# Numerical solution of fuzzy differential equations by Milne's predictor-corrector method and the dependency problem

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***ABSTRACT***

*The study of this paper suggests on dependency problem in fuzzy computational method by using the numerical solution of Fuzzy differential equations(FDEs) in Milne's predictor-corrector method. This method is adopted to solve the dependency problem in fuzzy computation. We solve some fuzzy initial value problems to illustrate the theory.*

***KEYWORDS***

*Fuzzy initial value problem, Dependency problem in fuzzy computation, Milnes predictor-corrector method.*

## 1. INTRODUCTION

Fuzzy Differential Equations (FDEs) are used in modeling problems in science and engineering. Most of the problems in science and engineering require the solutions of FDEs which are satisfied by fuzzy initial conditions, therefore a Fuzzy Initial Value Problem(FIVP) occurs and should be solved. Fuzzy set was first introduced by Zadeh[22]. Since then, the theory has been developed and it is now emerged as an independent branch of Applied Mathematics. The elementary fuzzy calculus based on the extension principle was studied by Dubois and Prade [14]. Seikkala[21] and Kaleva[16] have discussed FIVP. Buckley and Feuring[13] compared the solutions of FIVPs which where obtained using different derivatives. The numerical solutions of FIVP by Euler's method was studied by Ma et al.[18]. Abbasbandy and Allviranloo [1, 2] proposed the Taylor method and the fourth order Runge-Kutta method for solving FIVPs. Palligkinis et al.[20] applied the Runge-Kutta method for more general problems and proved the convergence for n-stage Runge-Kutta method. Allahviranloo et. al.[8] and Barnabas Bed [10] to solve the numerical solution of FDEs by predictor-corrector method. The dependency problem in fuzzy computation was discussed by Ahmad and Hasan[4] and they used Euler's method based on Zadeh's extension principle for finding the numerical solution of FIVPs. Omar and Hasan[7] adopted the same computation method to derive the fourth order Runge-Kutta method for FIVP. Latterly Ahmad and Hasan[4] investigate the dependency problem in fuzzy computation based on Zadeh extension principle. In this paper we study the dependency problem in fuzzy computations by using Milne's predictor-corrector method.

## 2. PRELIMINARY CONCEPTS

In this section, we give some basic definitions.

***Definition 2.1*** Subset Ã of a universal set Y is said to be a fuzzy set if a membership function $\mu_{\tilde{A}}(y)$ takes each object in Y onto the interval [0,1]. The function $\mu_{\tilde{A}}(y)$ is the possibility degrees to which each object is compatible with the properties that characterized the group.
A fuzzy set $\tilde{A} \subseteq Y$ can also be presented as a set of ordered pairs

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)) : y \in Y\}, \tag{1}$$

The support, the core and the height of A are respectively

$$supp(\tilde{A}) = \{y \in Y: y > \mu_{\tilde{A}}(y)\}, \tag{2}$$
$$core(\tilde{A}) = \{y \in Y: \mu_{\tilde{A}}(y) = 1\}, \tag{3}$$
$$hgt(\tilde{A}) = \sup_{y \in Y} \mu_{\tilde{A}}(y). \tag{4}$$

***Definition 2.2*** A fuzzy number is a convex fuzzy subset $A$ of $R$, for which the following conditions are satisfied:

(i) $\tilde{A}$ is normalized. i.e. $hgt(\tilde{A}) = 1$;
(ii) $\mu_{\tilde{A}}(y)$ are upper semicontinuous;
(iii) $\{y \in R: \mu_{\tilde{A}}(y) = \alpha\}$ are compact sets for $0 < \alpha \leq 1$, and
(iv) $\overline{\{y \in R: \mu_{\tilde{A}}(y) = \alpha\}}$ are also compact sets for $0 < \alpha \leq 1$.

***Definition 2.3*** If $F(R)$ is the set of all fuzzy numbers, and $\tilde{A} \in F(R)$, we can characterize $\tilde{A}$ by its α-levels by the following closed-bounded intervals:

$$[\tilde{A}]^{\alpha} = \underline{\{y \in R: \mu_{\tilde{A}}(y) \geq \alpha\}} = \underline{[a_1^{\alpha}, a_2^{\alpha}]}, \qquad 0 < \alpha \leq 1 \tag{5}$$
$$[\tilde{A}]^{\alpha} = \overline{\{y \in R: \mu_{\tilde{A}}(y) \geq \alpha\}} = \overline{[a_1^{\alpha}, a_2^{\alpha}]}, \qquad 0 < \alpha \leq 1 \tag{6}$$

Operations on fuzzy numbers can be described as follows: If $\tilde{A}, \tilde{B} \in F(R)$, then for $0 < \alpha 1$

1. $[\tilde{A} + \tilde{B}]^{\alpha} = [a_1^{\alpha} + b_1^{\alpha}, a_2^{\alpha} + b_2^{\alpha}]$;
2. $[\tilde{A} - \tilde{B}]^{\alpha} = [a_1^{\alpha} - b_1^{\alpha}, a_2^{\alpha} - b_2^{\alpha}]$;
3. $[\tilde{A} \cdot \tilde{B}]^{\alpha} = [min\{a_1^{\alpha} \cdot b_1^{\alpha}, a_1^{\alpha} \cdot b_2^{\alpha}, a_2^{\alpha} \cdot b_1^{\alpha}, a_2^{\alpha} \cdot b_2^{\alpha}\}, max\{a_1^{\alpha} \cdot b_1^{\alpha}, a_1^{\alpha} \cdot b_2^{\alpha}, a_2^{\alpha} \cdot b_1^{\alpha}, a_2^{\alpha} \cdot b_2^{\alpha}\}]$;
4. $\left[\frac{A}{B}\right]^{\alpha} = \left[min\left\{\frac{a_1^{\alpha}}{b_1^{\alpha}}, \frac{a_1^{\alpha}}{b_2^{\alpha}}, \frac{a_2^{\alpha}}{b_1^{\alpha}}, \frac{a_2^{\alpha}}{b_2^{\alpha}}\right\}, max\left\{\frac{a_1^{\alpha}}{b_1^{\alpha}}, \frac{a_1^{\alpha}}{b_2^{\alpha}}, \frac{a_2^{\alpha}}{b_1^{\alpha}}, \frac{a_2^{\alpha}}{b_2^{\alpha}}\right\}\right]$ here $0 \notin [\tilde{B}]^{\alpha}$;
5. $[s\tilde{A}]^{\alpha} = s[\tilde{A}]^{\alpha}$ where $s$ is scalar and
6. $[a_1^{\alpha_i}, a_2^{\alpha_i}] = [a_1^{\alpha_j}, a_2^{\alpha_j}]$ for $0 < \alpha_i \leq \alpha_j$.

***Definition 2.4*** A fuzzy process is a mapping $\tilde{y}: I \to F(R)$, where $I$ is a real interval [17]. This process can be denoted as:

$$[\tilde{y}(t)]^{\alpha} = [y_1^{\alpha}(t), y_2^{\alpha}(t)], \qquad t \in I \quad and \quad 0 < \alpha \leq 1. \tag{7}$$

The fuzzy derivative of a fuzzy process *x(t)* is defined by

$$[\tilde{y}(t)]^{\alpha} = \left[y_1'^{\alpha}(t), y_2'^{\alpha}(t)\right], \qquad t \in I \quad and \qquad 0 < \alpha \leq 1. \tag{8}$$

***Definition 2.5*** Triangular fuzzy number are those fuzzy sets in $F(R)$ in which are characterized by an ordered triple $\left(y^l, y^c, y^r\right) \in R^3$ with $y^l \leq y^c \leq y^r$ such that $[U]^0 = \left[y^l, y^r\right]$ and $[U]^1 = \left[y^c\right]$ then

$$[U]^{\alpha} = \left[\, y^c - (1-\alpha)\left(y^c - y^l\right), y^c + (1-\alpha)\left(y^r - y^c\right)\right], \tag{9}$$

for any $\alpha \in R$

## 3. Fuzzy Initial Value Problem

The FIVP can be considered as follows

$$\frac{dy(t)}{dt} = f\left(t, y(t)\right), \qquad y(0) = \tilde{Y}_0, \tag{10}$$

Where $f: R_+ \times R \to R$ is a continuous mapping and $\tilde{Y}_0 \in F(R)$ with $\alpha$-level interval

$$[\tilde{y}_0]^{\alpha} = [y_{01}^{\alpha}, y_{02}^{\alpha}] \qquad 0 < \alpha \leq 1. \tag{11}$$

When $y = y(t)$ is a fuzzy number, the extension principle of Zadeh leads to the following definition:

$$f(t, y)(s) = sup\{y(\tau) : \ s = f(t, \tau)\}, \qquad s \in R \tag{12}$$

It follows that

$$[f(t, y(t))]^{\alpha} = [f_1^{\alpha}(t, y(t)), f_2^{\alpha}(t, y(t))], \qquad 0 < \alpha \leq 1, \tag{13}$$

Where

$$f_1^{\alpha}(t, y(t)) = min\{f(t, w) : w \,\epsilon [y_1^{\alpha}(t), y_2^{\alpha}(t)]\}, \qquad 0 < \alpha \leq 1 \tag{14}$$

$$f_2^{\alpha}(t, y(t)) = max\{f(t, w) : w \,\epsilon [y_1^{\alpha}(t), y_2^{\alpha}(t)]\}, \qquad 0 < \alpha \leq 1. \tag{15}$$

***Theorem 3.1*** Let $f$ satisfy

$$|f(t, y) - f(t, y^*)| \leq g(t, |y - y^*|), \qquad t \geq 0 \qquad y, y^* \in R \tag{16}$$

Where $g: R_+ \times R_+ \to R_+$ is a continuous mapping such that $r \to g(t, r)$ is non decreasing, the IVP

$$z'(t) = g\big(t, z(t)\big), \qquad z(0) = z_0, \tag{17}$$

Has a solution on $R_+$ for $z_0 > 0$ and that $z(t) \equiv 0$ is the only solution of equation (17) for $z_0 = 0$. Then the FIVP (10) has a unique fuzzy solution.

***Proof*** **.See [17]**

In the fuzzy computation, the dependency problem arises when we apply the straightforward fuzzy interval arithmetic and Zadeh's extension principle by computing the interval separately. For the dependency problem we refer [7].

## 4. THE MILNE'S PREDICTOR-CORRECTOR METHOD IN DEPENDENCY PROBLEM

We consider the IVP in equation (10) but with crisp initial condition $y(t_0) = y_0 \epsilon R$ and $t \epsilon [t_0, T]$. The formula for Milne's predictor-corrector method is follows:

$$\begin{aligned} y_{r+1,P} &= y_{r-3} + \frac{4h}{3}\left[2f(t_r, y_r) - f(t_{r-1}, y_{r-1}) + 2f(t_{r-2}, y_{r-2})\right], \\ y_{r+1,C} &= y_{r-1} + \frac{h}{3}\left[f(t_{r-1}, y(t_{r-1})) + 4f(t_r, y(t_r)) + f(t_{r+1}, y_{r+1,P}(t_{r+1}))\right], \\ y(t_{r-3}) &= y_{r-3}, y(t_{r-2}) = y_{r-2}, y(t_{r-1}) = y_{r-1}, y(t_r) = y_r, \end{aligned} \tag{18}$$

Where $h = \frac{T - t_0}{N} = t_{r+1} - t_r,\ r = 0,1,\ldots,N.$ We consider the right-hand side of equation (18), we modify the Milne's predictor-corrector method by using dependency problem in fuzzy computation as one function

$$V(t_r, h, y_r) = y(t_{r-1}) + \frac{h}{3}\left[f(t_{r-1}, y(t_{r-1})) + 4f(t_r, y(t_r)) + f(t_{r+1}, y_{r+1,P}(t_{r+1}))\right], \tag{19}$$

By the equivalent formula

$$y_{r+1,C} = y(t_{r-1}) + \frac{h}{3}\left\{\begin{aligned} &f(t_{r-1}, y(t_{r-1})) + 4f(t_r, y(t_r)) \\ &+ f\left(t_{r+1}, y_{r-3} + \frac{4h}{3}\left[2f(t_r, y_r) - f(t_{r-1}, y_{r-1}) + 2f(t_{r-2}, y_{r-2})\right]\right) \end{aligned}\right\} \tag{20}$$

Now, let $\tilde{Y} \in F(R)$, the formula

$$V(t_r, h, \tilde{Y}_r)(v_r) = \begin{cases} sup_{y_r \in V^{-1}(t_r, h, v_r)} \tilde{Y}_r(y_r), & if\ v_r \in range(V); \\ 0, & if\ v_r \notin range(V), \end{cases} \tag{21}$$

Can extend equation (20) in the fuzzy setting.

Let $[\tilde{Y}_r]^\alpha = [y_{r,1}^\alpha, y_{r,2}^\alpha]$ represent the $\alpha$-level of the fuzzy number defined in equation (21). We rewrite equation (21) using the $\alpha$-level as follows:

$$V\big(t_r, h, [\tilde{Y}_r]^\alpha\big) = \big[min\{t_r, h, y\big|y \in y^\alpha_{r,1}, y^\alpha_{r,2}\}, max\{t_r, h, y\big|y \in y^\alpha_{r,1}, y^\alpha_{r,2}\}\big] \tag{22}$$

By applying equation (22) in (18) we get

$$[\tilde{Y}_{r+1}]^\alpha = \big[y^\alpha_{r+1,1}, y^\alpha_{r+1,2}\big], \tag{23}$$

Where

$$y^\alpha_{r+1,1} = min\{V(t_r, h, y)\big|y \in [y^\alpha_{r,1}, y^\alpha_{r,2}]\}, \tag{24}$$

$$y^\alpha_{r+1,2} = max\{V(t_r, h, y)\big|y \in [y^\alpha_{r,1}, y^\alpha_{r,2}]\}. \tag{25}$$

Therefore

$$y^\alpha_{r+1,1} = \min\left\{y(t_{r-1}) + \frac{h}{3}\big[f(t_{r-1}, y(t_{r-1})) + 4f(t_r, y(t_r)) + f(t_{r+1}, y_{r+1,P}(t_{r+1}))\big]\Big| y \in \big[y^\alpha_{r,1}, y^\alpha_{r,2}\big]\right\}, \tag{26}$$

$$y^\alpha_{r+1,2} = \max\left\{y(t_{r-1}) + \frac{h}{3}\big[f(t_{r-1}, y(t_{r-1})) + 4f(t_r, y(t_r)) + f(t_{r+1}, y_{r+1,P}(t_{r+1}))\big]\Big| y \in \big[y^\alpha_{r,1}, y^\alpha_{r,2}\big]\right\}, \tag{27}$$

By using the computational method proposed in [5], we compute the minimum and maximum in equations (26), (27) as follows

$$x^{\alpha_i}_{r+1,1} = min\left[\min_{x \in \left[x^{\alpha_i}_{r,1}, x^{\alpha_{i+1}}_{r,1}\right]} V(t,h,x), \cdots, \min_{x \in \left[x^{\alpha_n}_{r,1}, x^{\alpha_{n+1}}_{r,2}\right]} V(t,h,x), \cdots, \min_{x \in \left[x^{\alpha_{i+1}}_{r,2}, x^{\alpha_i}_{r,2}\right]} V(t,h,x)\right] \tag{28}$$

$$x^{\alpha_i}_{r+1,2} = max\left[\max_{x \in \left[x^{\alpha_i}_{r,1}, x^{\alpha_{i+1}}_{r,1}\right]} V(t,h,x), \cdots, \max_{x \in \left[x^{\alpha_n}_{r,1}, x^{\alpha_{n+1}}_{r,2}\right]} V(t,h,x), \cdots, \max_{x \in \left[x^{\alpha_{i+1}}_{r,2}, x^{\alpha_i}_{r,2}\right]} V(t,h,x)\right] \tag{29}$$

## 5. NUMERICAL EXAMPLES

In this section, we present some numerical examples including linear and nonlinear FIVPs.

**Example 5.1** Consider the following FIVP.

$$\begin{cases} x'(t) = x(1-2t), & t \in [0,2]; \\ \tilde{X}^0(w) = \begin{cases} 0, & if \quad w < -0.5; \\ 1-4w^2, & if -0.5 \le w \le 0.5; \\ 0, & if\ w > 0.5; \end{cases} \end{cases} \tag{30}$$

The exact solution of equation (30) is given by

$$[X(t)]^{\alpha} = \left[\left(-\frac{\sqrt{(1-\alpha)}}{2}\right)e^{t-t^2}, \left(\frac{\sqrt{(1-\alpha)}}{2}\right)e^{t-t^2}\right]. \qquad (31)$$

The absolute results of the numerical fuzzy Milne's predictor-corrector method approximated solutions at $t_{20} = 2.$ See Table 1 and Figure 1 and 2.

**TABLE 1**

The error of the obtained results with the exact solution at t=2.

| α | PC Approximation | | MPC Approximation | | Exact | | Error PC | | Error MPC | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ |
| 0.0 | -0.0676 | 0.0676 | -0.0677 | 0.0677 | -0.0677 | 0.0677 | -8.05e-5 | 8.05e-5 | -2.53e-6 | 2.53e-6 |
| 0.1 | -0.0641 | 0.0641 | -0.0642 | 0.0642 | -0.0642 | 0.0642 | -7.63e-5 | 7.63e-5 | -2.40e-6 | 2.40e-6 |
| 0.2 | -0.0605 | 0.0605 | -0.0605 | 0.0605 | -0.0605 | 0.0605 | -7.20e-5 | 7.20e-5 | -2.26e-6 | 2.26e-6 |
| 0.3 | -0.0565 | 0.0565 | -0.0566 | 0.0566 | -0.0566 | 0.0566 | -6.73e-5 | 6.73e-5 | -2.11e-6 | 2.11e-6 |
| 0.4 | -0.0524 | 0.0524 | -0.0524 | 0.0524 | -0.0524 | 0.0524 | -6.23e-5 | 6.23e-5 | -1.96e-6 | 1.96e-6 |
| 0.5 | -0.0478 | 0.0478 | -0.0479 | 0.0479 | -0.0478 | 0.0478 | -5.69e-5 | 5.69e-5 | -1.79e-6 | 1.79e-6 |
| 0.6 | -0.0427 | 0.0427 | -0.0428 | 0.0428 | -0.0428 | 0.0428 | -5.09e-5 | 5.09e-5 | -1.60e-6 | 1.60e-6 |
| 0.7 | -0.0370 | 0.0370 | -0.0371 | 0.0371 | -0.0371 | 0.0371 | -4.41e-5 | 4.41e-5 | -1.38e-6 | 1.38e-6 |
| 0.8 | -0.0302 | 0.0302 | -0.0303 | 0.0303 | -0.0303 | 0.0303 | -3.60e-5 | 3.60e-5 | -1.13e-6 | 1.13e-6 |
| 0.9 | -0.0214 | 0.0214 | -0.0214 | 0.0214 | -0.0214 | 0.0214 | -2.54e-5 | 2.54e-5 | -8.00e-7 | 8.00e-7 |
| 1.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |

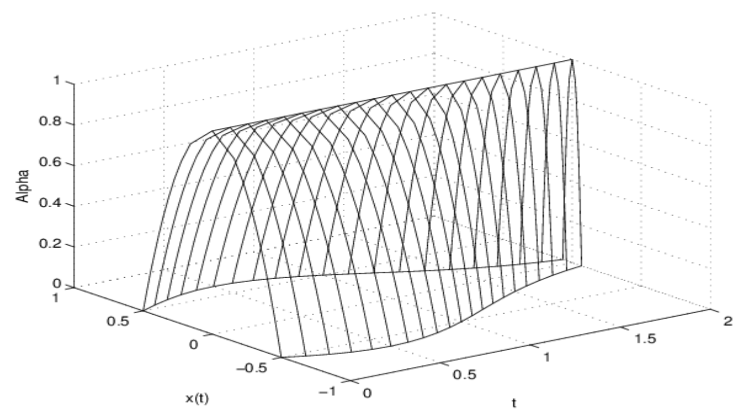


**Figure 1**: The approximation of fuzzy solution by Milne's predictor-corrector (h=0.1)

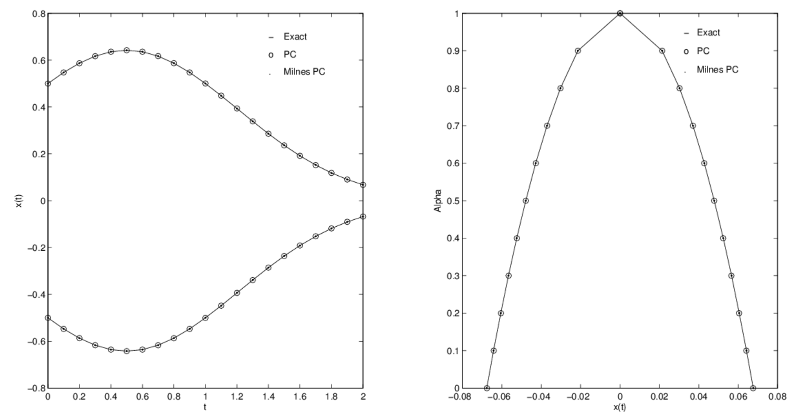


**Figure 2**: Comparison between the exact, Milne's predictor-corrector, predictor-corrector

In this example, the comparison of the absolute local error between Milne's predictor-corrector method with the fuzzy exact solution is given in Table 1 for various values of $\alpha$ -level $\alpha = 0, 0.1, \ldots, 0.9, 1$ and fixed value of $h(t_{20} = 2)$. The results shows that Milne's predictor-corrector method is more accurate than predictor-corrector method shows the graphical comparison of a fuzzy solution between exact, Milne's predictor-corrector, predictor-corrector at fixed $h(t_{10} = 1)$. The behaviour solutions of the end points of the fuzzy intervals of a fuzzy exact solution, Milne's predictor-corrector and predictor-corrector fuzzy approximated solutions are plotted and compared in Figure 2 at $\alpha_1 = 0$. Figure 2 clearly show that Milne's predictor-corrector provides a more accurate results than predictor-corrector method.

**Example 5.2** Consider the following FIVP

$$\begin{cases} x'(t) = x(t^2 - 4t + 3), & t \in [0,2]; \\ \tilde{X}^0(w) = \begin{cases} 0, & if \quad w < -0.5; \\ 1 - 4w^2, & if - 0.5 \leq w \leq 0.5; \\ 0, & if\ w > 0.5; \end{cases} \end{cases} \tag{32}$$

The exact solution of equation (32) is given by

$$[X(t)]^\alpha = \left[\left(-\frac{\sqrt{(1-\alpha)}}{2}\right)e^{\frac{t^3}{3}-2t^2+3t}, \left(\frac{\sqrt{(1-\alpha)}}{2}\right)e^{\frac{t^3}{3}-2t^2+3t}\right]. \tag{33}$$

The absolute results of the numerical fuzzy Milne's predictor-corrector method approximated solutions at $t_{20} = 2$. See Table 2 and Figure 3 and 4.

**TABLE 2**

The error of the obtained results with the exact solution at t=2.

| α | PC Approximation | | MPC Approximation | | Exact | | Error PC | | Error MPC | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ | $x_1(t_i;\alpha)$ | $x_2(t_i;\alpha)$ |
| 0.0 | -0.9740 | 0.9740 | -0.9738 | 0.9738 | -0.9739 | 0.9739 | -1.33e-4 | 1.33e-4 | -3.70e-5 | 3.70e-5 |
| 0.1 | -0.9240 | 0.9240 | -0.9239 | 0.9239 | -0.9239 | 0.9239 | -1.26e-4 | 1.26e-4 | -3.51e-5 | 3.51e-5 |
| 0.2 | -0.8712 | 0.8712 | -0.8710 | 0.8710 | -0.8711 | 0.8711 | -1.19e-4 | 1.19e-4 | -3.31e-5 | 3.31e-5 |
| 0.3 | -0.8149 | 0.8149 | -0.8148 | 0.8148 | -0.8148 | 0.8148 | -1.11e-4 | 1.11e-4 | -3.10e-5 | 3.10e-5 |
| 0.4 | -0.7545 | 0.7545 | -0.7543 | 0.7543 | -0.7544 | 0.7544 | -1.03e-4 | 1.03e-4 | -2.87e-5 | 2.87e-5 |
| 0.5 | -0.6887 | 0.6887 | -0.6886 | 0.6886 | -0.6886 | 0.6886 | -9.41e-4 | 9.41e-4 | -2.62e-5 | 2.62e-5 |
| 0.6 | -0.6160 | 0.6160 | -0.6159 | 0.6159 | -0.6159 | 0.6159 | -8.42e-4 | 8.42e-4 | -2.34e-5 | 2.34e-5 |
| 0.7 | -0.5335 | 0.5335 | -0.5334 | 0.5334 | -0.5334 | 0.5334 | -7.29e-4 | 7.29e-4 | -2.03e-5 | 2.03e-5 |

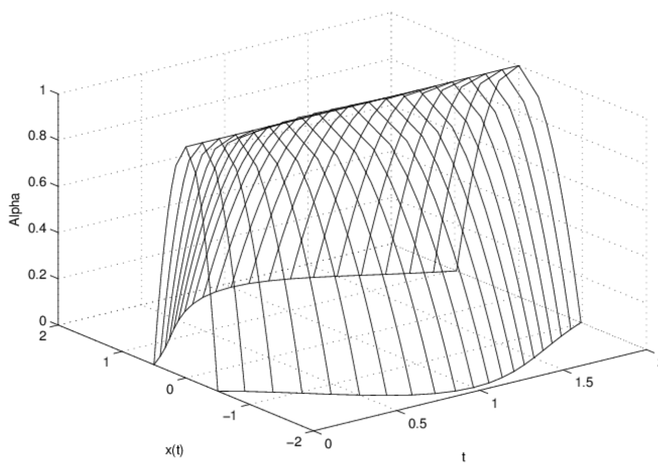


**Figure 3**: The approximation of fuzzy solution by Milne's predictor-corrector (h=0.1)

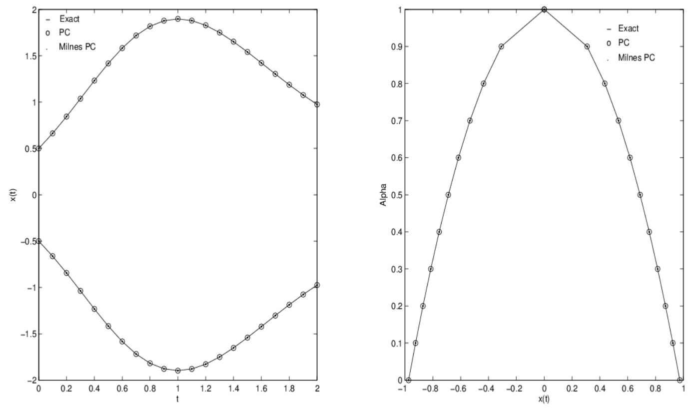


**Figure 4**: Comparison between the exact, Milne's predictor-corrector, predictor-corrector

In this example, we compare the solution obtained by Milne's predictor-corrector method with the exact solution and predictor-corrector. We have given the numerical values in Table 2 fixed value of $t_{20} = 2$. and for different values of $\alpha$.

## 6. CONCLUSION

In this paper we used the Milne's predictor-corrector method for solving FIVP by considering the dependency problem in fuzzy computation. We compared the solutions obtained in two numerical examples.

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