MODELING OF REDISTRIBUTION OF INFUSED DOPANT IN A MULTILAYER STRUCTURE DOPANT UNDER INFLUENCE OF VARIATION OF PRESSURE OF VAPOR OF THE DOPANT

E.L. Pankratov[1], E.A. Bulaeva[2]

[1] Nizhny Novgorod State University, 23 Gagarin avenue, Nizhny Novgorod, 603950, Russia
[2] Nizhny Novgorod State University of Architecture and Civil Engineering, 65 Il'insky street, Nizhny Novgorod, 603950, Russia

ABSTRACT

In this paper we used an analytical approach to model nonlinear diffusion of dopant in a multilayer structure with account nonstationary annealing of the dopant. The approach do without crosslinking solutions at the interface between layers of the multilayer structure. In this paper we analyzed influence of pressure of vapor of infusing dopant during doping of multilayer structure on values of optimal parameters of technological process to manufacture p-n-junctions. It has been shown, that doping of multilayer structures by diffusion and optimization of annealing of dopant gives us possibility to increase sharpness of p-n-junctions (single p-n-junctions and p-n-junctions within transistors) and to increase homogeneity of dopant distribution in doped area.

KEYWORDS

Dopant diffusion; multilayer structure; p-n-heterojunctions; optimization of technological process; influence of pressure of vapor

1. INTRODUCTION

One of actual problems of solid state electronics is increasing of degree of integration of elements of integrated circuits (*p-n-* junctions, field and bipolar transistors, thyristors, …) [1-9]. At the same time one can find decreasing dimensions of the elements. To decrease the dimensions different approaches are used. One group of the approaches including into itself laser and microwave types of annealing [10-12]. One can found that during laser and microwave types of annealing inhomogenous distribution of temperature is generated. In this situation dimensions of elements of integrated circuits decreases due to Arrhenius law. To decrease dimensions elements of integrated circuits it could be also used of inhomogeneity of heterostructures [13-15]. However it is necessary to optimize technological process in this case [16,17]. It is known, that radiation damage of semiconductor materials leads to changing of distribution of dopant concentration in *p-n-* junctions and transistors [9,13,15,18]. In this situation radiation damage of semiconductor materials attracted an interest [19].

This paper is about doping by diffusion of a heterostructure, which consist of a substrate with known type of conductivity (*p* or *n*) and epitaxial layer (see Fig. 1). A dopant has been infused in

the epitaxial layer from gaseous source to produce required type of conductivity (*n* or *p*). It is known, that under special conditions sharpness of *p-n-* junctions increases [16,17]. Main our aim framework the present paper is analysis of influence of pressure of vapor in source of dopant on distribution of concentration of dopant in *p-n-* junction.
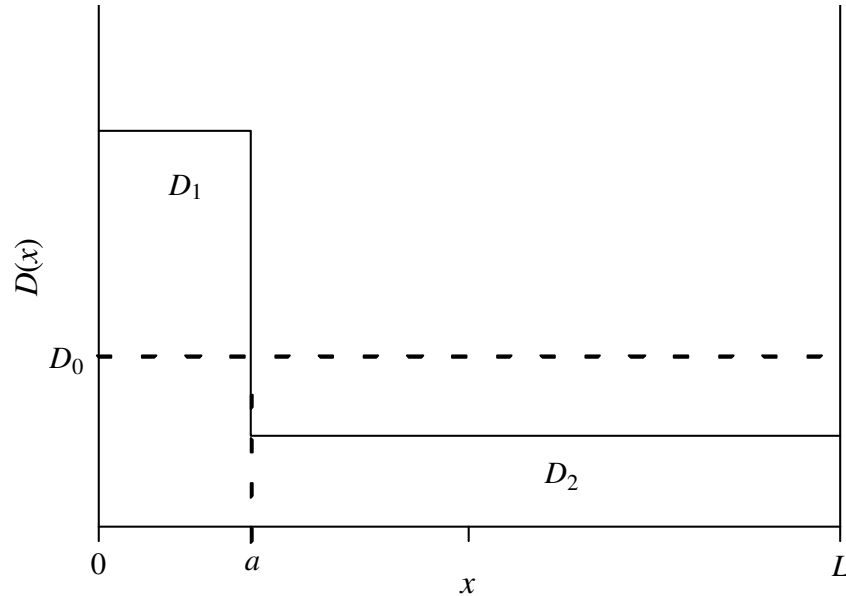

Fig. 1. Heterostructure, which consist of a substrate and an epitaxial layer

2. Method of solution

We solve our aim by analysis of spatio-temporal distribution of concentration of dopant. To determine the spatio-temporal distribution we solve the second Fick's law [8,9,13]

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,t)}{\partial x}\right] \tag{1}$$

with boundary and initial conditions

$$C(0,t)=N, \left.\frac{\partial C(x,t)}{\partial x}\right|_{x=L} = 0, \, C(x>0,0)=0. \tag{2}$$

We assume, that dopant infusing from infinite source with near-boundary concentration *N*, which is essentially larger, than limit of solubility of dopant *P*. In the Eq.(1) and conditions (2) we used the following notations: *C* (*x*,*t*) is the spatio-temporal distribution of concentration of dopant; *T* is the temperature of annealing; D_C is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on temperature through the Arrhenius law (during heating and cooling of heterostructure) and properties of materials of layers of heterostructure. Properties of materials depends on level of doping of materials. Approximation of dependences of dopant diffusion coefficient on parameters could be written as [8]

$$D_C = D_L(x,T)\left[1 + \xi \frac{C^\gamma(x,t)}{P^\gamma(x,T)}\right]. \tag{3}$$

Here $D_L(x,T)$ taking into account the spatial (i.e. accounting several layers in heterostructure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficient; $P(x,T)$ is the dependence of limit of solubility of dopant on coordinate and temperature; parameter γ depends on properties of materials and could be integer in the following interval $\gamma \in [1,3]$ [8]. Concentrational dependence of dopant diffusion coefficient has been described in details in [8].

To solve our aim let us determine solution of Eq.(1) and make analysis of dynamics of dopant. To calculate analytical solution of Eq.(1) we used recently elaborated approach [16,17,19]. Framework the approach we transform approximation of dopant diffusion coefficient to the following form: $D_C = D_{0L}[1 + \varepsilon \, \eta(x,T)][1 + \xi \, C^\gamma(x,t)/P^\gamma(x,T)]$, where $0 \leq \varepsilon < 1$, $|\eta(x,T)| \leq 1$, D_{0L} is the average value of dopant diffusion coefficient. Farther we solve Eq.(1) in the following form

$$C(x,t) = \sum_{k=0}^{\infty} \varepsilon^k \sum_{m=0}^{\infty} \xi^m C_{km}(x,t). \tag{4}$$

Functions $C_{km}(x,t)$ could be determine by solution of the following system of equation

$$\frac{\partial C_{00}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{00}(x,t)}{\partial x^2}$$

$$\frac{\partial C_{k0}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{k0}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left[\eta(x,T)\frac{\partial C_{k-10}(x,t)}{\partial x^2}\right], k \geq 1$$

$$\frac{\partial C_{0m}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{0m}(x,t)}{\partial x^2} + D_{0L} \frac{\partial}{\partial x}\left\{\left[\frac{C_{00}(x,t)}{P(x,T)}\right]^\gamma \frac{\partial C_{0m-1}(x,t)}{\partial x^2}\right\}, m \geq 1 \tag{5}$$

$$\frac{\partial C_{11}(x,t)}{\partial t} = D_{0L} \frac{\partial^2 C_{11}(x,t)}{\partial x^2} + D_{0L} \frac{\partial^2}{\partial x^2}\left[C_{10}(x,t)\frac{C_{00}^\gamma(x,t)}{P^\gamma(x,T)}\right] +$$

$$+ D_{0L} \frac{\partial}{\partial x}\left[\eta(x,T)\frac{\partial C_{01}(x,t)}{\partial x^2}\right] + D_{0L} \frac{\partial}{\partial x}\left[\eta(x,T)\left(\frac{C_{00}(x,t)}{P(x,T)}\right)^\gamma \frac{\partial C_{00}(x,t)}{\partial x}\right]$$

with boundary and initial conditions

$$C_{00}(0,t)=N, \frac{\partial C_{km}(x,t)}{\partial x}\bigg|_{x=0} = 0, k \geq 1, m \geq 1; C_{km}(0,t)=0, C_{km}(x>0,0)=0, k \geq 1, m \geq 1;$$

$$C_{00}(x,0)=0, C_{00}(0,0)=N; C_{km}(x,0)=0, k \geq 1, m \geq 1. \tag{6}$$

Solutions of the system of equations (5) with account conditions (6) could be obtain by standard approaches [20,21] and could be written as

$$C_{00}(0,t)=N, \; C_{00}(x>0,t)= P_0\left[1+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{\sin(v_{n+0.5}x)e_{n+0.5}(t)}{n+0.5}\right],$$

where $v_n=\pi n L^{-1}$, $F_n = \int_0^L f(u)\cos(v_n u)d u$, $e_n(t)=\exp(-v_n^2 D_{0L}t)$, P_0 is the average value of limit solubility of dopant

$$C_{10}(x,t)= \frac{2D_0\pi}{L^3}\sum_{n=0}^{\infty}(n+0.5)\sin(v_{n+0.5}x)e_{n+0.5}(t)\times$$

$$\times\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)e_{n+0.5}(u)[H_{n+m+1}(u)+H_{n-m}(u)]d u ,$$

$$C_{20}(x,t)=-\frac{2}{L^6}\sum_{k=0}^{\infty}(k+0.5)\sin(v_{k+0.5}x)e_{k+0.5}(t)\sum_{n=0}^{\infty}(n+0.5)^2\sum_{m=1}^{\infty}\int_0^t e_{k+0.5}(-u)e_{n+0.5}(u)\times$$

$$\times[H_{n-k}(u)+H_{n+k+1}(u)]\int_0^u e_{n+0.5}(-\tau)e_{m+0.5}(\tau)[H_{n-m}(\tau)+H_{n+m+1}(\tau)]d\tau\, d u\,\pi^3 D_0^2 ,$$

where $H_n(t)=\int_0^L \eta(u,T)P(u,T)\sin(v_n u)d u$.

$$C_{01}(x,t)=-\gamma\alpha_1 - \alpha_2 ,$$

where $\alpha_1 = \dfrac{2}{\pi^3}\sum_{n=0}^{\infty}(n+0.5)^2\sin(v_{n+0.5}x)\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\dfrac{e_{n+0.5}(t)[e_{km}(t)-e_{n+0.5}(t)]}{(n+0.5)^2-(m+0.5)^2-(k+0.5)^2}\times$

$\times\dfrac{1}{k+0.5}\left\{\left[(m+0.5)^2-(n-k)^2\right]^{-1}-\left[(m+0.5)^2-(n+k+1)^2\right]^{-1}\right\},$ $\qquad e_{km}(t)=e_{k+0.5}$

$(t)e_{m+0.5}(t),$

$$\alpha_2 = \begin{cases} 0, \gamma<3 \\ \dfrac{1}{\pi^5}\sum_{n=0}^{\infty}(n+0.5)^3\sin(v_{n+0.5}x)e_{n+0.5}(t)\sum_{k=0}^{\infty}\dfrac{1}{k+0.5}\sum_{l=0}^{\infty}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\{\left[nn-(k-l+i-j)^2\right]^{-1}+ \right. \end{cases}$$

$$+\left[(n+0.5)^2-(k-l-i+j)^2\right]^{-1}-\left[(n+0.5)^2-(k-l+i+j+1)^2\right]^{-1}-\left[(n+0.5)^2-\right.$$

$$-(k-l-i-j+1)^2\right]^{-1}-\left[(n+0.5)^2-(k+l+i-j+1)^2\right]^{-1}-$$

$$-\left[(n+0.5)^2-(i-j-k-l+1)^2\right]^{-1}+$$

$$+\left[(n+0.5)^2-(i+j-k-l)^{2-1}\right]+\left[(n+0.5)^2-(i+j+k+l-2)^2\right]^{-1}\right\}\times$$

$$\times\frac{(l+0.5)^{-1}(i+0.5)^{-1}(j+0.5)^{-1}\left[e_{klij}(t)-e_{n+0,5}(t)\right]}{(n+0.5)^2-(k+0.5)^2-(l+0.5)^2-(i+0.5)^2-(j+0.5)^2} , \gamma =3,$$

$$C(x,t)=-\gamma^2\alpha_3 - \alpha_4 ,$$

where

$$\alpha_3 = \frac{4}{\pi^5} \sum_{n=0}^{\infty} (n+0.5)^2 \sin(v_{n+0.5}x) e_{n+0.5}(t) \sum_{k=1}^{\infty} \frac{1}{k+0.5} \sum_{l=0}^{\infty} \left[(n-k)^2 - (n+k+1)^2 \right] \times$$

$$\times \left[(l+0.5)^2 - (n-k)^2 \right]^{-1} \left[(l+0.5)^2 - (n+k+1)^2 \right]^{-1} \sum_{m=1}^{\infty} \sum_{i=0}^{\infty} \frac{1}{i+0.5} \sum_{j=0}^{\infty} \left[(l+i+1)^2 - (l-i)^2 \right] \times$$

$$\times \left[(l+0.5)^2 - (i+0.5)^2 - (j+0.5)^2 \right]^{-1} \left[(j+0.5)^2 - (l+i+1)^2 \right]^{-1} \left[(j+0.5)^2 - (l-i)^2 \right]^{-1} \times$$

$$\times (l+0.5)^3 \sum_{i=0}^{\infty} \frac{1}{i+0.5} \sum_{j=0}^{\infty} \frac{(l+i+1)^2 - (l-i)^2}{\left[(l+0.5)^2 - (i+0.5)^2 - (j+0.5)^2 \right] \left[jj - (l+i+1)^2 \right] \left[jj - (l-i)^2 \right]} \times$$

$$\times \left[\frac{e_{klj}(t) - e_{n+0.5}(t)}{(n+0.5)^2 - (k+0.5)^2 - (i+0.5)^2 - (j+0.5)^2} - \frac{e_{kl}(t) - e_{n+0.5}(t)}{(n+0.5)^2 - (k+0.5)^2 - (l+0.5)^2} \right],$$

$$\alpha_4 = \begin{cases} 0, \gamma = 1, \gamma = 2 \\ \dfrac{3}{\pi^7} \displaystyle\sum_{n=0}^{\infty} (n+0.5)^4 \sin(v_{n+0.5}x) e_{n+0.5}(t) \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \dfrac{(j+0.5)^2}{i+0.5} \left\{ \left[(n+0.5)^2 - (i-j)^2 \right]^{-1} - \right. \end{cases}$$

$$\left[(n+0.5)^2 - (i+j+1)^2 \right]^{-1} \sum_{k=0}^{\infty} \frac{1}{k+0.5} \sum_{l=0}^{\infty} \frac{1}{l+0.5} \sum_{m_1=0}^{\infty} \frac{1}{m_1+0.5} \sum_{m_2=0}^{\infty} \left\{ \left[(j+0.5)^2 - \right. \right.$$

$$- (k-l+m_1-m_2)^2 \right]^{-1} + \left[(j+0.5)^2 - (k-l-m_1+m_2)^2 \right]^{-1} - \left[(j+0.5)^2 - \right.$$

$$- (k-l+m_1+m_2)^2 \right]^{-1} - \left[(j+0.5)^2 - (k-l-m_1-m_2-1)^2 \right]^{-1} - \left[(j+0.5)^2 - \right.$$

$$- (k+l+m_1-m_2-1)^2 \right]^{-1} - \left[(j+0.5)^2 - (k+l-m_1+m_2-1)^2 \right]^{-1} + \left[(j+0.5)^2 - \right.$$

$$- (k+l-m_1-m_2)^2 \right]^{-1} + \left[(j+0.5)^2 - (k+l+m_1+m_2+2)^2 \right]^{-1} \right\} \times$$

$$\times \left\{ \frac{e_{iklm_1m_2}(t) - e_n(t)}{(n+0.5)^2 - (i+0.5)^2 - (j+0.5)^2 - (k+0.5)^2 - (l+0.5)^2 - (m_1+0.5)^2 - (m_2+0.5)^2} - \right.$$

$$- \frac{e_{ij}(t) - e_n(t)}{(n+0.5)^2 - (i+0.5)^2 - (j+0.5)^2} \left. \right\} \frac{1}{m_2+0.5} + \frac{4}{\pi^7} \sum_{n_1=0}^{\infty} (n_1+0.5)^3 \sin(v_{n_1+0.5}x) \times$$

$$\times \sum_{n_2=0}^{\infty} \frac{1}{n_2+0.5} \sum_{n_3=0}^{\infty} \frac{1}{n_3+0.5} \sum_{n_4=0}^{\infty} \frac{1}{n_4+0.5} \sum_{n_5=0}^{\infty} \left\{ \left[(n_1+0.5)^2 - (n_2-n_3-n_4+n_5)^2 \right]^{-1} + \right.$$

$$+ \left[(n_1+0.5)^2 - (n_2-n_3+n_4-n_5)^2 \right]^{-1} - \left[(n_1+0.5)^2 - (n_2+n_3+n_4-n_5+1)^2 \right]^{-1} -$$

$$- \left[(n_1+0.5)^2 - (n_2+n_3-n_4+n_5+1)^2 \right]^{-1} + \left[(n_1+0.5)^2 - (n_2-n_3+n_4-n_5)^2 \right]^{-1} -$$

$$- \left[(n_1+0.5)^2 - (n_2+n_3+n_4-n_5+1)^2 \right]^{-1} - \left[(n_1+0.5)^2 - (n_2+n_3-n_4+n_5+1)^2 \right]^{-1} -$$

$$- \left[(n_1+0.5)^2 - (n_2-n_3+n_4+n_5+1)^2 \right]^{-1} - \left[(n_1+0.5)^2 - (n_2-n_3-n_4-n_5-1)^2 \right]^{-1} +$$

$$+ \left[(n_1+0.5)^2 - (n_2+n_3-n_4-n_5)^2 \right]^{-1} + \left[(n_1+0.5)^2 - (n_2+n_3+n_4+n_5+2)^2 \right]^{-1} \right\} \times$$

$$\times \sum_{k=0}^{\infty} \frac{(n_5+0.5)^2}{k+0.5} \sum_{m=0}^{\infty} \left[\frac{1}{(m+0.5)^2 - (n_5-k)^2} - \frac{1}{(m+0.5)^2 - (n_5+k+1)^2} \right] \times$$

$$\times \left[\frac{e_{n_2 n_3 n_4 km}(t) - e_{n_1}(t)}{(n_1+0.5)^2 - (n_2+0.5)^2 - (n_3+0.5)^2 - (n_4+0.5)^2 - (k+0.5)^2 - (m+0.5)^2} - \right.$$

$$\left. - \frac{e_{n_5}(t) - e_{n_1}(t)}{(n_1+0.5)^2 - (n_5+0.5)^2} \right] \frac{1}{(n_5+0.5)^2 - (k+0.5)^2 - (m+0.5)^2}.$$

$$C_{11}(x,t) = \alpha_5 + 2\alpha_6 + 2\alpha_7 - 2\alpha_8 + 2\alpha_9 + \alpha_{10} + 4\alpha_{11},$$

where

$$\alpha_5 = \frac{2\gamma D_{0L}}{\pi L^3} \sum_{l=0}^{\infty} (l+0.5) \sin(v_{l+0.5}x) e_{l+0.5}(t) \sum_{n=0}^{\infty} (n+0.5)^3 \sum_{k=0}^{\infty} \frac{1}{k+0.5} \sum_{m=0}^{\infty} \left\{ \left[(m+0.5)^2 - \right. \right.$$

$$- (n-k)^2 \right]^{-1} - \left[(m+0.5)^2 - (n+k+1)^2 \right]^{-1} \right\} \left[(n+0.5)^2 - (k+0.5)^2 - (n+0.5)^2 \right]^{-1} \times$$

$$\times \int_0^t \left[H_{l-n}(u) + H_{l+n+1}(u) \right] \left[e_{l+0.5}(-u) e_{k+0.5}(u) e_{m+0.5}(u) - e_{l+0.5}(-u) e_{n+0.5}(u) \right] du,$$

$$\alpha_6 = \frac{\gamma D_{0L}^2}{L^5} \sum_{n=0}^{\infty} (n+0.5)^2 \sin(v_{n+0.5}x) e_{n+0.5}(t) \sum_{k=0}^{\infty} (k+0.5)^3 \sum_{l=0}^{\infty} \left\{ \left[(l+0.5)^2 - (n-k)^2 \right]^{-1} - \right.$$

$$-\left[(l+0.5)^2-(n+k+1)^2\right]^{-1}\Big\}\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)e_{k+0.5}(u)e_{l+0.5}(u)\int_0^u\left[H_{l-m}(\tau)+H_{l+m+1}(\tau)\right]\times$$

$$\times e_{l+0.5}(-\tau)\,e_{m+0.5}(\tau)\,d\,\tau\,d\,u\,(l+0.5)^2\,,$$

$$\alpha_7=D_{0L}\frac{\pi}{L^3}\sum_{n=0}^{\infty}(n+0.5)e_{n+0.5}(t)\sin(v_{n+0.5}x)\times$$

$$\times\sum_{m=0}^{\infty}\int_0^t e_{n+0.5}(-u)e_{m+0.5}(u)\left[H_{m-n}(u)+H_{m+n+1}(u)\right]d\,u\,,$$

$$\alpha_8=\frac{\gamma\,\pi\,D_{0L}}{L^3}\sum_{n=0}^{\infty}(n+0,5)\sin(v_{n+0,5}x)e_{n+0,5}(t)\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{k+0,5}\int_0^t e_{n+0,5}(-u)e_{km}(u)\times$$

$$\times[H_{n+k-m-0,5}(u)+H_{n-k+m+0,5}(u)-H_{n-k-m-0,5}(u)-H_{n+k+m+1,5}(u)]d\,u\,,$$

$$\alpha_9=\begin{cases}0,\gamma=1\\[2mm]\dfrac{\gamma\,D_{0L}}{\pi\,L^3}\displaystyle\sum_{n=0}^{\infty}(n+0.5)e_{n+0.5}(t)\sin(v_{n+0.5}x)\sum_{k=0}^{\infty}\dfrac{1}{k+0.5}\sum_{l=0}^{\infty}\dfrac{1}{l+0.5}\sum_{m=0}^{\infty}\int_0^t e_m(-\tau)\times\end{cases}$$

$$\times e_{klm}(\tau)[I_{k-l-n+m}(T)+I_{k-l+n-m}(T)+I_{k-l-n-m-1}(T)+I_{k-l+n+m+1}(T)-I_{k-l-n-m-1}(T)-$$
$$-I_{k+l+n-m+1}(T)-I_{m-k-l+n-}(T)-I_{m+k+l+n+2-}(T)]d\,\tau,\ \gamma>2,$$

where $I_n(T)=\int_0^L\eta(u,T)\cos(v_n u)d\,u$,

$$\alpha_{10}=\begin{cases}0,\gamma<3\\[2mm]\dfrac{2D_{0L}}{\pi^3 L^3}\displaystyle\sum_{i=0}^{\infty}(i+0.5)e_{i+0.5}(t)\sin(v_{i+0.5}x)\sum_{n=0}^{\infty}(n+0.5)^2\int_0^t\left[I_{i-n}(T)+I_{i+n+1}(T)\right]\times\end{cases}$$

$$\times\sum_{k=0}^{\infty}\frac{1}{k+0.5}\sum_{l=0}^{\infty}\frac{1}{l+0.5}\sum_{m_1=0}^{\infty}\frac{1}{m_1+0.5}\sum_{m_2=0}^{\infty}\frac{1}{m_2+0.5}\Big\{\left[(n+0.5)^2-(k-l+m_1-m_2)^2\right]^{-1}+$$

$$+[(n+0.5)^2-(k-l-m_1+m_2)^2]^{-1}-[(n+0.5)^2-(k-l+m_1+m_2+1)]^{-1}-[(n+0.5)^2-(k-l-m_1-m_2-1)]^{-1}-$$

$$-[(n+0.5)^2-(k+l+m_1--m_2+1)]^{-1}-[(n+0.5)^2-(k+l-m_1+m_2++1)]^{-1}+[(n+0.5)^2-(k+l-m_1-$$

$$-m_2)]^{-1}+[(n+0.5)^2-(k+l+m_1+m_2+2)]^{-1}\Big\}[(n+0.5)^2-(k+0.5)^2-(l+0.5)^2-(m_1+0.5)^2-$$

$$-\left(m_2+0.5\right)^{-1}\left]\left[e_i(-u)e_{klm_1m_2}(u)-e_i(-u)e_n(u)\right]d\,u\,,\ \gamma>3,$$

$$\alpha_{11} = \begin{cases} 0, \gamma < 3 \\ \dfrac{D_{0L}^2}{\pi^3 L^5} \sum_{i=0}^{\infty} (i+0.5)^3 e_{i+0.5}(t) \sin(v_{i+0.5}x) \sum_{j=0}^{\infty} \dfrac{1}{j+0.5} \sum_{k=0}^{\infty} \dfrac{1}{k+0.5} \sum_{l=0}^{\infty} \dfrac{1}{l+0.5} \sum_{n=0}^{\infty} (n+0.5) \times \end{cases}$$

$$\times \{(i+0.5)^2 - (j-k+l-n)^2]^{-1} + [(i+0.5)^2 - (j-k-l+n)^2]^{-1} - [(i+0.5)^2 - (j+k-l+n+1)^2]^{-1} -$$

$$- [(i+0.5)^2 - (j+k-l-n+1)^2]^{-1} - [(i+0.5)^2 - (j+k-l+n+1)^2]^{-1} + [(i+0.5)^2 - (j+k-l-n)^2]^{-1} -$$

$$[(i+0.5) - (j+k+l+n+1)^2]^{-1} \} \sum_{m=0}^{\infty} \int_0^t \int_0^u e_{n+0.5}(-\tau) e_m(\tau) [I_{n-m}(T) + I_{n+m+1}(T)] d\tau \times$$

$$\times e_{i+0.5}(-u) e_{jk\ln}(u) \, d u \,, \gamma = 3$$

$$\alpha_{11} = \begin{cases} 0, \gamma < 3 \\ \dfrac{D_{0L}^2}{\pi^2 L^3} \sum_{n=0}^{\infty} (n+0.5) e_{n+0.5}(t) \sin(v_{n+0.5}x) \sum_{k=0}^{\infty} \dfrac{1}{k+0.5} \sum_{l=0}^{\infty} \dfrac{1}{l+0.5} \sum_{m=0}^{\infty} \dfrac{1}{m+0.5} \times \end{cases}$$

$$\times \sum_{i=0}^{\infty} \int_0^t e_{n+0.5}(-u) \, e_{klmi}(u) \{ H_{k-n+i-l+m+0.5}(u) + H_{k-n-i+l-m+0.5}(u) + H_{k-n+i+l-m+0.5}(u) +$$

$$+ H_{k+n-i-l+m+0.5}(u) - H_{k+n-i-l+m-0.5}(u) - H_{k-n+l+l-m+1.5}(u) - H_{k-n+i-l+m-0.5}(u) -$$

$$- H_{k+n-i+l-m+1.5}(u) + H_{k+n+i-l+m+1.5}(u) + H_{k-n-i+l-m-0.5}(u) + + H_{k-n-i-l+m-0.5}(u) +$$

$$+ H_{k+n+i+l-m+1.5}(u) - H_{k+n+i-l-m+0.5}(u) - H_{k-n-i+l+m+0.5}(u) - H_{k-n-i-l-m-1.5}(u) -$$

$$- H_{k+n+l+l+m+2.5}(u) \} d u.$$

Analysis of spatiotemporal distributions of dopant concentrations has been done analytically by using the second-order approximation of dopant concentration on parameters ε and ξ. Farther the distribution has been amended numerically.

3. DISCUSSION

In this section we analyzed dynamics of redistribution of dopant in heterostructure from Fig. 1 based on calculated in previous section relations. Spatial distributions of concentration of dopant in the considered heterostructure are presented on Fig. 2 for fixed value of annealing time. The Fig. 2 shows, that increasing of difference between values of dopant diffusion coefficient in the substrate and in the epitaxial layer gives us possibility to increase sharpness of *p-n*-junction and at the same time to increase homogeneity of dopant distribution in doped area. However using this type of doping leads to necessity in optimization of annealing time. Reason of this optimization is following. If annealing time is small, dopant cannot achieves interface between layers of heterostructure. In this situation homogeneity of distribution of concentration of dopant became less, than in heterostructure. If annealing time is large, distribution of concentration of dopant became overly homogenous. We determine optimal annealing time by using recently introduced criterion [16,17,19,22]. Framework the approach we approximate real distribution of concentration of do-

pant by step-wise function (see Fig. 3). Farther we determine optimal value of annealing time by minimization of the following mean-squared error

$$U = \frac{1}{L}\int_0^L [C(x,\Theta) - \psi(x)]\,dx. \qquad (7)$$

Here $\psi(x)$ is the approximation time. Θ is the optimal value of annealing time. Dependences of optimal value of annealing time on parameters are presented in Fig. 4.
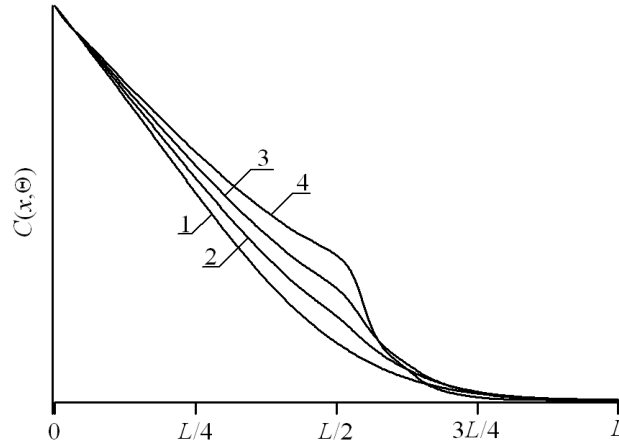
Fig. 2. Distributions of concentration of dopant in heterostructure from Fig. 1 in direction x. The direction x is perpendicular to interface between layers. Increasing of number of curves corresponds to increasing of value of parameter ε. This curves correspond to situation, when value of dopant diffusion coefficient in the epitaxial layer is larger, than in the substrate
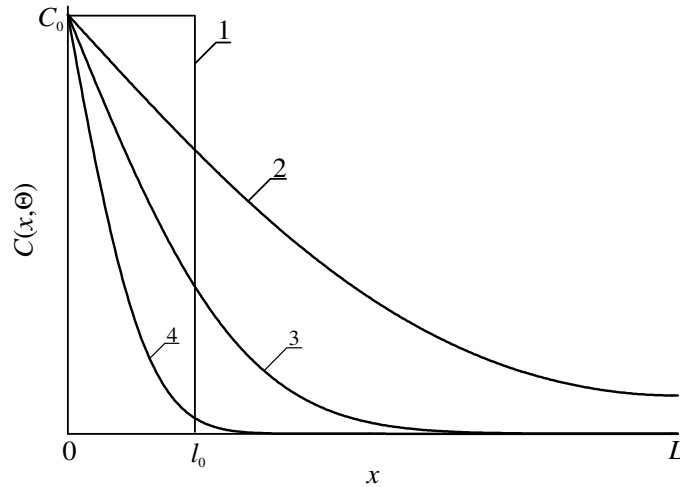
Fig.3. Spatial distributions of concentration of dopant. Curve 1 is required idealized distribution of concentration of dopant. Curves 2-4 are real distributions of concentration of dopant.

Increasing of number of curves corresponds to increasing of annealing time

Farther we analyzed influence of value of pressure of vapor of dopant from infinite source on distribution of concentration of the dopant in the considered heterostructure. We assume, that ga-

seous source of dopant is ideal gas. In this case pressure of gas and surficial concentration of dopant are correlated with each other by linear law: $p M = R TN$, where M is the molar mass, $R = 8.31$ J/(mole·K) is the gas constant, p is the pressure of gas. In this situation increasing of pressure of gas in source of dopant leads to proportional increasing of surficial concentration of dopant. However dependence of optimal value of annealing time on pressure is not so simple due to non-linearity of criterion of estimation of the time [16,17,19,22,23]. Analysis of dynamics of redistribution of dopant shows, that variation of value of dynamics of vapor leads to quantitative variation of distribution of concentration of dopant inside of the considered heterostructure, but not to quantitative variation.
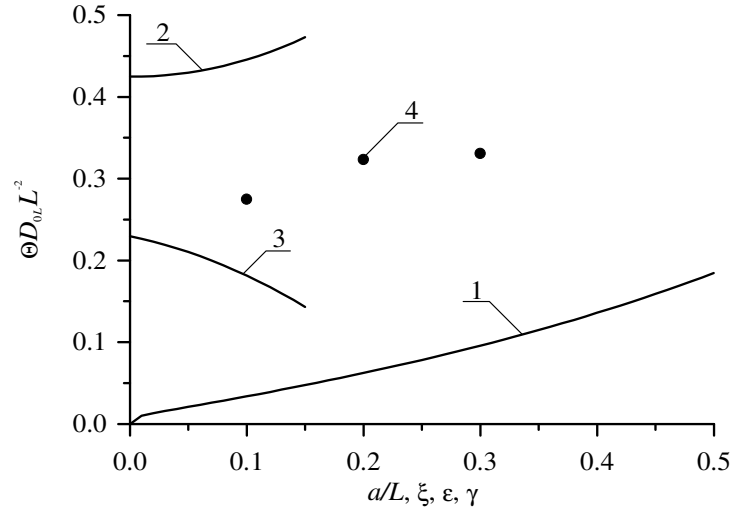

Fig.4. Dependences of dimensionless optimal annealing time, which have been obtained by minimization of mean-squared error (7), on several parameters. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for $a/L = 1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for $a/L = 1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for $a/L = 1/2$ and $\varepsilon = \xi = 0$

4. CONCLUSION

In this paper we analyzed influence of changing of pressure of vapor of dopant from infinite source on distribution of concentration of the dopant in the p-n- heterojunction. It has been shown, that the changing of pressure leads to some quantitative variation of distribution of concentration of dopant, but not to quantitative variation.

Acknowledgements

REFERENCES

[1] A. Kerentsev, V. Lanin. "Design and technological features of MOSFET-transistors" Power Electronics. Issue 1. P. 34 (2008).

[2] A.O. Ageev, A.E. Belyaev, N.S. Boltovets, V.N. Ivanov, R.V. Konakova, Ya.Ya. Kudrik, P.M. Litvin, V.V. Milenin, A.V. Sachenko. "Au–TiBx–n-6H-SiC Schottky barrier diodes: the features of current flow in rectifying and nonrectifying contacts". Semiconductors. Vol. 43 (7). P. 897-903 (2009).

[3] Jung-Hui Tsai, Shao-Yen Chiu, Wen-Shiung Lour, Der-Feng Guo. "High-performance InGaP/GaAs pnp δ-doped heterojunction bipolar transistor". Semiconductors. Vol. 43 (7). P. 971-974 (2009).

[4] E.I. Gol'dman, N.F. Kukharskaya, V.G. Naryshkina, G.V. Chuchueva. "The manife-station of excessive centers of the electron-hole pair generation, appeared as a result to field and thermal stresses, and their subsequent annihilation in the dynamic current-voltage characteristics of Si-MOS-structures with the ultrathin oxide". Semiconduc-tors. Vol. 45 (7). P. 974-979 (2011).

[5] T.Y. Peng, S.Y. Chen, L.C. Hsieh C.K. Lo, Y.W. Huang, W.C. Chien, Y.D. Yao. "Impedance behavior of spin-valve transistor". J. Appl. Phys. Vol. 99 (8). P. 08H710-08H712 (2006).

[6] W. Ou-Yang, M. Weis, D. Taguchi, X. Chen, T. Manaka, M. Iwamoto. "Modeling of threshold voltage in pentacene organic field-effect transistors". J. Appl. Phys. Vol. 107 (12). P. 124506-124510 (2010).

[7] J. Wang, L. Wang, L. Wang, Z. Hao, Yi Luo, A. Dempewolf, M. M ller, F. Bertram, J rgen Christen. "An improved carrier rate model to evaluate internal quantum effi-ciency and analyze efficiency droop origin of InGaN based light-emitting diodes". J. Appl. Phys. Vol. 112 (2). P. 023107-023112 (2012).

[8] Z.Yu. Gotra, Technology of microelectronic devices (Radio and communication, Moscow, 1991).

[9] V.I. Lachin, N.S. Savelov, Electronics (Phoenix, Rostov-na-Donu, 2001).

[10] K.K. Ong, K.L. Pey, P.S. Lee, A.T.S. Wee, X.C. Wang, Y.F. Chong. "Dopant distri-bution in the re-crystallization transient at the maximum melt depth induced by laser annealing". Appl. Phys. Lett. 89 (17), 172111-172114 (2006).

[11] H.T. Wang, L.S. Tan, E. F. Chor. "Pulsed laser annealing of Be-implanted GaN". J. Appl. Phys. 98 (9), 094901-094905 (2006).

[12] Yu. V. Bykov, A. G. Eremeev, N. A. Zharova, I. V. Plotnikov, K. I. Rybakov, M. N. Drozdov, Yu. N. Drozdov and V. D. Skupov, "Diffusion processes in semiconductor structures during microwave annealing". Radiophys. Quant. Electron., vol. 46 (8). P. 749 (2003).

[13] I.P. Stepanenko. Basis of Microelectronics (Soviet Radio, Moscow, 1980).

[14] A.G. Alexenko, I.I. Shagurin. Microcircuitry (Radio and communication, Moscow, 1990).

[15] N.A. Avaev, Yu.E. Naumov, V.T. Frolkin. Basis of microelectronics (Radio and communication, Moscow, 1991).

[16] E.L. Pankratov. "Influence of the spatial, temporal, and concentrational dependence of the diffusion coefficient on dopant dynamics: optimization of annealing time". Phys. Rev. B. Vol. 72 (7). P. 075201-075208 (2005).

[17] E.L. Pankratov. "Dopant Diffusion Dynamics and Optimal Diffusion Time as Influ-enced by Diffusion-Coefficient Nonuniformity". Russian Microelectronics. 2007. V.36 (1). P. 33-39.

[18] V.V. Kozlivsky. Modification of semiconductors by proton beams (Nauka, Sant-Peterburg, 2003, in Russian).

[19] E.L. Pankratov, E.A. Bulaeva. " Doping of materials during manufacture p-n-junctions and bipolar transistors. Analytical approaches to model technological ap-proaches and ways of optimization of distributions of dopants ". Reviews in Theoreti-cal Science. Vol. 1 (1). P. 58-82 (2013).

[20] A.N. Tikhonov, A.A. Samarskii. The mathematical physics equations (Moscow, Nauka 1972) (in Russian).

[21] H.S. Carslaw, J.C. Jaeger. Conduction of heat in solids. - Oxford: At the Clarendon Press. 1964. 488 p.

[22] E.L. Pankratov, E.A. Bulaeva. "Application of native inhomogeneities to increase compactness of vertical field -effect transistors". J. Comp. Theor. Nanoscience. Vol. 10 (4). P. 888-893 (2013).

[23] E.L. Pankratov, E.A. Bulaeva. "Optimal Criteria to Estimate Temporal Characteris-tics of Diffusion Process in a Media with Inhomogenous and Nonstationary Parame-ters. Analysis of Influence of Variation of Diffusion Coefficient on Values of Time Characteristics". Reviews in Theoretical Science. Vol. 1 (3). P. 305-316 (2013).

Authors

Pankratov Evgeny Leonidovich was born at 1977. From 1985 to 1995 he was educated in a secondary school in Nizhny Novgorod. From 1995 to 2004 he was educated in Nizhny Novgorod State University: from 1995 to 1999 it was bachelor course in Radiophysics, from 1999 to 2001 it was master course in Radiophysics with specialization in Statistical Radiophysics, from 2001 to 2004 it was PhD course in Radiophysics. From 2004 to 2008 E.L. Pankratov was a leading technologist in Institute for Physics of Microstructures. From 2008 to 2012 E.L. Pankratov was a senior lecture/Associate Professor of Nizhny Novgorod State University of Architecture and Civil Engineering. Now E.L. Pankratov is in his Full Doctor course in Radiophysical Department of Nizhny Novgorod State University. He has 96 published papers in area of his researches.

Bulaeva Elena Alexeevna was born at 1991. From 1997 to 2007 she was educated in secondary school of village Kochunovo of Nizhny Novgorod region. From 2007 to 2009 she was educated in boarding school "Center for gifted children". From 2009 she is a student of Nizhny Novgorod State University of Architecture and Civil Engineering (spatiality "Assessment and management of real estate"). At the same time she is a student of courses "Translator in the field of professional communication" and "Design (interior art)" in the University. E.A. Bulaeva was a contributor of grant of President of Russia (grant № MK-548.2010.2). She has 35 published papers in area of her researches.