OPTIMAL PREDICTION OF THE EXPECTED VALUE OF ASSETS UNDER FRACTAL SCALING EXPONENT

Bright O. Osu[1] and Joy I. Adindu-Dick [2]

[1]Department of Mathematics, Abia State University, P M B 2000, Uturu, Nigeria.
[2]Department of Mathematics, Imo State University, Nigeria.

ABSTRACT

In this paper, the optimal prediction of the expected value of assets under the fractal scaling exponent is considered. We first obtain a fractal exponent, then derive a seemingly Black-Scholes parabolic equation. We further obtain its solutions under given conditions for the prediction of expected value of assets given the fractal exponent.

Keywords:

Fractal scaling exponent, Hausdorff dimension, Black-Scholesequation.

1. INTRODUCTION

Financial economist always strive for better understanding of the market dynamics of financial prices and seek improvement in modeling them. Many studies have found that the multi-fractal is more reasonable to describe the financial system than the monofractal.

The concept of "fractal world" was proposed by Mandelbrot in 1980's and was based on scale-invariant statistics with power law correlation (Mandelbrot, 1982). In subsequent years, this new theory was developed and finally it brought a more general concept of multi-scaling. It allows one to study the global and local behavior of a singular measure or in other words, the mono-and multi-fractal properties of a system. In economy, multi-fractal is one of the well-known stylized facts which characterized non-trivial properties of financial time series (Eisler,2004).

The multi-fractal model fundamentally differs from previous volatility models in its scaling properties. The emphasis on scaling originates in the work of Mandelbrot (1963), for extreme variations and Mandelbrot (1965),and Mandelbrot and Van ness (1968) for long memory. Multi-fractality is a form of generalized scaling that includes both extreme variations and long memory.

Several studies have examined the cyclic long-term dependence property of financial prices, including stock prices (Aydogan and Booth, (1988);Greene and Fielitz, (1977)). These studies used the classical rescaled range (R/S) analysis, first proposed by Hurst (1951) and later refined by Mandelbrot and Wallis and Matalas (1970), among others. A problem with the classical R/S analysis is that the distribution of its regression-based test statistics is not well defined. As a result, Lo (1991) proposed the use of a modified R/S procedure with improved robustness. The modified R/S procedure has been applied to study dynamic behavior of stock prices (Lo, 1991; Cheung, Lai, and Lai, 1994).

The problem associated with random behavior of stock exchange has been addressed extensively by many authors (see for example, Black and Scholes, 1973 and Black and Karasinski, 1991).Hull and White (1987) among others followed the traditional approach to pricing options on stocks with stochastic volatility which starts by specifying the joint process for the stock price and its volatility risk. Their models are typically calibrated to the prices of a few options or estimated from the time series of stock prices. Ugbebor et al (2001) considered a stochastic model of price changes at the floor of stock market. On the other hand, Osu and Adindu-Dick (2014) examined multi-fractal spectrum model for the measurement of random behavior of asset price returns. They investigated the rate of returns prior to market signals corresponding to the value for packing dimension in fractal dispersion of Hausdorff measure. They went a step further to give some conditions which determine the equilibrium price, the future market price and the optimal trading strategy.

In this paper we present the optimal prediction of the expected value of assets under the fractal scaling exponent. We first obtain a fractal exponent, then derive a seemingly Black-Scholes parabolic equation. We further obtain its solutions undergiven conditions for the prediction of expected value of assets given the fractal exponent.

2. THE MODEL

Consider the average fractal dimension which is the optimal extraction part to be

$$f(x) = \frac{1}{\Delta\alpha}\int_{\alpha_{min}}^{\alpha_{max}} f(\alpha)d\alpha. \tag{2.1}$$

Here, α is the singularity strength or the holder exponent, while $f(\alpha)$ is the dimension of the subset of series characterized by α and $f(x)$ is the average fractal dimension of all subsets.

$$\Delta f(\alpha) = f(\alpha_{max}) - f(\alpha_{min})$$

$$\Delta\alpha = (\alpha_{max} - \alpha_{min}).$$

If the process follows the Hausdorff multi-fractal process we have

$$f(x) = \frac{1}{\Delta\alpha}\int_{\alpha_{min}}^{\alpha_{max}} f(\alpha)\, d\alpha \;= limsup\; \frac{\mu(B(x,r))}{r^{\alpha}(/logr/)} = U(p) = \int_E \; \emptyset r_{pq}\, d\mu(q). \tag{2.2}$$

Let $(R^n, \beta(R^n))$ be a measurable space and $f\colon \beta(R^n) \to R$ be a measurable functionL

et λ be a real valued function on $\beta(R^n)$, then the multi-fractal spectrum with respect to the functions $f and$ λis given by

$$D(a) = \lambda\{x\epsilon\beta(R^n: f(x) = a\} \quad (2.3)$$

whereλ is taken to be the Hausdorff dimension. Xiao (2004), defined

$$f(x) = limsup \frac{\mu(B(x,r))}{r^\alpha(/logr/)} = V_\mu^\emptyset(E) \quad (2.4)$$

from which Uzoma (2006), derive another gauge function to be

$$f(x)_\lambda = limsup \frac{\mu\big(B(x,r)\big)}{r^\alpha(/logr/)^\lambda} = C^\emptyset(E). \quad (2.5)$$

Let$D(a)$be multi-fractal thick points of μ and X_t be Brownian motion in R^n if $n > 3$ then for all $0 \le a \le \frac{4}{q_n^2}$, Xiao (2004) showed that

$$dim\left\{x\epsilon R^n: \lim_{r\to 0} sup = \frac{\mu(B(x,r))}{r^\alpha(/logr/)} = a\right\} = 2 - \frac{aq_n^2}{2} \quad (2.6)$$

with$q_n > 0$ a Bessel function given as$\frac{J_n}{2-2(x)}$, $\mu\big(B(x,r)\big)$ is the sojourn time,a the singularity strength and$r$radius of the ball. We assume that

$$f(x) - f(x)_\lambda \Rightarrow C^\emptyset(E) - V_\mu^\emptyset(E), \quad (2.7)$$

where λ = distortion parameter defined on $\beta(R^n), distributed\ as\ \mu$ dynamics and is governed by the useful techniques for Hausdorff dimension. For (2.7) is not equal to zero, we obtain in the sequel its value of which we shall call the fractal exponent α.

2.1.1 ESTIMATION OFα

Given a real function Q_t, which is continuous and monotonic decreasing for $t > 0$ with $\lim_{t\to 0} Q_t = +\infty$

Frostman (1935), defined capacity with respect to Q_t : suppose E is bounded borel set in $E_k and\ Q_t$ then μ is a measureable distribution function defined for Borel subsets of E such that

$$\mu(E) = 1$$

$$U(p) = \int_E \emptyset r_{pq}\, d\mu(q) \Rightarrow f(x) = \frac{1}{\Delta\alpha}\int_{\alpha_{min}}^{\alpha_{max}} f(\alpha)\, d\alpha$$

Where r_{pq} denotes the distance between p and q, exists for $p\ \epsilon\ E_k$ and is finite or $+\infty$. U(p) is $\emptyset$- potential with respect to the distribution μ. Define $\emptyset - capacity$ of E denoted$C^\emptyset(E)$ by

(i) if $V^\emptyset(E) = \infty\ then\ C^\emptyset(E) = 0$

(ii) if $V^\emptyset(E) < \infty\ then\ C^\emptyset(E) - V^\emptyset(E) \neq 0.$

Given R_t as the closure of x, it is clear that if x is not in R_t then

$$D_\mu(x) = \lim_{r\to 0} sup \frac{\mu(B(x,r))}{h(2r)} = 0 \qquad (2.8)$$

Thus if

$$E = \left\{x\epsilon R^n : \lim_{r\to 0} sup \frac{\mu\left(B(x,r)\right)}{h(2r)} = 0\right\} \qquad (2.9)$$

Then $E \cap R_t = \emptyset$

Applying (2.3) and (2.8) we see that

$$D(0) = \lambda\{x\epsilon\, R^n : f(x) = 0\} \qquad (2.10)$$

gives the Hausdorff dimension of $E, E \cap R_t = \emptyset$.

From the gauge function $C^\emptyset(E) = D_\mu$ (x)

$$h(r) = r^2 (log\frac{1}{r})^\lambda,\ \lambda > 1 \qquad (2.11)$$

is the correct gauge function such that $f(x) = 0$.

Note that the occupation measure associated with Brownian motion in $n \geq 3$ has a simple meaning for it becomes

$$\lim_{r\to 0} sup \frac{T(r)}{h(2r)}, \qquad (2.12)$$

Where

$$T(r) = \int_0^1 I_{B(x,r)}\, X(s) dt \qquad (2.13)$$

is the total time spent in $B(x,r)$ up to time 1.

THEOREM 1

Let $V^\emptyset(E)$be as in (2.4) and define $C^\emptyset(E)$ capacity of E to be $C^\emptyset(E)$ as in (2.5). If $V^\emptyset(E) = \infty$ then $C^\emptyset(E) = 0$ and given $V^\emptyset(E) < \infty$, then the dimension capacity $\emptyset$ (equivalent to our fractal exponent) is given by $C^\emptyset(E) - V^\emptyset(E) \neq 0 \Longrightarrow \frac{aq_n^2}{2}$.

Proof

We shall proof this in two parts; the value of $C^{\emptyset}(E)$and $V^{\emptyset}(E)$.

For $C^{\emptyset}(E)$, Let there be a Brownian motion in $R^n, n \geq 2$, then there exist a positive constant c such that for $\mathbb{Z} \geq \mathbb{Z}_0 > 0$, $p\{T(r) \geq \mathbb{Z}r^2\} \leq \exp(-cz)$ (Taylor, 1967) .Let X_t be a Brownian motion in $R^n, n \geq 3$.

Suppose $h(r) = r^2(log\frac{1}{r})^{\lambda},\ \lambda > 1$.

Then following Uzoma (2006), we have

$$\mathrm{Lim}_{r\to 0}\, sup\frac{T(r)}{h(2r)} = 0. \tag{2.14}$$

For a fixed $\epsilon > 0$ and $a_k \to 0$ as $k \to \infty$define$E_{\lambda} = \left\{T(a_k) \geq \epsilon\ \ a_k^2(log\frac{1}{a_k})^{\lambda}\right\}$ by

$$P(E_{\lambda}) \leq exp\left\{-c(log\frac{1}{a_k})^{\lambda}\right\} \leq exp\left\{-c(log\frac{1}{a_k})^{\lambda\epsilon}\right\} = (log\frac{1}{a_k})^{-\lambda\epsilon c}, \tag{2.15}$$

hence $\sum P(E_{\lambda}) < \infty, if\ \lambda > \frac{1}{c\epsilon} > 1$.

Thus by Borel Cantelli lemma, we have $P(E_{\lambda}\, , i.0) = 0$ therefore there exist a_0 such that

$$\left\{T(a_k) < \epsilon\ \ a_k^2(log\frac{1}{a_k})^{\lambda}, i.0\right\} \text{ for some } a_k \leq a_0 \text{ so that}$$

$$\lim_{a_k\to 0} sup\frac{T(a_k)}{a_k^2(log\frac{1}{a_k})^{\lambda}} \leq \epsilon\ for\ \lambda > 1$$

Allowing $\epsilon \to 0, shows\ that$

$$P\left[\lim_{a_k\to 0} sup\frac{T(a_k)}{a_k^2(log\frac{1}{a_k})^{\lambda}} = 0\right] > 0, \lambda > 1$$

By the Blumenthal zero- one law, we have

$$P\left[\lim_{a_k\to 0} sup\frac{T(a_k)}{a_k^2(log\frac{1}{a_k})^{\lambda}} = 0\right] = 1, \lambda > 1 \tag{2.16}$$

Hence, by monotonicity of T and h, we have

$$\lim_{r\to 0} sup\frac{T(r)}{h(r)} \leq (I+\epsilon)\lim_{k\to\infty} sup\frac{T(a_k)}{(a_k^2 log\frac{1}{a_k})^{\lambda}}{}' \lambda > 1$$

and the result is established

Thus if

$$\lambda > 1 \text{ and } E = \left\{x\epsilon R^n : \lim_{r\to 0} \sup\frac{\mu(B(x,r))}{r^2/\log r/^{\lambda}} = 0\right\}$$

then from (2.6)

$$\dim E = \left\{x\epsilon R^n : \lim_{r\to 0} \sup\frac{\mu(B(x,r))}{r^2/\log r/^{\lambda}} = 0\right\} = 2 \text{ when } n > 3 \text{ a.s} \qquad (2.17)$$

For the second part, it has been shown that$V^{\emptyset}(E) = 2 - \frac{aq_n^2}{2}$ (Xiao, 2004). If $V^{\emptyset}(E) = \infty$ then $C^{\emptyset}(E) = 0$ and given $V^{\emptyset}(E) < \infty$, then the dimension capacity $\emptyset$ is given by $C^{\emptyset}(E) - V^{\emptyset}(E)$.

Put$V_{\mu}^{\emptyset}(E) = \frac{\mu(B(x,r))}{r^{\alpha}(|log r|)} = Inf_{\mu} V_{\mu}^{\emptyset}(E)$ where $\mu$is a measure with respect to $\emptyset$-capacity of E on the function $V^{\emptyset}(E)$and $C_{\mu}^{\emptyset}(E) = \frac{\mu(B(x,r))}{r^{\alpha}(\log r)^{\lambda}} = Sup_{\mu} C_{\mu}^{\emptyset}(E)$, then

$$C_{\mu}^{\emptyset}(E) - V_{\mu}^{\emptyset}(E) = Sup_{\mu} C_{\mu}^{\emptyset}(E) - Inf_{\mu} V_{\mu}^{\emptyset}(E)$$

$$= 2 - \left(2 - \frac{aq_n^2}{2}\right) = \frac{aq_n^2}{2} \quad , \qquad (2.18)$$

as required.

3. Optimal expected value of assets under fractal scaling exponent

Consider a portfolio comprising h unit of assets in long position and one unit of the option in short position. At time T the value of the portfolio is

$$hS - V, \qquad (3.1)$$

measured by the fractal index $C^{\emptyset}(E) - V^{\emptyset}(E) \neq 0$.

After an elapse of time Δt the value of the portfolio will change by the rate $h(\Delta S + D\Delta t) - \Delta V$in view of the dividend received on h units held. By Ito's lemma this equals

$$h(\mu s\Delta t + \sigma S\Delta z + D\Delta t) - \left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}\mu S + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial v}{\partial S}\sigma S\Delta z)$$

or

$$(h\mu S + hD) \ -\left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}\mu S + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)\Delta t + (h\sigma S - \frac{\partial v}{\partial S}\sigma S)\Delta z$$

If we take

$$h = \frac{\partial v}{\partial s} \qquad (3.2)$$

the uncertainty term disappears, thus the portfolio in this case is temporarily riskless. It should therefore grow in value by the riskless rate in force i.e.

$$(h\mu S + hD) \ -\left(\frac{\partial v}{\partial t} + \frac{\partial v}{\partial s}\mu S + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right)\Delta t = (hS - V)r\Delta t$$

Thus

$$D\frac{\partial V}{\partial S} \ -\left(\frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2\right) = (\frac{\partial v}{\partial S}S - V)r$$

So that

$$\frac{\partial V}{\partial t} + (rs - D)\frac{\partial V}{\partial S} + \frac{1}{2}\frac{\partial^2 v}{\partial S^2}\sigma^2 S = rV\ . \qquad (3.3)$$

Proposition 1: Let $D = 0$ (where D is the market price of risk), then the solution of (3.3) which coincides with the solution of

$$\frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\sigma^2 S^2 = 0 \qquad (3.4a)$$

is given by

$$\mathrm{V(S,t)} = \mathrm{V_0 exp}\left\{\frac{-2\alpha t\bar{S}^{-2}}{\sigma^2} + \lambda\bar{S}\right\}e^{rt}. \qquad (3.4b)$$

For proof see (Osu and Adindu –Dick, 2014).

Proposition 2: For $D \neq 0$, the solution of (3.3) is given as:

$$V(s) = \left(\frac{aq_n^2}{2s}\right)^{\beta}\left\{Ae^{\lambda_1\frac{aq_n^2}{2s}} + Be^{\lambda_2\frac{aq_n^2}{2s}}\right\}, \qquad (3.5a)$$

Where

$$\lambda_1 = -\frac{2}{z} \pm \sqrt{\frac{4}{z^2} + \frac{8r}{z^2\sigma^2}} \text{and } \lambda_2 = \pm\frac{1}{z}\sqrt{4 + \frac{8r}{\sigma^2}} \quad (3.5b).$$

Proof

We take

$$Z = \frac{\alpha}{S}; V(s) = Z^{\beta} W(Z). \quad (3.6)$$

Thus

$$\frac{dz}{ds} = -\frac{\alpha}{S^2} = -\frac{1}{\alpha} Z^2$$

$$\frac{dv}{ds} = \frac{dV}{dZ} . \frac{dZ}{dS}$$

$$= -\frac{1}{\alpha} Z^2 (\beta Z^{\beta-1} W + Z^{\beta} \frac{dW}{dZ})$$

$$= -\frac{1}{\alpha} (\beta Z^{\beta+1} W + Z^{\beta+2} \frac{dW}{dZ}).$$

Hence

$$\frac{d^2V}{dS^2} = \frac{d}{dS}(\frac{dV}{dZ}).\frac{dZ}{dS} = -\frac{1}{\alpha} Z^2 (\beta(\beta+1)Z^{\beta} W + \beta Z^{\beta+1}\frac{dW}{dZ} + (\beta+2)Z^{\beta+1}\frac{dW}{dZ} + Z^{\beta+2}\frac{d^2W}{dZ^2}).$$

In this case V is not dependent on r. Substituting into the given differential equation we have

$$rZ^{\beta} W = \frac{\sigma^2}{2}(\beta(\beta+1)Z^{\beta} W + \beta Z^{\beta+1}\frac{dW}{dZ} + (\beta+2)Z^{\beta+1}\frac{dW}{dZ} + Z^{\beta+2}\frac{d^2W}{dZ^2})$$
$$+\left(\frac{r\alpha}{Z} - D\right)\left(\frac{-1}{\alpha}\right)(\beta Z^{\beta+1} W + Z^{\beta+2}\frac{dW}{dZ})$$

Cancelling by Z^{β} and collecting like terms we have

$$0 = \frac{\sigma^2}{2} Z^2 \frac{d^2W}{dZ^2} + \frac{dW}{dZ}\left(\sigma^2(\beta+1)Z - rZ + \frac{D}{\alpha}Z^2\right) + W\left(\frac{\sigma^2}{2}\beta(\beta+1) - r\beta + \beta\frac{D}{\alpha}Z\right) - rw$$
$$= \frac{\sigma^2}{2} Z^2 \frac{d^2W}{dZ^2} + \frac{dW}{dZ} Z\left(\sigma^2(\beta+1) - r + \frac{D}{\alpha}Z\right) + W\left(\frac{\sigma^2}{2}\beta(\beta+1) - r(\beta+1) + \beta\frac{D}{\alpha}Z\right)$$

Let

$$\beta = 0. r = \frac{D}{\alpha} Z \quad (3.7)$$

We obtain

$$Z^2 \frac{d^2W}{dz^2} + 2Z\frac{dW}{dz} - \frac{2Wr}{\sigma^2} = 0. \tag{3.8}$$

Let λ_1 and λ_2 be the roots of the equation, then

$$\lambda_1 + \lambda_2 = -\frac{2}{z}$$

$$\lambda_1\lambda_2 = -\frac{2r}{Z^2\sigma^2}$$

Now,

$$\frac{d^2W}{dz^2} - (\lambda_1 + \lambda_2)\frac{dW}{ds} - \lambda_1\lambda_2 W = 0$$

or

$$\frac{d}{dz}\left(\frac{dW}{dZ} - \lambda_2 W\right) = \lambda_1\left(\frac{dW}{ds} - \lambda_2 W\right)$$

Then

$$\frac{dW}{dZ} = Y, \qquad Y = \left(\frac{dW}{ds} - \lambda_2 W\right)$$

Which gives $Y = Ce^{\lambda_2 z}$with solution

$$e^{-\lambda_1 z}W = \int C\, e^{(\lambda_1 - \lambda_2)z}dz + B \tag{3.9}$$

(Where C and B are arbitrary constants). Hence

$$W(z) = Ae^{\lambda_1 z} + Be^{\lambda_2 z} \tag{3.10}$$

$$V(s) = Z^{\beta}W(Z)$$

$$= \left(\frac{\alpha}{s}\right)^{\beta}\left\{Ae^{\lambda_1\frac{\alpha}{s}} + Be^{\lambda_2\frac{\alpha}{s}}\right\}$$

$$= \left(\frac{aq_n^2}{2s}\right)^{\beta}\left\{Ae^{\lambda_1\frac{aq_n^2}{2s}} + Be^{\lambda_2\frac{aq_n^2}{2s}}\right\} \tag{3.11}$$

4. Conclusion

The Models: (3.4b) and (3.5a) suggest the optimal prediction of the expected value of assets under fractal scaling exponent$(C - V) = \frac{aq_n^2}{2}$ which we obtained. We derived a seemingly Black Scholes parabolic equation and its solution under given conditions for the prediction of assets values given the fractal exponent. Considering (3.4b), we observed that when $a = 0\,, \alpha = 0$,the equation reduces to $V(s,t) = V_0 e^{`rt}$.This means that the expected value is being determined by the interest rate r and time t. If $a = 4, \alpha = 2q_n^2$,(3.4b) reduces to

$V(s,t) = V_0 exp\left\{\frac{-4q_n^2 tS^{-2}}{\sigma^2} \pm \sqrt{2}q_n\right\} e^{rt}$ thisalso means that the growth rate depends on price, time, and interest rate.

Considering (3.5a), we also observed that when $a = 0$, the equation becomes $V(s) = 0$, this signifies no signal. If $a = 4$, (3.5a) becomes $V(s) = \left(\frac{2q_n^2}{s}\right)^{\beta} \left\{Ae^{\lambda_1 \frac{2q_n^2}{s}} + Be^{\lambda_2 \frac{2q_n^2}{s}}\right\}$,this implies that there is signal. We now further look at it when $q = 1$ to have $V(s) = \left(\frac{2}{s}\right)^{\beta} \left\{Ae^{\frac{2\lambda_1}{s}} + Be^{\frac{2\lambda_2}{s}}\right\}$.

Hence, if $\lambda_1\; and\; \lambda_2$are negative, the equation decays exponentially. On the otherhand if $\lambda_1 and\; \lambda_2$are positive , the equation grows exponentially.

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