AGGREGATION OF OPINIONS FOR SYSTEM SELECTION USING APPROXIMATIONS OF FUZZY NUMBERS

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ABSTRACT

In this article we assume that experts express their view points by way of approximation of Triangular fuzzy numbers. We take the help of fuzzy set theory concept to model the situation and present a method to aggregate these approximations of triangular fuzzy numbers to obtain an overall approximation of triangular fuzzy number for each system and then linear ordering done before the best system is chosen. A comparison has been made betweenapproximation of triangular fuzzy systems and the corresponding fuzzy triangular numbers systems. The notions like fuzziness and ambiguity for the approximation of triangular fuzzy numbers are also found.

***Keywords* :**

Approximations ofTriangular fuzzy numbers, similarity of fuzzy numbers, Relative similarity degree.

1. INTRODUCTION

In any decision making problem, when one has to select froma finite number of systems, opinions of different decision makers are sought. Each expert/decision makers has his own way of assessing a system and thus provides his own rating or grading for that system. Each expert may prefer to express their view point in an imprecise manner rather than an exact manner. It is because of this imprecision or vagueness inherent in the subject assessment of different decision makers/experts, that the help of fuzzy set theory is sought. Once opinions are expressed by the decision makers, the question arises, how best to aggregate these individual opinions into a general consensus opinion. Aggregation operations on fuzzy sets are operations by which several fuzzy sets are combined to produce a single fuzzy set. Different ways of aggregating opinions have been suggested by many works. Nurmi, (1981) [7], Tanino (1984) [8], Fedrizzi, and kacprazyk (1988) [3] proposed that each expert assigns a fuzzy preference relation and these individual fuzzy preference relations were then aggregated into a group fuzzy preference relation in order to determine the best alternative. Bardossy et al. (1993) [2] proposed five aggregation techniques, namely crisp weighting, fuzzy weighting, minimal fuzzy extension, convex fuzzy extension and mixed linear extension method. Hsu and Chen (1996)[4] suggested a method of aggregation by which a consensus opinion is arrived at, on evaluating positive trapezoidal fuzzy numbers that represent an individual's subjective estimate. They employed the method called Similarity Aggregation Method (SAM) to find out an agreement between the experts. In one of our recent works, we have suggested some refinement in the procedure given by Hsu and Chen (1996) [4] which iscomputationally simpler and is easy to understand.

Once aggregated opinions for each system is obtained, a selection of the best system has to be made.

2. PRELLIMINARIES

2.1. FUZZY SETS

Let X denote a universal set i.e., x={x}; then the characteristic function which assigns certain values or a membership grade to the elements of this universal set within a specified range {0, 1} is known as the membership function and the set thus defined is called a fuzzy set. The membership graders correspond to the degree to which an element is compatible with the concept represented by the fuzzy set

If $\mu_{\tilde{A}}$is the membership function definingÃ fuzzy set Ã, then,

$$\mu_{\tilde{A}}: X \to [0, 1]$$

Where [0, 1] developed the interval of real numbers from 0 to 1.

2.2 α-CUT

An α-cut of a fuzzy set Ã is a crisp set $\tilde{A}_\alpha$ that contains all the elements of the universal set X that have a membership grade in A greater than or equal the specified value of α. Thus,
Thus, $\tilde{A} = \{x \in X; \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq x \leq 1\}$

2.3 FUZZY NUMBER

A fuzzy subject Ãof the real line R with membership function $\mu_{\tilde{A}}: X \to [0, 1]$ is called a fuzzy number if,

a) Ã is normal, i.e., there exist an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$.
b) Ã is fuzzy convex, i.e.,$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \geq \mu_{\tilde{A}}(x_1) \wedge \mu_{\tilde{A}}(x_2)\ \forall x_1, x_2 \in R.$
c) $\mu_{\tilde{A}}$ is a upper semi continuous and
d) Sup Ã is bounded, where sup Ã = $\{x \in R; \mu_{\tilde{A}}(x) \geq 0\}$

2.4 POSITIVE FUZZY NUMBER

A fuzzy number A is called positive fuzzy number if its member ship function is such that $\mu_{\tilde{A}}(x) = 0, \forall x < 0$.

This is denoted by Ã> 0.

2.5 TRIANGULAR FUZZY NUMBER

A triangular fuzzy number Ã is denoted as Ã = (a_1, a_2, a_3) and is defined by the membership function as,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \dfrac{x-a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \le x \le a_3 \\ 0 & \text{if } x > a_3 \end{cases}$$

It can be characterized by defining the interval of confidence at level α. Thus for all $\alpha \in [0, 1]$

$\tilde{A}_\alpha = [(a_2-a_1)\,\alpha + a_1,\ a_3-(a_3-a_2)\alpha]$

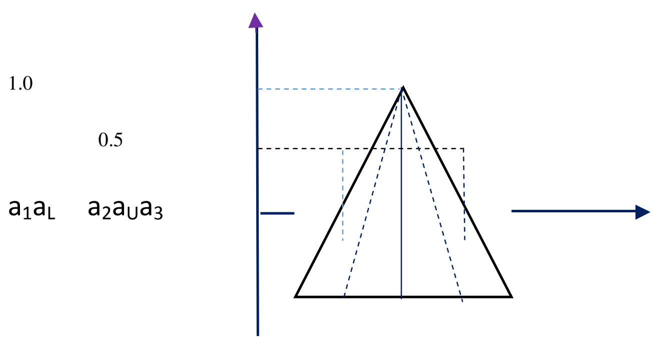
2.6 APPROXIMATION OF TRIANGULAR FUZZY NUMBER

Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number and the approximation of triangular fuzzy number is defined as $\tilde{A}^* = (t_1, t_2, t_3)$

Where $t_1 = a_L = \inf\{x \in A / \mu_{\tilde{A}}(x) \ge 0.5\} = \dfrac{a_1 + a_2}{2}$

$t_2 = a_2 = \{x \in A / \mu_{\tilde{A}}(x) = 1\}$

$t_3 = a_U = \sup\{x \in A / \mu_{\tilde{A}}(x) \ge 0.5\} = \dfrac{a_2 + a_3}{2}$



2.7 AMBIGUITY OF FUZZY NUMBER [9]

Let $\tilde{A}$ be a fuzzy number with α-cut representation $(A_L(\alpha))$, $A_U(\alpha)$, then the ambiguity of $\tilde{A}$ is defined as,

$$\text{Amb}(\tilde{A}) = \int_0^1 \alpha[A_U(\alpha) - A_L(\alpha)]dx$$

2.8 FUZZINESS OF FUZZY NUMBER

Let $\tilde{A}$ be a fuzzy number with α-cut representation $(A_L(\alpha))$, $A_U(\alpha)$, then the fuzziness of $\tilde{A}$ is defined as,

$$\text{Fuzz}(\tilde{A}) = \int_0^{\frac{1}{2}} [A_U(\alpha) - A_L(\alpha)]dx + \int_{\frac{1}{2}}^{1} [A_L(\alpha) - A_U(\alpha)]dx$$

2.9 ASSOCIATED APPROXIMATION OF TRIANGULAR FUZZY NUMBER\

If $\tilde{A}^* = (t_1, t_2, t_3)$ is an approximation of Triangular fuzzy number, then its associated value is given by $\hat{A}^* = \frac{t_1 + 2t_2 + t_3}{4}$

3. AGGREGATION OF OPINIONS

3.1 NOTATIONS

$\mu_{\tilde{A}}(x)$ – Membership function for the element with respect to the fuzzy subset A.
$\tilde{A}_i^*$ – Approximation of Triangular fuzzy number corresponding to system $\tilde{A}_i$.
$\tilde{A}_{i\alpha}^*$ – α-cut of the approximation of Triangular fuzzy number $\tilde{A}_i$ or the interval of confidence at level $\alpha \in [0, 1]$ of the approximation of Triangular fuzzy number $\tilde{A}_i^*$.
LD $(\tilde{A}_i^*, \tilde{A}_j^*)$ – Left distance between two approximation of Triangular fuzzy number $\tilde{A}_i^*$ and $\tilde{A}_j^*$.
$\partial(\tilde{A}_i^*, \tilde{A}_j^*)$ – Normalized distance between two approximation of Triangular fuzzy number $\tilde{A}_i^*$ and $\tilde{A}_j^*$.
D $(\tilde{A}_i^*, \tilde{A}_j^*)$ – level of similarity between two triangular fuzzy number $\tilde{A}_i^*$ and $\tilde{A}_j^*$.
E_i - i^{th} expert / decision maker
$A(E_i)$ – Average similarity degree of expert $E_i(i=1, . . . n)$.
RDS_i – Relative similarity degree of expert $E_i(i=1, . . . n)$.
$\tilde{A}^*$ – Overall Approximation of Triangular fuzzy number.
Pi – weights given to the expert $E_i(i=1, . . . n)$ according to his importance.
Wi – Relative degree of importance of the expert $E_i(i=1, . . . n)$
CDC_i – consensus similarity degree coefficient of the expert $E_i(i=1, . . . n)$
γ - Weights attached to the relative degree of importance of the expert $E_i(i=1, . . . n)$
$(1-\gamma)$ –Weight attached to the relative, similarity degree of the expert $E_i(i=1, . . . n)$
$(\beta_1-\beta_2)$ – Arbitrary values at the left and right respectively of triangular fuzzy number, chosen such that

$$(\beta_2-\beta_1) \geq \frac{[LD(\tilde{A}_i, \tilde{A}_j) + RD(\tilde{A}_i, \tilde{A}_j)]}{2}$$

(.) – Product operator of fuzzy numbers.

3.2 ALGORITHMS

3.2.1 Aggregation of opinions in terms of approximations of triangular fuzzy is to select the best system when each expert is equally important.

Step 1:

A finite set of experts / decision makers $E_i(i=1, . . . n)$ give their subjective estimates of the alternatives R, P, Q,M, . . . in terms of approximation of triangular fuzzy numbers $\tilde{R}*_i, \tilde{P}*_i, \tilde{Q}*_i, \tilde{M}*_i$ .write down these numbers in terms of intervals of confidence

$$\tilde{R}*_{i\alpha}, \tilde{P}*_{i\alpha}, \tilde{Q}*_{i\alpha}, \tilde{M}*_{i\alpha}, ...$$

Step 2:

Calculate normalized distance

$\partial(\tilde{R}*_i, \tilde{R}*_j)$, $\partial(\tilde{P}*_i, \tilde{P}*_j)$, $\partial(\tilde{Q}*_i, \tilde{Q}*_j)$, $\partial(\tilde{M}*_i, \tilde{M}*_j)$, ... between every pair of approximation of Triangular fuzzy numbers $\tilde{R}*_i, \tilde{R}*_j$ (for system R), $\tilde{P}*_i, \tilde{P}*_j$ (for system P), $\tilde{Q}*_i, \tilde{Q}*_j$ (for system Q), $\tilde{M}*_i, \tilde{M}*_j$ (for system M) and so on (i, j = 1 . . . .n) where, for approximation of Triangular fuzzy numbers $\tilde{A}*_i$.

$$\partial(\tilde{A}*_i - \tilde{A}*_j) = \frac{1}{2(\beta_2 - \beta_1)}[LD(\tilde{A_i}*, \tilde{A_j}*) + RD(\tilde{A_i}*, \tilde{A_j}*)], 0 \le \partial < 1$$

(kaufmann et al. (1985)[ ])

If the interval of confidence of approximation of triangular fuzzy number $\tilde{A}*_i$ and $\tilde{A}*_j$ be, respectively,

$$\tilde{A}*_{i\alpha} = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha]$$

$$\tilde{A}*_{i\alpha} = [(b_2 - b_1)\alpha + b_1, b_3 - (b_3 - b_2)\alpha]$$

Then

$$\text{LD}(\tilde{A}*_i, \tilde{A}*_j) = [(a_2 - a_1)\alpha + a_1 - (b_2 - b_1)\alpha - b_1]$$

And

$$\text{RD}(\tilde{A}*_i, \tilde{A}*_j) = [-(a_3 - a_2)\alpha + a_3 + (b_3 - b_2)\alpha - b_3]$$

Step 3:

Calculate the levels of similarity D($\tilde{R}*_i$, $\tilde{R}*_j$), D($\tilde{P}*_i$, $\tilde{P}*_j$), D($\tilde{Q}*_i$, $\tilde{Q}*_j$), D($\tilde{M}*_i$, $\tilde{M}*_j$) .

For approximation of triangular fuzzy numbers ($\tilde{A}*_i, \tilde{A}*_j$)

$$D(\tilde{A}*_i, \tilde{A}*_j) = 1 - \partial(\tilde{A}*_i, \tilde{A}*_j)$$

$$= 1 - \frac{[LD(\tilde{A}*_i, \tilde{A}*_j) + RD(\tilde{A}*_i, \tilde{A}*_j)]}{2(\beta_2 - \beta_1)} 0 \le \partial < 1$$

Step 4:

Construct a similarity matrix SM where E_1 . . . . $.E_2$. . . . $.E_i$. . . . $.E_j$. . . . $.E_n$

$$SM = \begin{array}{c} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{array} \begin{bmatrix} 1 & D_{12}\ldots\ldots & D_{1i}\ldots\ldots & D_{1j}\ldots\ldots & D_{1n} \\ D_{21} & 1 & D_{2i}\ldots\ldots & D_{2j}\ldots\ldots & D_{2n} \\ \vdots & & & & \\ D_{i1} & D_{i2} & 1\ldots\ldots & D_{ij}\ldots\ldots & D_{in} \\ \vdots & & & & \\ D_{n1} & D_{n2} & D_{in}\ldots\ldots & D_{nj}\ldots\ldots & 1 \end{bmatrix}$$

Where $D_{ii}=1$ $\forall i=1$ . . . .n and $D_{ij}=D(\tilde{A}^*_i, \tilde{A}^*_j)$ $\forall i \neq 1$

It is to be noted that SM is a symmetric matrix.

Step5:

Calculate average similarity degree of expert E_i(i=1, ... n) as

$$A(Ei) = \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq 1}}^{n} D_{ij}$$

Step 6:

Calculate the relative similarity degree RSD_i as

$$RSD_i = \frac{A(E_i)}{\sum_{j=1}^{n} A(E_i)}$$

Step 7:

Overall fuzzy number or aggregated consensus opinion is given by

$$\tilde{A} = \sum_{i=1}^{n} (RSD_i(.)\tilde{A}^*_i)$$

Step 8:

Find out the associated ordinary number corresponding to each overall Approximation of fuzzy number, where the

$$A^* = \frac{t_1 + 2t_2 + t_3}{4}$$

Step 9:

Arrange them in descending order. The one coming at the 1st place is best alternative and is selected.

3.2.2 Aggregation of opinions of experts and selection of the best alternative when some experts are more important than others.

Let there be some experts who are more important than the others. Let these experts be given a weight P_i=1. Then the steps of this procedure are as follows.

Step 1:

Find out the relative importance weight P_i of all other experts with respect to most Important experts.

Step 2:

Calculate the relative degree of importance W_i of each experts as

$$W_i = \frac{P_i}{\sum_{i=1}^{n} P_i}, \qquad \sum_{i=1}^{n} W_i = 1$$

Step 3:

Repeat steps 1 to 6 of case I

Step 4:

Calculate consensus similarity degree coefficient as

$$CDC_i = \gamma w_i + (1-\gamma)RSDi,\ 0 \le \gamma \le 1$$

Step 5:

The overall Approximationof fuzzy number of the combination of experts opinion is then given by

$$\tilde{A} = \sum_{i=1}^{n} (CDC_i (.) \tilde{A}_i)$$

Step 6:

Repeat steps 8 and 9 for selection of the best alternative CDC_i is convex combination of weights attached to an expert and his relative similarity degree. The weights attached to them are$\gamma$and (1-γ) respectively. One has to make a judicious choice ofγ, keeping in mind clearly. The importance of relative similarity degree and weight of the expert. Further if $\gamma = 0$, then the determining factor is only relative similarity degree which is the case when all the experts are equally important ($w_1 = w_2 = ...w_n = 1/n$) where as if γ=1, full weight is attached to the relative degree of importance w_i and the relative similarity degree RSD_i becomes irrelevant. For the purpose of illustration we take the same value of γ as has been used in Hsu and Chen (1996) [4].

4. Illustrations

(Case I)

Let us consider an illustration from [1]. Let there be three experts E_1,E_2 andE_3 and four alternatives/systems, R, P, Q, and M. Let the experts give their opinion an each alternatives interms of Triangular fuzzy numbers as follows,

System Expert	R	P	Q	M
E_1	(2,4,5)	(1,2.5,4)	(4,5,6)	(3.5,5.5,6.5)
E_2	(2.5,4,6)	(1.5,3,5)	(3,5,8)	(2,6,8)
E_3	(3,5,7)	(2,3.25,6)	(2,4,5)	(1,3,6)

Then the approximation of the above corresponding triangular fuzzy numbers as follows

System Expert	R*	P*	Q*	M*
E_1	(3,4,4.5)	(1.75, 2.5, 3.25)	(4.5, 5, 5.5)	(4.5, 5.5, 6)
E_2	(3.25, 4.5)	(2.25, 3,4)	(4,5,6.5)	(4,6,7)
E_3	(4, 5, 6)	(2.62, 3.25, 4.62)	(3, 4, 4.5)	(2, 3, 5.5)

Out of the four alternations, the best alternative/system has to be chosen. Expressing the approximation of triangular fuzzy numbers in terms of confidence at level α, α∈ [0,1], we have s∀

$\tilde{R}^*_{1\alpha} = (3+\alpha, 4.5-(0.5)\alpha)$; $\tilde{R}^*_{2\alpha} = (3.25+(0.75\ )5-\alpha)$; $\tilde{R}^*_{3\alpha} = (4+\alpha, 6-\alpha)$

$\tilde{Q}^*_{1\alpha} = (4.5+(0.5)\alpha, 5.5-6.5)\alpha)$; $\tilde{Q}^*_{2\alpha} = (4+\alpha, 6.5-(1.5)\alpha)$; $\tilde{Q}^*_{3\alpha} = (3+\alpha, 4.5-(6.5)\alpha)$

$\tilde{P}^*_{1\alpha} = (1.75+(0.75\ )\alpha, 3.25-(0.75)\alpha)$; $\tilde{P}^*_{2\alpha} = (2.25+(0.75\ )\alpha, 4-\alpha)$; $\tilde{P}^*_{3\alpha} = (2.65+(0.63)\alpha, 4.62-(1.32)\alpha)$

$\tilde{M}^*_{1\alpha} = (4.5+\alpha, 6-(6.5)\alpha)$; $\tilde{M}^*_{2\alpha} = (4.5+\alpha, 6-(6.5)\alpha)$; $\tilde{M}^*_{3\alpha} = (2+\alpha, 5.5-(2.5)\alpha)$

Since distance are always positive, whenever, we get a negative value it will be taken as positive. Now we calculate normalized distances between every pair of experts and for each system. Using the formulas given in kaufmann and Gupta (1985) [6] (using α=0.5) δ ($\tilde{R}^*_i, \tilde{R}^*_j$),δ ($\tilde{P}^*_i$, $\tilde{P}^*_j$),δ ($\tilde{Q}_i$, $\tilde{Q}_j$), and δ ($\tilde{M}_i$, $\tilde{M}_j$) can easily be computed because in ATFN's, we have only straight lines.

For system R^*,we take the arbitrary values of$\beta_1$as 3 (minimum value of the ATFN) and β_2 as 6 (Maximum value of ATFN).

LD ($\tilde{R}^*_1, \tilde{R}^*_2$)= 3+α-3.25-(0.75)α=-0.25+(0.25) α

RD ($\tilde{R}^*_1$, $\tilde{R}^*_2$) = 4.5-(0.5) α-5+α = -0.5+(0.5) α

$$\partial_{12} = \frac{-0.25 + (0.25)\alpha + (0.5)\alpha - 0.5}{2(6-3)} = \frac{-0.75 + (0.75)\alpha}{6} = 0.0625$$

And $D_{12}=1-\partial_{12}=1-0.0625=0.9375$

$\partial_{13} = 0.456$ $\partial_{23} = 0.3125$
$\partial_{13} = 0.542$ $\partial_{23} = 0.6875$

Hence, the similarity Matrix can be written as

$$SM= \begin{array}{c c} & \begin{array}{ccc} E1 & E2 & E3 \end{array} \\ \begin{array}{c} E_1 \\ E_2 \\ E_3 \end{array} & \begin{bmatrix} 1 & 0.937 & 0.542 \\ 0.937 & 1 & 0.687 \\ 0.542 & 0.687 & 1 \end{bmatrix} \end{array}$$

The average similarity degree is calculated as

A (E_1) = ½ [0.937+0.542] =0.7395
A (E_2) = 0.812
A (E_3) = 0.614

Then the relative similarity degree is given by

$$RSD_1 = \frac{A(E_i)}{\sum_{i=1}^{3} A(E_i)} = \frac{0.7395}{2.16} = 0.3423$$

RSD_2= 0.3759
RSD_3= 0.2842

So the overall fuzzy number or aggregated fuzzy opinion is

$\tilde{R}^*$ = 0.3453 (3, 4, 4.5) +0.3759 (3.25, 4, 5) +0.2842(4, 5, 6)
= (3.3853, 4.2938, 5.125)

And

$$\tilde{R}^*_\alpha = [3.3853 + (0.9085)\alpha, 5.125 - (0.8312)\alpha]$$

The associated ordinary number corresponding to this overall approximation fuzzy number is

$\hat{R}$*= 0.3428(3, 4, 4.5) +0.3759(3.25, 4.5) +0.2842(4, 5, 6)
= (3.3853, 4.2938, 5.125)

And

$$\tilde{R}^*_\alpha = [3.3853 + (0.9085)\alpha, 5.125 - (0.8312)\alpha]$$

The associated ordinary number corresponding to this overall approximation of triangular fuzzy number is

$$\hat{R}^* = \frac{3.3853 + 2(4.2938) + 5.9896}{4}$$

$$\hat{R}^* = 4.4906$$

For system P*, we have β_1=1.75 and β_2 = 4.62

∂_{12} = 0.1959; D_{12} = 0.8041
∂_{13} = 0.325; D_{13} = 0.6742
∂_{23} = 0.0540; D_{23} = 0.946

Hence the similarity matrix is

$$\text{SM} = \begin{bmatrix} 1 & 0.8041 & 0.6742 \\ 0.8041 & 1 & 0.9542 \\ 0.6742 & 0.9542 & 1 \end{bmatrix}$$

By using the algorithm,

The overall fuzzy number is given as

$\tilde{P}$ *=0.3041 (1.75, 2.5, 3.25) +0.3617(2.25, 3, 4) +0.3350 (2.62, 3.25, 4.62)

The associated ordinary number is given by

$\hat{P}$ *=2.9014

For system Q*, we have β_1=3 and β_2 = 6.5.

∂_{12} = 0.1071; D_{12} = 0.8928
∂_{13} = 0.1785; D_{13} = 0.8215
∂_{23} = 0.2142; D_{23} = 0.7858

Hence the similarity matrix is

$$\text{SM} = \begin{bmatrix} 1 & 0.8928 & 0.8215 \\ 0.8928 & 1 & 0.7858 \\ 0.8215 & 0.7858 & 1 \end{bmatrix}$$

By using the algorithm,
The overall fuzzy number is given as

$$\tilde{Q}^* = 0.3428\,(4.5, 5, 5.5) + 0.3357(4, 5, 6.5) + 0.3214(3, 4, 4.5)$$
$$= (3.8496, 4.6781, 5.5137)$$

The associated ordinary number is given by

$$\hat{Q}^* = 4.6798$$

For system M*, we have $\beta_1=2$ and $\beta_2 = 7$

$$\partial_{12} = 0.075;\ D_{12} = 0.925$$
$$\partial_{13} = 0.4;\ D_{13} = 0.6$$
$$\partial_{23} = 0.475;\ D_{23} = 0.525$$

Hence the similarity matrix is

$$SM = \begin{bmatrix} 1 & 0.925 & 0.6 \\ 0.925 & 1 & 0.525 \\ 0.6 & 0.525 & 1 \end{bmatrix}$$

By using the algorithm,
The overall fuzzy number is given as

$$\tilde{M}^* = 0.3719(4.5, 5.5, 6) + 0.3536(4, 6, 7) + 0.2741(2, 3, 5.5)$$
$$= (3.6361, 4.9893, 6.2141)$$

The associated ordinary number is given by

$$\hat{M}^* = 4.9572$$

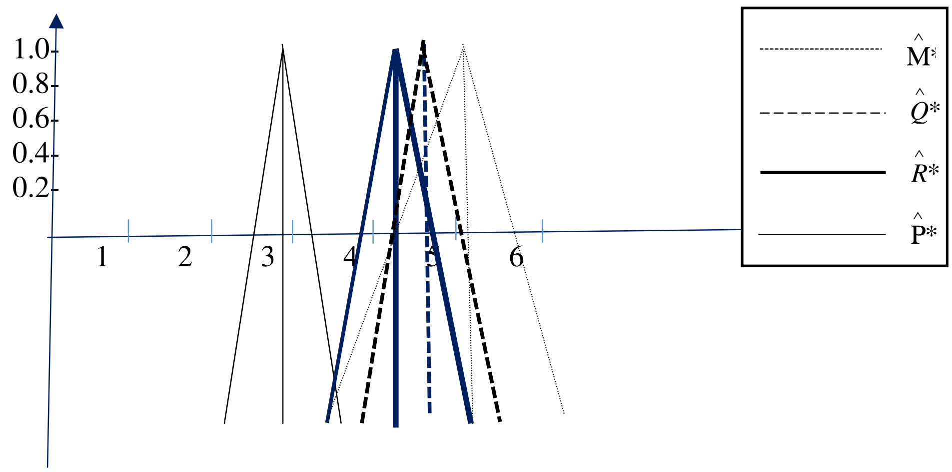
Collecting all the associated numbers, one each for each system, we have

$$\hat{R}^* = 4.4906 \qquad \hat{Q}^* = 4.6798\,\hat{R}$$
$$\hat{P}^* = 2.9014 \qquad \hat{M}^* = 4.9572$$

Ordering them linearly in decreasing order i.e., the one having the maximum value is placed first, we have

$$\hat{M}^* > \hat{R}^* > \hat{Q}^* > \hat{P}^*$$

Hence the system $\hat{M}^*$ is chosen. If the system M* is somehow not available, then the next one i.e., Q* is chosen, and so on.



Case (II)

Let us consider the same illustration as in case I. Out of the 3 experts E_1, E_2 and E_3.Let the expert E_1 be most important. Hence we give a weight P_1=1 to him.

The importance of other experts relative to him are say for E_2, P_3=0.6 and for E_3, β_3=0.2
Then the relative degrees of importance are

$$W_1 = \frac{1}{1+0.6+0.2} = 0.5555 = 0.56$$

$$W_2 = 0.33;\ W_3 = 0.11,\text{s.t } \sum_{i=1}^{3} w_i = 1$$

Considering the values of RSD_i's from illustration I case I, for system R*, the consensus degree coefficient for experts E_1, E_2 and E_3 are, respectively.

CDC_1 = 0.4 x 0.56 + 0.6 x 0.3423 = 0.4293
CDC_2 = 0.4 x 0.33 + 0.6 x 0.3759 = 0.3575
CDC_3 = 0.2145

Where γ = 0.4 (as w_1> RSD_1)
Thus, the overall fuzzy number is given us

$\tilde{R}$ *= 0.4293(3, 4, 5,) +0.3575(3.25, 4, 5) +0.2145(4, 5, 6)
= (3.3075, 4.2197, 5.0063)

And

$\tilde{R}_\alpha$ *= (3.3075+ (0.9122)α, 5.0063-(0.7866)α)
Therefore the associated ordinary number corresponding to this triangular fuzzy number is

$$\tilde{R}^* = \frac{a_1 + 2a_2 + a_3}{4} = \frac{3.3075 + 2(4.2197) + 5.0063}{4}$$

$$\hat{R}^* = \frac{8.3138 + 8.4394}{4} = 4.1883$$

Similarly for system P*, we have
The associated ordinary number corresponding to this triangular fuzzy number is

$$\hat{P}^* = 2.2963$$

Similarly for system Q*, we have
The associated ordinary number corresponding to this number is

$$\hat{Q}^* = 4.7742$$

Similarly for the system M*, we have
The associated ordinary number is

$$\hat{M}^* = 5.0850$$

Collecting all the associated ordinary numbers of overall fuzzy numbers, one each for each system, we have

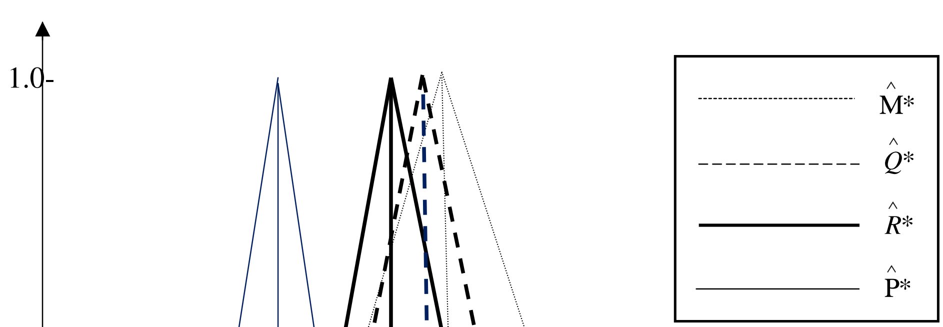
$$\hat{R}^* = 4.1883; \quad \hat{Q}^* = 4.7742$$

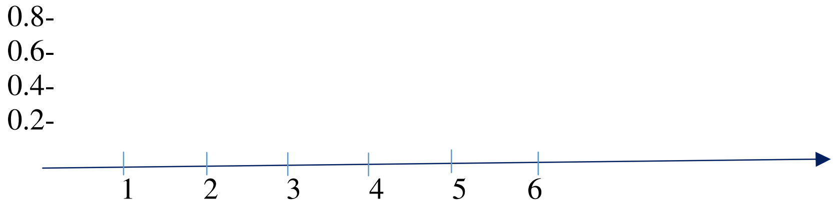
$$\hat{P}^* = 2.2963; \quad \hat{M}^* = 5.0850$$

Ordering them in descending order, we get

$$\hat{M}^* > \hat{Q}^* > \hat{R}^* > \hat{P}^*$$

Hence, the system M* is to be chosen. If system M* is not available, system Q* is chosen and so an.





5. Comparison between fuzzy numbers and its approximation of fuzzy numbers

Case (i)

System	TFN	ATFN
R*	$\text{Amb}(\tilde{R}) = 0.5824$ $\text{Fuzz}(\tilde{R}) = 0.8737$	$\text{Amb}(\tilde{R}^{*}) = 0.29$ $\text{Fuzzy}(\tilde{R}^{*})=$ 0.124
P*	$\text{Amb}(\tilde{P}) = 0.5836$ $\text{Fuzz}(\tilde{P}) = 0.876$	$\text{Amb}(\tilde{P}^{*}) =$ 0.2150 $\text{Fuzz}(\tilde{P}^{*})=$ 0.3226
Q*	$\text{Amb}(\tilde{Q}) = 0.5553$ $\text{Fuzz}(\tilde{Q})= 0.8335$	$\text{Amb}(\tilde{Q}^{*})=$ 0.2773 $\text{Fuzz}(\tilde{Q}^{*})=$ 0.4160
M*	$\text{Amb}(\tilde{M})= 0.7729$ $\text{Fuzz}(\tilde{M}) = 1.1594$	$\text{Amb}(\tilde{M}^{*})=$ 0.4296 $\text{Fuzz}(\tilde{M}^{*}) =$ 0.6445

Table. 5a

Case (ii)

System	TFN	ATFN
R*	$\text{Amb}(\tilde{R}) = 0.5678$	$\text{Amb}(\tilde{R}^{*}) = 0.2831$

	Fuzz($\tilde{R}$) = 0.8517	Fuzzy($\tilde{R}^*$) = 0.4247
P*	Amb($\tilde{P}$) = 0.5685 Fuzz($\tilde{P}$) = 0.8528	Amb($\tilde{P}^*$) = 0.2850 Fuzz($\tilde{P}^*$) = 0.4276
Q*	Amb($\tilde{Q}$) = 0.5413 Fuzz($\tilde{Q}$) = 0.5120	Amb($\tilde{Q}^*$) = 0.2697 Fuzz($\tilde{Q}^*$) = 0.4045
M*	Amb($\tilde{M}$) = 0.7460 Fuzz($\tilde{M}$) = 1.1190	Amb($\tilde{M}^*$)= = 0.4054 Fuzz($\tilde{M}^*$)= = 0.6081

Table. 5b

6. CONCLUSION

A comparison has been made between approximation of triangular fuzzy number systems and the corresponding fuzzy triangular numbers systems, with the aid of notions like fuzziness and ambiguity for the approximation of fuzzy numbers. It can also be seen that from the section V,the fuzziness and ambiguity of theapproximationof triangular fuzzy number system is very less than the triangular fuzzy number system which would be an important point to be noted for the future works on the approximations of fuzzy numbers.

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