DEPENDENCE OF CHARGE CARRIERS MOBILITY IN THE *P-N*-HETEROJUNCTIONS ON COMPOSITION OF MULTI-LAYER STRUCTURE

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ABSTRACT

In this paper we consider manufacturing a p-n-junctions by dopant diffusion or ion implantation into a multilayer structure. We introduce an approach to increase sharpness of these p-n-junctions and at the same time to increase homogeneity of distributions of dopants in enriched by these dopants areas. We consider influence of the above changing of distribution of dopant on charge carrier mobility. We also consider an approach to decrease value of mismatch-induced stress in the considered multilayer structure by using a buffer layer. The decreasing gives a possibility to increase value of charge carrier mobility.

KEYWORDS:

Multilayer structure; charge carriers mobility; p-n-heterojunctions

1. INTRODUCTION

In the present time one of the actual questions is increasing of performance of solid state electronic devices (diodes, field-effect and bipolar transistors, ...) [1-6]. To increase the performance it is attracted an interest searching of materials with higher values of the charge carrier mobility [7-10]. An alternative approach to increase the performance is development of new technological processes and optimization of existing one. Framework this paper we introduce an approach of manufacturing of heterodiodes. The approach gives a possibility to decrease of switching time. At the same using the approach gives a possibility to decrease dimensions of these diodes. With decreasing of switching time and dimensions of these diodes value of mismatch- induced stress also decreases.

In this paper we consider a heterostructure on Fig. 1. The heterostructure includes into itself a substrate and an epitaxial layer. A buffer layer has been grown between these substrate and epitaxial layer. We consider doping of the epitaxial layer by diffusion or ion implantation after manufacturing the heterostructure. The doping gives a possibility to produce required type of conductivity (*p* or *n*). We assume, that type of conductivity of substrate is known: *p* or *n*. After doping of the epitaxial layer optimized annealing of dopant and/or radiation defects has been considered. In this paper we analyzed possibility to increase sharpness of *p*-*n*- junction with increasing of homogeneity of concentration of dopant in enriched by the dopant area. Framework the paper we analyzed influence of changing of distribution of concentration of dopant in space and time on charge carrier mobility with account mismatch-induced stress in the considered heterostructure.

2. METHOD OF SOLUTION

To analyze influence of changing of distribution of concentration of dopant in space and time on charge carrier mobility we solve the second Fick's law in the following form [1,11-15]

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+ \quad (1)$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}C(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}C(x,y,W,t)\,dW\right].$$

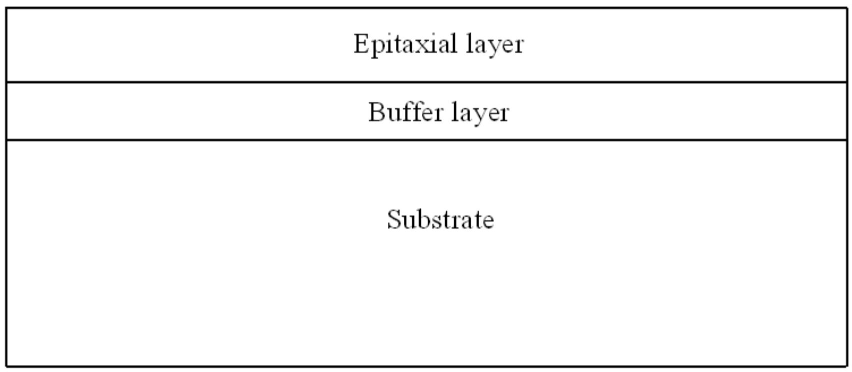

Fig. 1. Heterostructure, which consist of a substrate, epitaxial layers and buffer layer

Boundary and initial conditions for our case could be written as

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ C x,y,z,0)=f_C(x,y,z).$$

The function $C(x,y,z,t)$ describes distribution of concentration of dopant in space and time. Ω is the atomic volume of dopant; ∇_s is the symbol of surficial gradient. The function $\int\limits_0^{L_z}C(x,y,z,t)\,dz$ describes distribution of surficial concentration of dopant in space and time on interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to interface between layers of heterostructure). The function $\mu_1(x,y,z,t)$ describes the chemical potential due to the presence of mismatch-induced stress in space and time. The functions D and D_S describe distributions of coefficients of volumetric and surficial diffusions on coordinate and temperature. Values of dopant diffusions coefficients depends on properties of materials of heterostructure, speed of heating and cooling of materials during annealing and spatio-temporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relations [13-15]

$$D_C = D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right],$$

$$D_S = D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]. \quad (2)$$

Here $D_L(x,y,z,T)$ and $D_{LS}(x,y,z,T)$ are the spatial (due to accounting all layers of heterostruicture) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficients; T is the temperature of annealing. Function $P(x,y,z,T)$ describes spatial and temperature dependence of the limit of solubility of dopant. Parameter $\gamma\in[1,3]$ with integer values depends on properties of materials [13]. The function $V(x,y,z,t)$ describes distribution of concentration of radiation vacancies in space and time; V^* is the equilibrium distribution of vacancies. One can find description of dependence of dopant diffusion coefficient on concentration of dopant in [13]. To determine distributions of concentration of point radiation defects in space and time we solve the following boundary problem [11,12,14,15]

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-k_{I,I}(x,y,z,T)\times$$
$$\times I^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)\,V(x,y,z,t)+ \quad (3)$$
$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-k_{V,V}(x,y,z,T)\times$$
$$\times V^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)\,V(x,y,z,t)+$$
$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0$$

$$I(x,y,z,0)=f_I(x,y,z),\ V(x,y,z,0)=f_V(x,y,z). \quad (4)$$

The function $I(x,y,z,t)$ describes distribution of concentration of radiation interstitials in space and time with the equilibrium distribution I^*. The functions $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, $D_{VS}(x,y,z,T)$ describe dependences of coefficients of volumetric and surficial diffusions of interstitials and vacancies on coordinate and temperature. Terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ gives a possibility to take into account generation of divacancies and diinterstitials [15]. The functions

$k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ describe dependences of parameters of recombination of point radiation defects and generation of their complexes on coordinate and temperature.
We determine spatio-temporal distributions of divacancies $\Phi_V(x,y,z,t)$ and diinterstitials $\Phi_I(x,y,z,t)$ by solving the following boundary problem [11,12,14,15]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)dW\right]+k_I(x,y,z,T)\times$$
$$\times I(x,y,z,t)+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_I(x,y,W,t)dW\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \quad (5)$$
$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)dW\right]+k_V(x,y,z,T)\times$$
$$\times V(x,y,z,t)+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\Phi_V(x,y,W,t)dW\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$
$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$
$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$
$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z). \quad (6)$$

The functions $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ describe dependences of coefficients of volumetric and surficial diffusions of complexes of radiation defects on coordinate and temperature. The functions $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ describe dependences of parameters of decay of complexes of radiation defects on coordinate and temperature.
Chemical potential μ_1 in Eq.(1) could be determine by the following relation [11]

$$\mu_1=E(z)\Omega\sigma_{ij}\,[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2, \quad (7)$$

where $E(z)$ is the Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The Eq. (3) could be transform to the following form

$$\mu\left(x,y,z,t\right)=\left[\frac{\partial u_i\left(x,y,z,t\right)}{\partial x_j}+\frac{\partial u_j\left(x,y,z,t\right)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i\left(x,y,z,t\right)}{\partial x_j}+\frac{\partial u_j\left(x,y,z,t\right)}{\partial x_i}\right]-\right.$$

$$\left.-\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k\left(x,y,z,t\right)}{\partial x_k}-3\varepsilon_0\right]-K\left(z\right)\beta\left(z\right)\left[T\left(x,y,z,t\right)-T_0\right]\delta_{ij}\right\}\frac{\Omega}{2}E\left(z\right),$$

where σ is Poisson coefficient; the mismatch parameter could be determined as $\varepsilon_0=(a_s-a_{EL})/a_{EL}$, where a_s is the lattice distances of the substrate, a_{EL} and the epitaxial layer; K is the modulus of uniform compression; parameter β describes thermal expansion coefficient, parameter T_r describes the equilibrium temperature. The equilibrium temperature coincides in our case with room temperature. Components of displacement vector could be obtained by solution of the following equations [12]

$$\rho\left(z\right)\frac{\partial^2 u_x\left(x,y,z,t\right)}{\partial t^2}=\frac{\partial\sigma_{xx}\left(x,y,z,t\right)}{\partial x}+\frac{\partial\sigma_{xy}\left(x,y,z,t\right)}{\partial y}+\frac{\partial\sigma_{xz}\left(x,y,z,t\right)}{\partial z}$$

$$\rho\left(z\right)\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial t^2}=\frac{\partial\sigma_{yx}\left(x,y,z,t\right)}{\partial x}+\frac{\partial\sigma_{yy}\left(x,y,z,t\right)}{\partial y}+\frac{\partial\sigma_{yz}\left(x,y,z,t\right)}{\partial z}$$

$$\rho\left(z\right)\frac{\partial^2 u_z\left(x,y,z,t\right)}{\partial t^2}=\frac{\partial\sigma_{zx}\left(x,y,z,t\right)}{\partial x}+\frac{\partial\sigma_{zy}\left(x,y,z,t\right)}{\partial y}+\frac{\partial\sigma_{zz}\left(x,y,z,t\right)}{\partial z},$$

where $\sigma_{ij}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_i\left(x,y,z,t\right)}{\partial x_j}+\frac{\partial u_j\left(x,y,z,t\right)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k\left(x,y,z,t\right)}{\partial x_k}\right]+\delta_{ij}\frac{\partial u_k\left(x,y,z,t\right)}{\partial x_k}\times$ $\times K\left(z\right)-\beta\left(z\right)K\left(z\right)\left[T\left(x,y,z,t\right)-T_r\right]$, $\rho\left(z\right)$ is the density of materials of heterostructure, δ_{ij} is the Kronecker symbol. With account the relation for σ_{ij} last system of equation could be written as

$$\rho(z)\frac{\partial^2 u_x\left(x,y,z,t\right)}{\partial t^2}=\left\{K\left(z\right)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_x\left(x,y,z,t\right)}{\partial x^2}+\left\{K\left(z\right)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial x\partial y}+$$

$$+\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial y^2}+\frac{\partial^2 u_z\left(x,y,z,t\right)}{\partial z^2}\right]+\left[K\left(z\right)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right]\frac{\partial^2 u_z\left(x,y,z,t\right)}{\partial x\partial z}-$$

$$-K\left(z\right)\beta\left(z\right)\frac{\partial T\left(x,y,z,t\right)}{\partial x}$$

$$\rho(z)\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial x^2}+\frac{\partial^2 u_x\left(x,y,z,t\right)}{\partial x\partial y}\right]-K\left(z\right)\beta\left(z\right)\frac{\partial T\left(x,y,z,t\right)}{\partial y}+$$

$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_y\left(x,y,z,t\right)}{\partial z}+\frac{\partial u_z\left(x,y,z,t\right)}{\partial y}\right]\right\}+\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial y^2}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K\left(z\right)\right\}+$$

$$+\left\{K\left(z\right)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial y\partial z}+K\left(z\right)\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial x\partial y} \qquad (8)$$

$$\rho(z)\frac{\partial^2 u_z\left(x,y,z,t\right)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2 u_z\left(x,y,z,t\right)}{\partial x^2}+\frac{\partial^2 u_z\left(x,y,z,t\right)}{\partial y^2}+\frac{\partial^2 u_x\left(x,y,z,t\right)}{\partial x\partial z}+\frac{\partial^2 u_y\left(x,y,z,t\right)}{\partial y\partial z}\right]+$$

$$+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}+$$

$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}.$$

Conditions for the system of Eq. (8) could be written in the form

$$\frac{\partial \vec{u}(0,y,z,t)}{\partial x}=0;\ \frac{\partial \vec{u}(L_x,y,z,t)}{\partial x}=0;\ \frac{\partial \vec{u}(x,0,z,t)}{\partial y}=0;\ \frac{\partial \vec{u}(x,L_y,z,t)}{\partial y}=0;$$

$$\frac{\partial \vec{u}(x,y,0,t)}{\partial z}=0;\ \frac{\partial \vec{u}(x,y,L_z,t)}{\partial z}=0;\ \vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0.$$

We solve the Eqs.(1), (3) and (5) by using standard method of averaging of function corrections [17]. Previously we transform the Eqs.(1), (3) and (5) to the following form with account initial distributions of the considered concentrations

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+ \quad (1a)$$

$$+f_C(x,y,z)\delta(t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]$$

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+f_I(x,y,z)\delta(t)+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t) \quad (3a)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+f_V(x,y,z)\delta(t)+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t) \qquad (5a)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t).$$

Farther we replace concentrations of dopant and radiation defects in right sides of Eqs. (1*a*), (3*a*) and (5*a*) on their average values $\alpha_{1\rho}$. The average values $\alpha_{1\rho}$ are not yet known. In this situation we obtain equations for the first-order approximations of the required concentrations in the following form

$$\frac{\partial C_1(x,y,z,t)}{\partial t}=\alpha_{1C}\Omega\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+f_C(x,y,z)\delta(t) \qquad (1b)$$

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+$$

$$+f_I(x,y,z)\delta(t)-\alpha_{1I}^2k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T) \qquad (3b)$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_V(x,y,z)\delta(t)-\alpha_{1V}^2k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_{\Phi_I}(x,y,z)\delta(t)+k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \qquad (5b)$$

$$\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_{\Phi_V}(x,y,z)\delta(t)+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t).$$

Integration of the left and right sides of the Eqs. (1*b*), (3*b*) and (5*b*) on time gives us possibility to obtain relations for above approximation in the final form

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int_0^t D_{SL}(x,y,z,T)\,\nabla_S\mu_1(x,y,z,\tau)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$
$$\times\frac{z}{kT}\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]d\tau+\alpha_{1C}\Omega\frac{\partial}{\partial y}\int_0^t D_{SL}(x,y,z,T)\nabla_S\mu_1(x,y,z,\tau)\frac{z}{kT}\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]\times$$
$$\times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]d\tau+f_C(x,y,z) \qquad (1c)$$

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1I}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$
$$+f_I(x,y,z)-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau \qquad (3c)$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1V}z\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$
$$+f_V(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$
$$+f_{\Phi_I}(x,y,z)+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)d\tau \qquad (5c)$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$
$$+f_{\Phi_V}(x,y,z)+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)d\tau\,.$$

We determine average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation [17]

$$\alpha_{1\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t\,. \qquad (9)$$

We obtain required average values by substitution of the relations (1*c*), (3*c*) and (5*c*) into relation (9). The obtained relations could be written as

$$\alpha_{1C}=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)d\,z\,d\,y\,d\,x,\;\;\alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+\Theta^2L_xL_yL_za_1}{a_4}\right)}-\frac{a_3+A}{4a_4},$$
$$\alpha_{1V}=\frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x-\alpha_{1I}S_{II00}-\Theta L_xL_yL_z\right].$$

Here $S_{\rho\rho ij}=\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{\rho,\rho}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t$, $a_4=\left(S_{IV00}^2-S_{II00}S_{VV00}\right)\times$

$$\times S_{II00},\ a_3 = S_{IV00}S_{II00} + S_{IV00}^2 - S_{II00}S_{VV00},\ a_2 = S_{IV00}S_{IV00}^2 \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_V(x,y,z)\,d\,z\,d\,y\,d\,x + \Theta L_x^2 L_y^2 L_z^2 \times$$
$$\times S_{IV00} + 2S_{VV00}S_{II00}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x - \Theta L_x^2 L_y^2 L_z^2 S_{VV00} - \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x \times$$
$$\times S_{IV00}^2,\ a_1 = S_{IV00}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x,\ a_0 = S_{VV00}\left[\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,d\,z\,d\,y\,d\,x\right]^2,$$

$$A = \sqrt{8y + \Theta^2\frac{a_3^2}{a_4^2} - 4\Theta\frac{a_2}{a_4}},\ B = \frac{\Theta a_2}{6a_4} + \sqrt[3]{\sqrt{q^2+p^3} - q} - \sqrt[3]{\sqrt{q^2+p^3} + q},\ q = \left(4a_0 - \Theta L_x L_y L_z\frac{a_1 a_3}{a_4}\right)\times$$
$$\times\frac{\Theta^3 a_2}{24a_4^2} - \Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2 - \Theta^2\frac{a_3^2}{a_4}\right) - \frac{\Theta^3 a_2^3}{54a_4^3} - L_x^2 L_y^2 L_z^2\frac{\Theta^4 a_1^2}{8a_4^2},\ p = \Theta^2\frac{4a_0 a_4 - \Theta L_x L_y L_z a_1 a_3}{12a_4^2} -$$
$$-\Theta a_2/18a_4,$$

$$\alpha_{1\Phi_I} = \frac{R_{I1}}{\Theta L_x L_y L_z} + \frac{S_{II20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_{\Phi_I}(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$\alpha_{1\Phi_V} = \frac{R_{V1}}{\Theta L_x L_y L_z} + \frac{S_{VV20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_{\Phi_V}(x,y,z)\,d\,z\,d\,y\,d\,x,$$

Here $R_{\rho i} = \int\limits_0^{\Theta}(\Theta - t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} k_I(x,y,z,T)\,I_1^i(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$.

The second-order approximations and approximations with higher orders of the considered concentrations have been calculated framework standard iterative procedure of method of averaging of function corrections [17]. The considered approximations could be obtained by replacement the required concentrations in the Eqs. (1*c*), (3*c*), (5*c*) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to following results

$$\frac{\partial C_2(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left(\left\{1 + \xi\frac{[\alpha_{2C} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\left[1 + \varsigma_1\frac{V(x,y,z,t)}{V^*} + \varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$
$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\left[1 + \varsigma_1\frac{V(x,y,z,t)}{V^*} + \varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,t)}{\partial y}\times\right.$$
$$\left.\times D_L(x,y,z,T)\left\{1 + \xi\frac{[\alpha_{2C} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right) + \frac{\partial}{\partial z}\left(\left[1 + \varsigma_1\frac{V(x,y,z,t)}{V^*} + \varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$
$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\left\{1 + \xi\frac{[\alpha_{2C} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right) + f_C(x,y,z)\delta(t) +$$
$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2C} + C(x,y,W,t)]\,dW\right\} +$$
$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}[\alpha_{2C} + C(x,y,W,t)]\,dW\right\} \quad (1d)$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)]^2-k_{I,V}(x,y,z,T)\times$$
$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\times\right.$$
$$\left.\times\frac{D_{IS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\right\} \tag{3d}$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)[\alpha_{1V}+V_1(x,y,z,t)]^2-k_{I,V}(x,y,z,T)\times$$
$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\times\right.$$
$$\left.\times\frac{D_{VS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\right\}$$

$$\frac{\partial\Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial y}\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$
$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_I(x,y,z,T)I(x,y,z,t)+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,t)}{\partial z}\right]+f_{\Phi_I}(x,y,z)\delta(t) \tag{5d}$$

$$\frac{\partial\Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial y}\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$
$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_V(x,y,z,T)V(x,y,z,t)+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,t)}{\partial z}\right]+f_{\Phi_V}(x,y,z)\delta(t).$$

Final relations for the second-order approximations could be obtained by integration of the left and the right sides of Eqs. (1*d*), (3*d*) and (5*d*). The final relations could be written as

$$C_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times\frac{\partial C_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,\tau)}{\partial y}\times$$

$$\times\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}+\frac{\partial}{\partial z}\int_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,\tau)}{\partial z}\times$$

$$\times D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}[\alpha_{2C}+C_1(x,y,W,\tau)]\times$$

$$\times\frac{D_S}{kT}dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_S}{kT}\int_0^{L_z}[\alpha_{2C}+C_1(x,y,W,\tau)]\,dW\,d\tau+f_C(x,y,z)\quad(1e)$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau+f_I(x,y,z)-\int_0^t k_{I,I}(x,y,z,T)\,[\alpha_{2I}+I_1(x,y,z,\tau)]^2\,d\tau-$$

$$-\int_0^t k_{I,V}(x,y,z,T)\,[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,\tau)\times$$

$$\times\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,\tau)]\,dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{IS}}{kT}\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,\tau)]\,dW\,d\tau\quad(3e)$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau+f_V(x,y,z)-\int_0^t k_{V,V}(x,y,z,T)\,[\alpha_{2V}+V_1(x,y,z,\tau)]^2\,d\tau-$$

$$-\int_0^t k_{I,V}(x,y,z,T)\,[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,\tau)\times$$

$$\times\int_0^{L_z}[\alpha_{2I}+V_1(x,y,W,\tau)]\,dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{VS}}{kT}\int_0^{L_z}[\alpha_{2I}+V_1(x,y,W,\tau)]\,dW\,d\tau$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial y}\times$$

$$\times D_{\Phi_I}(x,y,z,T)d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_I S}}{kT}\times$$

$$\times\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\times$$

$$\times\frac{D_{\Phi_I S}}{kT}d\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)\,d\tau+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau+f_{\Phi_I}(x,y,z)\quad(5e)$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial y}\times$$

$$\times D_{\Phi_V}(x,y,z,T)\,d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial z}\,d\tau+\Omega\frac{\partial}{\partial x}\int_0^t \nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_V S}}{kT}\times$$

$$\times\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t \nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\times$$

$$\times\frac{D_{\Phi_V S}}{kT}\,d\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\tau+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau+f_{\Phi_V}(x,y,z).$$

Average values of the second-order approximations of required approximations by using the following standard relation [17]

$$\alpha_{2\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]dz\,dy\,dx\,dt. \qquad (10)$$

Substitution of the relations (1*e*), (3*e*), (5*e*) into relation (10) gives us possibility to obtain relations for required average values $\alpha_{2\rho}$

$$\alpha_{2C}=0,\ \alpha_{2\Phi I}=0,\ \alpha_{2\Phi V}=0,\ \alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}},$$

where $b_4=\frac{1}{\Theta L_xL_yL_z}S_{IV00}^2S_{VV00}-\frac{1}{\Theta L_xL_yL_z}S_{VV00}^2S_{II00}$, $b_3=-\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)+$

$+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+S_{IV01}+\Theta L_xL_yL_z\right)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-\frac{S_{IV00}^2S_{IV10}}{\Theta^3L_x^3L_y^3L_z^3}$,

$b_2=\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{VV02}+S_{IV11}+C_V\right)-\left(S_{IV10}-2S_{VV01}+\Theta L_xL_yL_z\right)^2+\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\times$

$\times\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}+\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+2S_{IV01}+\Theta L_xL_yL_z\right)^2+\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}+$

$+\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+2S_{IV01}+\Theta L_xL_yL_z\right)\left(2S_{VV01}+\Theta L_xL_yL_z+S_{IV10}\right)-\left(C_V-S_{VV02}-S_{IV11}\right)\times$

$\times\frac{S_{IV00}^2}{\Theta L_xL_yL_z}+\frac{C_IS_{IV00}^2}{\Theta^2L_x^2L_y^2L_z^2}-2S_{IV01}\frac{S_{IV10}S_{IV00}}{\Theta L_xL_yL_z}$, $b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\right.$

$\left.+\Theta L_xL_yL_z\right)+\frac{S_{IV01}}{\Theta L_xL_yL_z}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\left(2S_{VV01}+S_{IV10}+\Theta LL_yL_z\right)-\frac{S_{IV10}S_{IV01}^2}{\Theta L_xL_yL_z}-$

$-\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(3S_{IV01}+2S_{II10}+\Theta L_xL_yL_z\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_IS_{IV00}S_{IV01}$, $b_0=\frac{S_{II00}}{\Theta L_xL_yL_z}\times$

$\times\left(S_{IV00}+S_{VV02}\right)^2-\frac{S_{IV01}}{L_xL_yL_z\Theta}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\left(C_V-S_{VV02}-S_{IV11}\right)-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\times$

$\times S_{IV01}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)+2C_IS_{IV01}^2$, $C_I=\frac{\alpha_{1I}\alpha_{1V}}{\Theta L_xL_yL_z}S_{IV00}+\frac{\alpha_{1I}^2S_{II00}}{\Theta L_xL_yL_z}-\frac{S_{II20}S_{II20}}{\Theta L_xL_yL_z}-$,

$-\frac{S_{IV11}}{\Theta L_xL_yL_z}$, $C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2S_{VV00}-S_{VV02}-S_{IV11}$, $E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}$, $F=\frac{\Theta a_2}{6a_4}+$

$$+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r}\ ,\ r=\frac{\Theta^3 b_2}{24b_4^2}\left(4b_0-\Theta L_x L_y L_z\frac{b_1 b_3}{b_4}\right)-\frac{\Theta^3 b_2^3}{54b_4^3}-\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)\times$$
$$\times b_0\frac{\Theta^2}{8b_4^2}-L_x^2L_y^2L_z^2\frac{\Theta^4 b_1^2}{8b_4^2}\ ,\ s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}\ .$$

Farther we determine solutions of Eqs.(8), i.e. components of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections we replace the required functions in the right sides of the equations by their not yet known average values α_i. The substitution leads to the following result

$$\rho(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},\ \rho(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},$$
$$\rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of the left and the right sides of the above relations on time t leads to the following result

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta,$$
$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta,$$
$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta.$$

Approximations of the second and higher orders of components of displacement vector could be determined by using standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$ [17]. The replacement leads to the following result

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}+$$
$$\times\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}+$$
$$+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}+$$
$$+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+$$
$$+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\right]\times$$

$$\times\frac{E(z)}{2\left[1+\sigma(z)\right]}+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}+\frac{E(z)}{6\left[1+\sigma(z)\right]}\times$$

$$\times\frac{\partial}{\partial z}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\Bigg\}-\frac{\partial T(x,y,z,t)}{\partial z}\times$$

$$\times K(z)\beta(z).$$

Integration of the left and right sides of the above relations on time t leads to the following result

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\left[K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right]\times$$

$$\times\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\left[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\times$$

$$\times\frac{E(z)}{2\rho(z)\left[1+\sigma(z)\right]}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}-K(z)\frac{\beta(z)}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{1}{\rho(z)}\times$$

$$\times\left\{K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial z^2}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{2\rho(z)}-\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial x\partial z}\int_0^{\infty}\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\frac{1}{\rho(z)}+u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]+$$

$$\times\frac{1}{1+\sigma(z)}+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+\right.$$

$$\left.+K(z)\right\}\frac{1}{\rho(z)}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}\times$$

$$\times\frac{1}{2\rho(z)}-K(z)\frac{\beta(z)}{\rho(z)}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-\left\{\frac{E(z)}{6\left[1+\sigma(z)\right]}-K(z)\right\}\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\frac{1}{\rho(z)}-\frac{E(z)}{2\rho(z)\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]-K(z)\times$$

$$\times\frac{\beta(z)}{\rho(z)}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^{\infty}\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+\right.$$

$$+K(z)\Bigg\}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\left\{\left[\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\times\right.$$

$$\times\frac{E(z)}{1+\sigma(z)}\Bigg\}\frac{1}{2\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial y\,\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y^2}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial x\,\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y\,\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial}{\partial z}\left\{\left[\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.\right.$$

$$\left.+\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]K(z)\Bigg\}\frac{1}{\rho(z)}+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\times\right.$$

$$\times\left[6\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta} u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta} u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\right.$$

$$\left.\left.-\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0z}.$$

In this paper we calculate distributions of concentration of dopant, concentrations of radiation defects and components of displacement vector in space and time by using the second-order approximation framework method of averaging of function corrections.

This approximation is usually enough good approximation to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.

3. DISCUSSION

Now we analyzed changing concentrations of dopant and radiation defects in space and time in a heterostructure with account mismatch-induced stress. Several spatial distributions of dopant concentration are presented on Fig. 2 for diffusion type of doping and on Fig. 3 for ion type of doping. In the both cases dopant diffusion coefficient in the epitaxial layer is larger, than in the substrate. One can find from these figures, that using in homogeneity of heterostructure leads to increasing of sharpness of *p-n*-junctions. In this situation the accompanying effect is increasing homogeneity of dopant distribution in doped part of heterostructure. Switching time of *p-n-* junction correlated with the junction sharpness and could be increased with increasing of the sharpness. Increasing homogeneity of dopant distribution leads to decreasing local overheat leads to decreasing local heating during functioning of *p-n*-junction or to decrease dimensions of the *p-n*-junction for fixed maximal value of local overheat. However framework this approach of manufacturing of *p-n*-junction and their systems (bipolar transistor and thyristors) it is necessary to optimize annealing of dopant and/or radiation defects. Reason of this optimization is following. If annealing time is small, the dopant did not achieve any interfaces between materials of heterostructure. In this situation one cannot find any modifications of distribution of concentration of dopant. Increasing of annealing time leads to increasing of homogeneity of distribution of concentration of dopant. We optimize annealing time framework recently introduces approach [16,18-24]. To use the criterion one shall to approximate real distribution of concentration of dopant by step-wise function (see Figs. 4 for diffusion doping and 5 for ion doping). The optimal values of annealing time have been calculated by minimization of the following mean-squared error

$$U=\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[C(x,y,z,\Theta)-\psi(x,y,z)\right]d\,z\,d\,y\,d\,x\,. \tag{15}$$

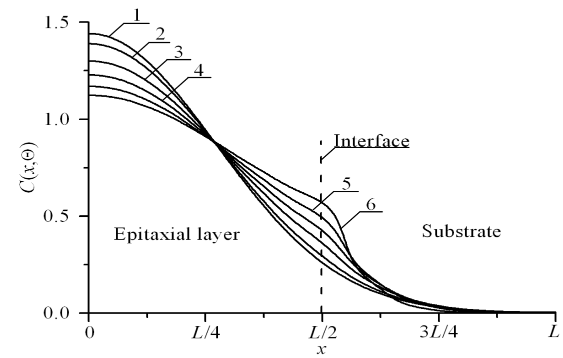

Fig.2. Distributions of concentration dopant in heterostructure from Fig. 1 for diffusion type of doping. Direction of diffusion is perpendicular to interface between layers of heterostructure. Dopant diffusion coefficient in the epitaxial layer is larger, than in the substrate and increasing with increasing of number of curves.

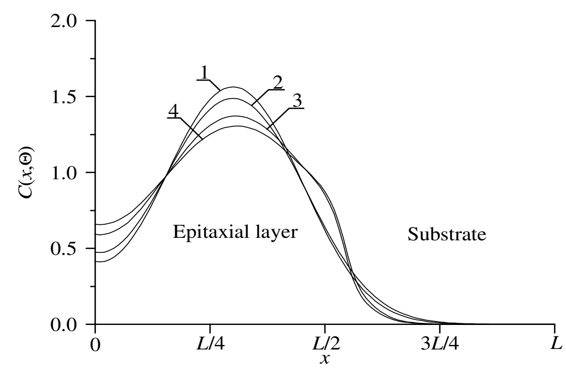

Fig.3. Distributions of concentration dopant in heterostructure from Fig. 1 for ion type of doping. Direction of diffusion is perpendicular to interface between layers of heterostructure. Curves 1 and 3 corresponds to annealing time $\Theta=0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 corresponds to annealing time $\Theta=0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 are the distributions of concentrations of dopant in homogenous sample. Curves 3 and 4 are the distributions of concentrations of dopant in the heterostructure. Dopant diffusion coefficient in the epitaxial layer is larger, than in the substrate and increasing with increasing of number of curves.

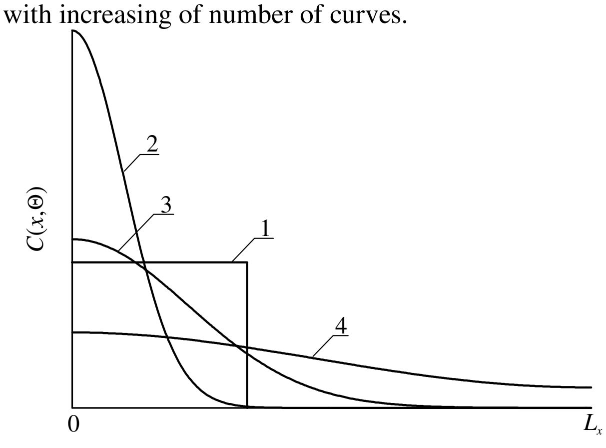

Fig. 4. Distributions of concentration dopant in heterostructure from Fig. 1 for diffusion type of doping. Curve 1 is idealized distribution of dopant. Curves 2-4 are the calculated distributions of dopant for several values of annealing time. Annealing time increased with increasing of number of curves.

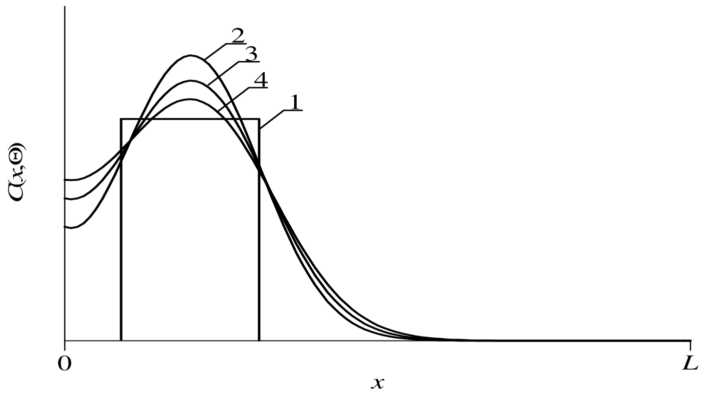

Fig. 5. Distributions of concentration dopant in heterostructure from Fig. 1 for ion type of doping. Curve 1 is idealized distribution of dopant. Curves 2-4 are the calculated distributions of dopant for several values of annealing time. Annealing time increased with increasing of number of curves.

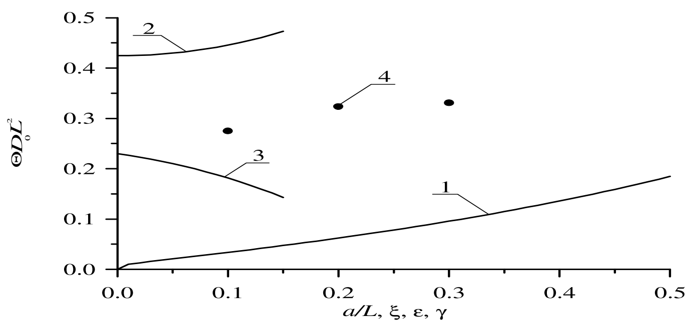

Fig.6. Dimensionless optimal annealing time of infused dopant. Curve 1 describes dependence of the annealing time on normalized thickness of epitaxial layer, $\xi = \gamma = 0$ and equal to each other values of dopant diffusion coefficient in all layers of heterostructure. Curve 2 describes dependence of the annealing time on normalized difference between diffusion coefficients $\varepsilon = (D_{EL}\text{-}D_S)/D_S$, a/L=1/2 and $\xi = \gamma = 0$. Curve 3 describes dependence of the annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon = \gamma = 0$. Curve 3 describes dependence of the annealing time on value of parameter γ for a/L=1/2 and $\varepsilon = \xi = 0$.

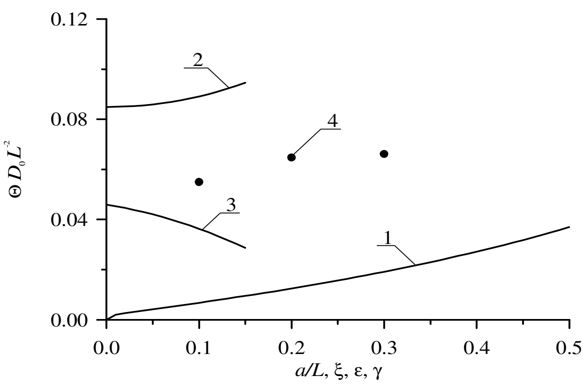

Fig.7. Dimensionless optimal annealing time of implanted dopant. Curve 1 describes dependence of the annealing time on normalized thickness of epitaxial layer, $\xi=\gamma=0$ and equal to each other values of dopant diffusion coefficient in all layers of heterostructure. Curve 2 describes dependence of the annealing time on normalized difference between diffusion coefficients $\varepsilon=(D_{EL}-D_S)/D_S$, a/L=1/2 and $\xi=\gamma$=0. Curve 3 describes dependence of the annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon=\gamma=0$. Curve 3 describes dependence of the annealing time on value of parameter γ for a/L=1/2 and $\varepsilon=\xi$=0.

Here $\psi(x,y,z)$ is the step-wise approximation function. We calculate dependences of optimal values of annealing time on parameters. These dependences are presented on Figs. 6 (for diffusion type of doping) and 7 (for ion types of doping). It is known, that radiation defects should be annealed after ion implantation. Distribution of concentration of these defects will be spreads during the annealing. If distribution of dopant concentration achieves nearest interface, than we obtain rare ideal case. If the dopant has not enough time for the achievement, it is practicably to additionally anneal the dopant. In this situation the considered the additional annealing time became smaller, than annealing time for diffusion type of doping.

Now we analyzed correlation between relaxations of mismatch-induced stress and distribution of concentration of dopant. If $\varepsilon_0 < 0$, one obtain compression of distribution of concentration of dopant. Contrary one obtain spreading of distribution of concentration of dopant. Laser annealing gives a possibility to decrease influence of mismatch-induced stress on distribution of concentration of dopant [22]. Using the annealing leads to acceleration of dopant diffusion in the irradiated area. Taking into account mismatch-induced stress leads to changing of optimal values of annealing time. Mismatch-induced stress could be used to increase density of elements of integrated circuits. On the other hand could leads to generation dislocations of the discrepancy. Fig. 8 shows distributions of component of displacement vector in direction, which is perpendicular to epitaxial layer and substrate.

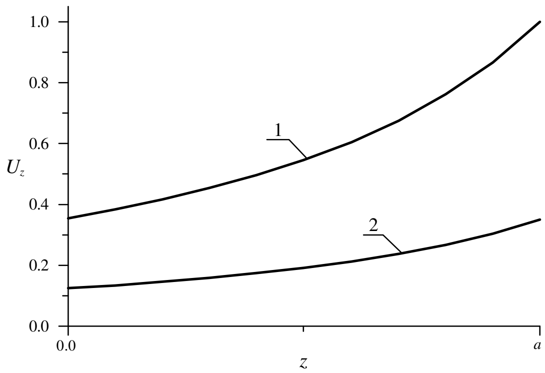

Fig. 8. Normalized dependences of component u_z of displacement vector on coordinate z for nonporous (curve 1) and porous (curve 2) epitaxial layers

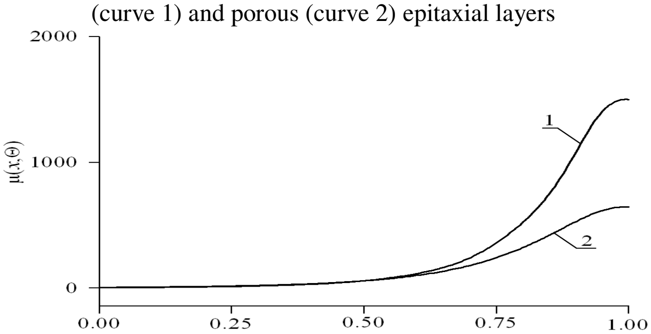

Fig. 9. Normalized distributions of charge carrier mobility in the considered heterostructure.

Curve 1 corresponds to the heterostructure, which has been considered in Fig. 1. Curve 2 correspond to a homogenous material with averaged parameters of heterostructure from Fig. 1

Now we consider the changing of charge carrier mobility with changing of distribution of concentration of dopant. It has been recently shown, that the mobility is a function of concentration of dopant: $\mu \sim C^{1/3}$ [25]. The situation is illustrated by Fig. 9. The figure shows normalized distributions of charge carrier mobility in the considered heterostructure. Curve 1 corresponds to the heterostructure, which has been considered in Fig. 1. Curve 2 correspond to material after averaging parameters of heterostructure from Fig. 1.

4. CONCLUSIONS

In this paper we consider a possibility to increase sharpness of diffusion-junction and implanted-junction heterorectifiers. At the same time homogeneity of distribution of concentration of dopant in enriched by the dopant increases. We also consider an approach to decrease mismatch-induced stress in the considered heterostructure. We analyzed influence of changing of concentration of dopant on value of charge carrier mobility.

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