{1-\beta_1}{2-\beta_1}\{\alpha_1(1-\beta_1)\}^{\frac{1}{1-\beta_1}} T^{\frac{1}{1-\beta_1}}$

$$\tilde{TC}_{GM2}(T) = \frac{A}{T} + (c_2\theta_2 + h_2)\frac{1-\beta_2}{2-\beta_2}\{\alpha_2(1-\beta_2)\}^{\frac{1}{1-\beta_2}} T^{\frac{1}{1-\beta_2}}$$

$$\tilde{TC}_{GM3}(T) = \frac{A}{T} + (c_3\theta_3 + h_1)\frac{1-\beta_3}{2-\beta_3}\{\alpha_3(1-\beta_3)\}^{\frac{1}{1-\beta_3}} T^{\frac{1}{1-\beta_3}}$$

And $\tilde{TC}_{GM4}(T) = \frac{A}{T} + (c_4\theta_4 + h_4)\frac{1-\beta_4}{2-\beta_4}\{\alpha_4(1-\beta_4)\}^{\frac{1}{1-\beta_4}} T^{\frac{1}{1-\beta_4}}$

From the equation (4.2.1.1), we get

$$\begin{aligned}\tilde{TC}_{GM}(T) = \frac{A}{T} &+ \frac{1}{6}(c_1\theta_1 + h_1)\frac{1-\beta_1}{2-\beta_1}\{\alpha_1(1-\beta_1)\}^{\frac{1}{1-\beta_1}} T^{\frac{1}{1-\beta_1}} \\ &+ \frac{1}{3}(c_2\theta_2 + h_2)\frac{1-\beta_2}{2-\beta_2}\{\alpha_2(1-\beta_2)\}^{\frac{1}{1-\beta_2}} T^{\frac{1}{1-\beta_2}} \\ &+ \frac{1}{3}(c_3\theta_3 + h_1)\frac{1-\beta_3}{2-\beta_3}\{\alpha_3(1-\beta_3)\}^{\frac{1}{1-\beta_3}} T^{\frac{1}{1-\beta_3}} \\ &+ \frac{1}{6}(c_4\theta_4 + h_4)\frac{1-\beta_4}{2-\beta_4}\{\alpha_4(1-\beta_4)\}^{\frac{1}{1-\beta_4}} T^{\frac{1}{1-\beta_4}} \qquad (4.2.1.2)\end{aligned}$$

The necessary condition for the minimization of the average cost $\tilde{TC}_{GM}(T)$ is

$$\frac{d\tilde{TC}_{GM}(T)}{dT} = 0$$

Or, $$-A + \frac{1}{6}\frac{c_1\theta_1 + h_1}{2-\beta_1}\{\alpha_1(1-\beta_1)\}^{\frac{1}{1-\beta_1}} T^{\frac{2-\beta_1}{1-\beta_1}} + \frac{1}{3}\frac{c_2\theta_2 + h_2}{2-\beta_2}\{\alpha_2(1-\beta_2)\}^{\frac{1}{1-\beta_2}} T^{\frac{2-\beta_2}{1-\beta_2}}$$

$$+ \frac{1}{3}\frac{c_3\theta_3 + h_1}{2-\beta_3}\{\alpha_3(1-\beta_3)\}^{\frac{1}{1-\beta_3}} T^{\frac{2-\beta_3}{1-\beta_3}} + \frac{1}{6}\frac{c_4\theta_4 + h_4}{2-\beta_4}\{\alpha_4(1-\beta_4)\}^{\frac{1}{1-\beta_4}} T^{\frac{2-\beta_4}{1-\beta_4}} = 0 \qquad (4.2.1.3)$$

Which gives the optimum values of T.

$\tilde{TC}_{GM}(T)$ is minimum only if $\frac{d^2\tilde{TC}_{GM}(T)}{dT^2}$ >0 would be satisfied for T>0.

The optimal total cost $\tilde{TC}^{*}_{GM}(T)$ is obtained by putting the optimal value T in the equation (4.2.1.2).

4.2.2 *Signed Distance Method (SD):*

By Signed Distance Method, the total cost is given by

$$\tilde{TC}_{SD}(T) = \frac{1}{4}[\tilde{TC}_{SD1}(T) + \tilde{TC}_{SD2}(T) + \tilde{TC}_{SD3}(T) + \tilde{TC}_{SD4}(T)] \tag{4.2.2.1}$$

where $\tilde{TC}_{SD1}(T) = \frac{A}{T} + (c_1\theta_1 + h_1)\frac{1-\beta_1}{2-\beta_1}\{\alpha_1(1-\beta_1)\}^{\frac{1}{1-\beta_1}} T^{\frac{1}{1-\beta_1}}$

$$\tilde{TC}_{SD2}(T) = \frac{A}{T} + (c_2\theta_2 + h_2)\frac{1-\beta_2}{2-\beta_2}\{\alpha_2(1-\beta_2)\}^{\frac{1}{1-\beta_2}} T^{\frac{1}{1-\beta_2}}$$

$$\tilde{TC}_{SD3}(T) = \frac{A}{T} + (c_3\theta_3 + h_1)\frac{1-\beta_3}{2-\beta_3}\{\alpha_3(1-\beta_3)\}^{\frac{1}{1-\beta_3}} T^{\frac{1}{1-\beta_3}}$$

And $\tilde{TC}_{SD4}(T) = \frac{A}{T} + (c_4\theta_4 + h_4)\frac{1-\beta_4}{2-\beta_4}\{\alpha_4(1-\beta_4)\}^{\frac{1}{1-\beta_4}} T^{\frac{1}{1-\beta_4}}$

From the equation (4.2.2.1), we get

$$\begin{aligned}\tilde{TC}_{SD}(T) = \frac{A}{T} &+ \frac{1}{4}(c_1\theta_1 + h_1)\frac{1-\beta_1}{2-\beta_1}\{\alpha_1(1-\beta_1)\}^{\frac{1}{1-\beta_1}} T^{\frac{1}{1-\beta_1}} \\ &+ \frac{1}{4}(c_2\theta_2 + h_2)\frac{1-\beta_2}{2-\beta_2}\{\alpha_2(1-\beta_2)\}^{\frac{1}{1-\beta_2}} T^{\frac{1}{1-\beta_2}} \\ &+ \frac{1}{4}(c_3\theta_3 + h_1)\frac{1-\beta_3}{2-\beta_3}\{\alpha_3(1-\beta_3)\}^{\frac{1}{1-\beta_3}} T^{\frac{1}{1-\beta_3}} \\ &+ \frac{1}{4}(c_4\theta_4 + h_4)\frac{1-\beta_4}{2-\beta_4}\{\alpha_4(1-\beta_4)\}^{\frac{1}{1-\beta_4}} T^{\frac{1}{1-\beta_4}}\end{aligned} \tag{4.2.2.2}$$

The necessary condition for the minimization of the average cost $\tilde{TC}_{SD}(T)$ is

$$\frac{d\tilde{TC}_{SD}(T)}{dT} = 0$$

Or,

$$\begin{aligned}-4A &+ \frac{c_1\theta_1 + h_1}{2-\beta_1}\{\alpha_1(1-\beta_1)\}^{\frac{1}{1-\beta_1}} T^{\frac{2-\beta_1}{1-\beta_1}} + \frac{c_2\theta_2 + h_2}{2-\beta_2}\{\alpha_2(1-\beta_2)\}^{\frac{1}{1-\beta_2}} T^{\frac{2-\beta_2}{1-\beta_2}} \\ &+ \frac{c_3\theta_3 + h_1}{2-\beta_3}\{\alpha_3(1-\beta_3)\}^{\frac{1}{1-\beta_3}} T^{\frac{2-\beta_3}{1-\beta_3}} + \frac{c_4\theta_4 + h_4}{2-\beta_4}\{\alpha_4(1-\beta_4)\}^{\frac{1}{1-\beta_4}} T^{\frac{2-\beta_4}{1-\beta_4}} = 0\end{aligned} \tag{4.2.2.3}$$

Which gives the optimum values of T.

$\tilde{TC}_{GM}(T)$ is minimum only if $\frac{\mathrm{d}^2 \tilde{TC}_{SD}(T)}{\mathrm{dT}^2}$ >0 would be satisfied for T>0.

The optimal total cost $\tilde{TC}^*_{SD}(T)$ is obtained by putting the optimal value T in the equation (4.2.2.2).

5. Numerical Examples:

To illustrate the preceding inventory model, the following examples are considered for the two inventory scenarios namely crisp model and fuzzy model.

Example 1: (Crisp Model):

The values of the parameters be as follows
A=1000 per order; α = 500 units/year; β = 0.01; θ = 0.02; c = \$ 8 per unit; h = \$ 10 per unit.
Solving the equation (4.1.9) with the help of computer using the above parameter values, we find the following optimum outputs

$$T^* = 0.611 \text{ year; } Q^* = 322.45 \text{ units and } TC^* = \$\ 3256.15$$

It is also checked that this solution satisfies the sufficient condition for optimality.

Example 2: (Fuzzy Model):

Consider the fuzzy parameters are
A=1000 per order; $\tilde{\alpha}$ = (400, 500, 600, 700); $\tilde{\beta}$ = (0.01, 0.02, 0.03, 0.04) ; $\tilde{\theta}$ = (0.02, 0.04, 0.06, 0.08) ; $\tilde{c}$ = (6, 8, 10, 12) ; $\tilde{h}$ = (8, 10, 12, 14).
Solving the equations (4.2.1.3) and (4.2.2.3) with the help of computer using the above values of fuzzy parameters, we find the following optimum outputs

Fuzzy Model	T^*	$\tilde{TC}^*$
Graded Mean Integration Method	0.512 year	\$ 3846.22
Signed Distance Method	0.507 year	\$ 3882.77

6. Sensitivity Analysis and Discussion:

We now study the effects of changes in the system fuzzy parameters A, $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\theta}$, $\tilde{c}$ and $\tilde{h}$ on the optimum length of order cycle (T^*) and the optimal total cost (TC^*) in the present inventory fuzzy model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A and table B.

Table A: Effect of changes in the fuzzy parameters for Graded Mean Integration Method:

Changing parameter	% change in the system parameter	Optimum values of	
		T^*	$TC^*_{GM}(T)$
A	-50	0.364	2705.92
	-20	0.459	3434.54
	+20	0.561	4218.95
	+50	0.626	4724.65
$\tilde{\alpha}$	-50	0.728	2705.92
	-20	0.574	3426.40
	+20	0.467	4218.95
	+50	0.417	4724.65
$\tilde{\beta}$	-50	0.533	3726.54
	-20	0.521	3797.58
	+20	0.504	3895.90
	+50	0.492	3972.47
$\tilde{\theta}$	-50	0.518	3802.75
	-20	0.515	3828.89
	+20	0.510	3863.46
	+50	0.507	3843.24
$\tilde{c}$	-50	0.518	3802.75
	-20	0.515	3830.75
	+20	0.510	3863.46
	+50	0.506	3889.18
$\tilde{h}$	-50	0.705	2794.32
	-20	0.587	3466.81
	+20	0.470	4191.91
	+50	0.428	4661.38

Table B: Effect of changes in the fuzzy parameters for Signed Distance Method:

Changing parameter	% change in the system parameter	Optimum values of	
		T^*	$TC^*_{GM}(T)$
A	-50	0.361	2731.00
	-20	0.455	3466.92
	+20	0.555	4259.30
	+50	0.619	4770.19
$\tilde{\alpha}$	-50	0.721	2731.00
	-20	0.568	3466.92
	+20	0.462	4259.30
	+50	0.413	4770.19
$\tilde{\beta}$	-50	0.529	3755.86
	-20	0.516	3831.13
	+20	0.498	3935.61
	+50	0.475	4018.42

$\tilde{\theta}$	-50 -20 +20 +50	0.513 0.510 0.505 0.501	3836.84 3864.46 3900.98 3928.13
$\tilde{c}$	-50 -20 +20 +50	0.513 0.510 0.505 0.501	3836.84 3865.95 3900.98 3928.13
$\tilde{h}$	-50 -20 +20 +50	0.697 0.563 0.466 0.419	2824.35 3499.18 4230.75 4703.38

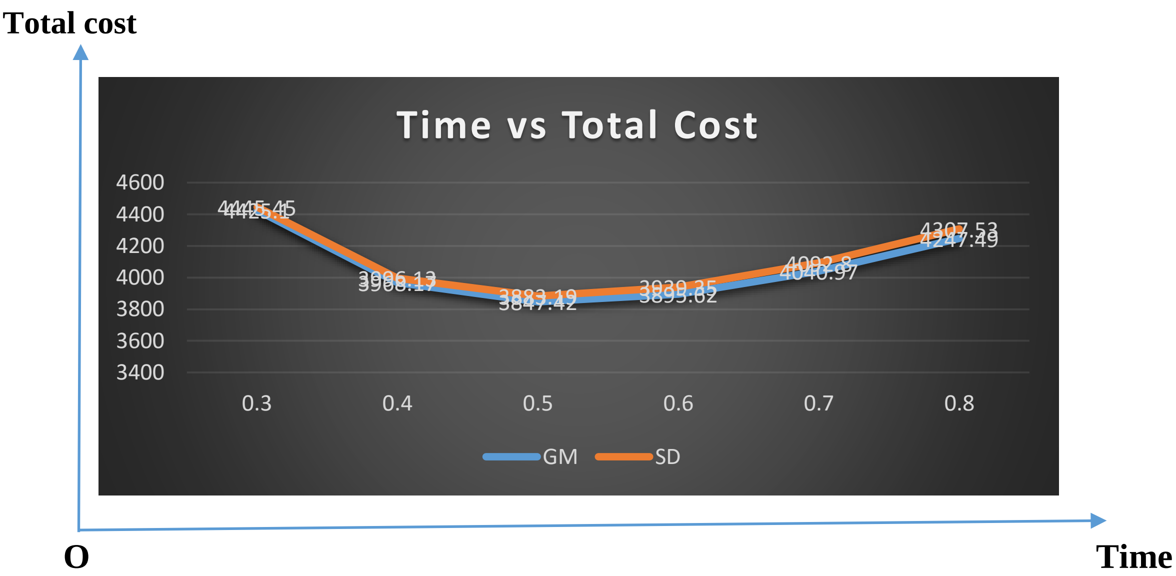
Analyzing the results of table A and table B, the following observations may be made:

(i) The optimum average cost TC^* increase or decrease with the increase or decrease in the values of the system parameters A. On the other hand TC^* increase or decrease with the decrease or increase in the values of the system parameters $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\theta}$, $\tilde{c}$ and $\tilde{h}$. The results obtained show that TC^* is very highly sensitive to changes in the value of parameters A, $\tilde{\alpha}$ and $\tilde{h}$; and moderate sensitive towards changes of parameters $\tilde{\beta}$, $\tilde{\theta}$ and $\tilde{c}$.

From the above analysis, it is seen that A, $\tilde{\alpha}$ and $\tilde{h}$ are very sensitive parameters in the sense that any error in the estimation of these parameters result in significant errors in the optimal solution. Hence estimation of the parameters A, $\tilde{\alpha}$ and $\tilde{h}$ need adequate attention.

7. Graphical representation :

A graphical presentation is furnished to compare the total costs under the above two mentioned methods in the fuzzy model



The above figure shows the comparison between the cycle time (T) and the total cost (TC) under the Graded Mean Integration Method and Signed Distance Method. It is observed that as the time (T) increases, the total cost decreases gradually up to a certain cycle period, after that it increases gradually. So the minimum cost attains at point of replenishment time, and it indicates that the proposed fuzzy model plays a significant and realistic behaviour in the present inventory model.

8. Concluding Remarks:

In the present paper, deteriorating inventory model with power demand and stock-dependent demand rate is developed. The holding cost is assumed as inventory dependent. Both crisp and fuzzy models are developed. We defuzzify the fuzzy total cost $\tilde{TC}(T)$ with the help of two methods i) **Graded Mean Integration Method and ii) Signed Distance Method.** Numerical examples in each model are given to illustrate the models. Sensitivity analysis is furnished for the fuzzy model. Lastly graphical behaviour is also discussed under the Graded Mean Integration Method and Signed Distance Method. We may extend this model by assuming the demand as quadratic function of time and shortages with fully or partially backlogged. No such weakness was found in the present paper.

Acknowledgement

The author is thankful to the referee for giving suggestions in improving the present paper

References

[1]. Ghare P.M.and Schrader (1963) " A model for exponentially decaying inventories", J. Ind. Eng., 14, pp. 238-243

[2]. Covert R.P. and Philip G.C. (1973) : "An EOQ model for items with Weibull distribution deterioration", AIIE Trans., 5, pp. 323-326.

[3]. Hwang H. and Hahn K. H.(2000) : "An optimal procurement policy for item with an inventory level dependent demand rate and fixed life time", Eur. J. Oper. Res, 127, pp. 537-545.

[4]. Ghiami Y, , Williams T. and Wu Y (2013): " A two-echelon inventory model for a deteriorating item with stock-dependent demand, partial backlogging and capacity constraints", Eur. J. Oper. Res., 231, pp. 587-597.

[5]. Rajoria Y.K., Saini S. and Singh S. R.(2015) : "EOQ model for deteriorating items with power demand, partial backlogging and inflation", Int. J. Appl. Eng. Res., 10(9), pp. 22861-22873.

[6]. Dr. Biswaranjan Mandal (2021) "Order level inventory model for perishable items with power demand and stock-dependent demand under variable cost functions", International Journal of Applied Mathematics and Statistical Sciences (IJAMSS), ISSN (P): 2319–3972; ISSN (E): 2319–3980, Vol. 10, Issue 2, pp. 139–150

[7]. Levin R. I., Mclaughlis C.P. and Kottas R. P. (1972) "Production Operation Management: Contemporary policy for managing operating system", 1st Edn. McGraw Hill, New York,, pp.373

[8]. T. K. Datta and A K Pal, "A note on inventory model with inventory level dependent demand rate", J. Oper. Res. Soc., 41, 1990, pp. 971-975.

[9]. Baker R.C. and Urban T. L. (1988) “An inventory model with time dependent demand, Weibull-distributed deterioration under salvage value and shortages”, IOSR Journal of Business and Management, 22(9), pp.59-64.

[10]. R. P. Tripathi, D. SINGH and A Surbhi, “Inventory models for stock-dependent demand and time varying holding cost under different trade credits”, Yugoslav Journal of Operations Research, DOI: https://doi.org/10.2298/YJOR160317018T, 2017, pp. 1-13.

[11]. Bellan R. E. and Zadeh L. A. (1970) : “ Decision-making in a fuzzy environment”, Mgmt. Sci., 17(4), pp. 141-164.

[12]. Yao J. S. and Lee H. M. (1999) : “ Fuzzy inventory with or without backorder for fuzzy order quantity with trapezoidal fuzzy number”, Fuzzy sets and systems., 105, pp. 311-337.

[13]. Yao J. S. and Chiang J. (2003) : “ Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance”, Eur. J. Oper. Res., 148, pp. 401-409.

[14]. Vujosevic M. and Petrovic D. (1996) : “ EOQ formula when inventory cost is fuzzy”, Int. J. Prod. Eco., 45, pp. 499-504.

[15]. Hsieh C. H. (2002) : “ Optimization of fuzzy production inventory models”, Information Sciences., 146, pp. 29-40

[16]. Chen S. H. (1985) : “ Operations on fuzzy numbers with function principle”, Tamkang J. of Mgmt. Sci., 6(1), pp. 13-25.

[17]. Park K.S. (1987) : “ Fuzzy set theoretic interpretation of economic order quantity”, IIIE Trans. On systems, Man and Cybernetics, 17, pp. 1082-1084

[18]. Kao C. and Hsu W. K. (2002) : “ Lot size-reorder point inventory model with fuzzy demands,”, Comp. and Math with Application, 43), pp. 1291-1302.

[19]. Sujit Kumar De (2021) : “ Solving an EOQ model under fuzzy reasoning”, Appl. Soft Computing, 99, https://doi.org/10.1016/j.asoc.2020.106892.

[20]. Zadeh (1965) : “ Fuzzy sets”, Information and Control, 8(3), pp. 338-353.