A Study on Intuitionistic Multi-Anti Fuzzy Subgroups

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ABSTRACT

For any intuitionistic multi-fuzzy set A = { < x , $\mu_A(x)$, $\nu_A(x)$ > : x∈X} of an universe set X, we study the set $[A]_{(\alpha, \beta)}$ called the (α, β)–lower cut of A. It is the crisp multi-set { x∈X : $\mu_i(x) \le \alpha_i$, $\nu_i(x) \ge \beta_i$, ∀i } of X. In this paper, an attempt has been made to study some algebraic structure of intuitionistic multi-anti fuzzy subgroups and their properties with the help of their (α, β)–lower cut sets.

Keywords

Intuitionistic fuzzy set (IFS), Intuitionistic multi-fuzzy set (IMFS), Intuitionistic multi-anti fuzzy subgroup (IMAFSG), Intuitionistic multi-anti fuzzy normal subgroup (IMAFNSG), (□, □)–lower cut, Homomorphism.

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1. INTRODUCTION

After the introduction of the concept of fuzzy set by Zadeh [14] several researches were conducted on the generalization of the notion of fuzzy set. The idea of Intuitionistic fuzzy set was given by Krassimir.T.Atanassov [1]. An Intuitionistic Fuzzy set is characterized by two functions expressing the degree of membership (belongingness) and the degree of non-membership (non-belongingness) of elements of the universe to the IFS. Among the various notions of higher-order fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainity and vagueness. An element of a multi-fuzzy set can occur more than once with possibly the same or different membership values. In this paper we study Intuitionistic multi-anti fuzzy subgroup with the help of some properties of their (α, β)–lower cut sets. This paper is an attempt to combine the two concepts: Intuitionistic Fuzzy sets and Multi-fuzzy sets together by introducing two new concepts called Intuitionistic Multi-fuzzy sets and Intuitionistic Multi-Anti fuzzy subgroups.

2. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition [14]

Let X be a non-empty set. Then a **fuzzy set** $\mu : X \to [0,1]$.

2.2 Definition [9]

Let X be a non-empty set. A **multi-fuzzy set** A of X is defined as $A = \{ < x, \mu_A(x) > : x \in X \}$ where $\mu_A = (\mu_1, \mu_2, \dots, \mu_k)$, that is, $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$ and $\mu_i : X \to [0,1]$, $\forall i=1,2,\dots,k$. Here k is the finite dimension of A. Also note that, for all i, $\mu_i(x)$ is a decreasingly ordered sequence of elements. That is, $\mu_1(x) \geq \mu_2(x) \geq \dots \geq \mu_k(x)$,$\forall x \in X$.

2.3 Definition [1]

Let X be a non-empty set. An **Intuitionistic Fuzzy Set (IFS)** A of X is an object of the form $A = \{< x, \mu(x), \nu(x) > : x \in X\}$, where $\mu : X \to [0, 1]$ and $\nu : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively with $0 \leq \mu(x) + \nu(x) \leq 1$, $\forall x \in X$.

2.4 Remark [1]

(i) Every fuzzy set A on a non-empty set X is obviously an intuitionistic fuzzy set having the form $A = \{< x, \mu(x), 1-\mu(x) > : x \in X\}$.
(ii) In the definition 2.3, When $\mu(x) + \nu(x) = 1$, that is, when $\nu(x) = 1-\mu(x) = \mu^c(x)$, A is called fuzzy set.

2.5 Definition [13]

Let $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X \}$ where $\mu_A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$ and $\nu_A(x) = (\nu_1(x), \nu_2(x), \dots, \nu_k(x))$ such that $0 \leq \mu_i(x) + \nu_i(x) \leq 1$, for all i, $\forall x \in X$. Here, $\mu_1(x) \geq \mu_2(x) \geq \dots \geq \mu_k(x)$, $\forall x \in X$. That is, μ_i 's are decreasingly ordered sequence. That is, $0 \leq \mu_i(x) + \nu_i(x) \leq 1, \forall x \in X$, for i=1, 2, … , k. Then the set A is said to be an **Intuitionistic Multi-Fuzzy Set (IMFS)** with dimension k of X.

2.6 Remark [13]

Note that since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.7 Definition [13]

Let $A = \{ < x , \mu_A(x), \nu_A(x) > : x \in X \}$ and $B = \{ < x , \mu_B(x), \nu_B(x) > : x \in X \}$ be any two IMFS's having the same dimension k of X. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.

(iii) $\neg A = \{ < x, \nu_A(x), \mu_A(x) > : x \in X \}$

(iv) $A \cap B = \{ < x, (\mu_{A\cap B})(x), (\nu_{A\cap B})(x) > : x \in X \}$, where

$(\mu_{A\cap B})(x) = \min\{ \mu_A(x), \mu_B(x) \} = (\min\{\mu_{iA}(x), \mu_{iB}(x)\})^k_{i=1}$ and

$(\nu_{A\cap B})(x) = \max\{ \nu_A(x), \nu_B(x) \} = (\max\{\nu_{iA}(x), \nu_{iB}(x)\})^k_{i=1}$

(v) $A \cup B = \{ < x, (\mu_{A\cup B})(x), (\nu_{A\cup B})(x) > : x \in X \}$, where

$(\mu_{A\cup B})(x) = \max\{ \mu_A(x), \mu_B(x) \} = (\max\{\mu_{iA}(x), \mu_{iB}(x)\})^k_{i=1}$ and

$(\nu_{A\cup B})(x) = \min\{ \nu_A(x), \nu_B(x) \} = (\min\{\nu_{iA}(x), \nu_{iB}(x)\})^k_{i=1}$

Here $\{ \mu_{iA}(x), \mu_{iB}(x) \}$ represents the corresponding i^{th} position membership values of A and B respectively. Also $\{ \nu_{iA}(x), \nu_{iB}(x) \}$ represents the corresponding i^{th} position non-membership values of A and B respectively.

2.8 Theorem [13]

For any three IMFS's A, B and C , we have :

1. Commutative Law

   $A \cup B = B \cup A$
   $A \cap B = B \cap A$

2. Idempotent Law

   $A \cup A = A$
   $A \cap A = A$

3. De Morgan's Laws

   $\neg(A \cup B) = (\neg A \cap \neg B)$
   $\neg(A \cap B) = (\neg A \cup \neg B)$

4. Associative Law

   $A \cup (B \cup C) = (A \cup B) \cup C$
   $A \cap (B \cap C) = (A \cap B) \cap C$

5. Distributive Law

   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2.9 Definition

Let $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in X \}$ be an IMFS and let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k) \in [0,1]^k$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_k) \in [0,1]^k$, where each α_i , $\beta_i \in [0,1]$ with $0 \le \alpha_i + \beta_i \le 1, \forall i$. Then **(α, β)-lower cut** of A is the set of all x such that $\mu_i(x) \le \alpha_i$ with the corresponding $\nu_i(x) \ge \beta_i$,$\forall i$ and is denoted by $[A]_{(\alpha, \beta)}$. Clearly it is a crisp multi-set.

2.10 Definition

Let $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in X \}$ be an IMFS and let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k) \in [0,1]^k$ and $\beta =$

$(\beta_1,\beta_2, \ldots , \beta_k)\in [0,1]^k$, where each α_i , $\beta_i\in [0,1]$ with $0 \le \alpha_i + \beta_i \le 1,\forall i$. Then **strong (α, β)-lower cut** of A is the set of all x such that $\mu_i(x) < \alpha_i$ with the corresponding $\nu_i(x) > \beta_i$,$\forall$i and is denoted by $[A]_{(\alpha, \beta)^*}$. Clearly it is also a crisp multi-set.

The following Theorem is an immediate consequence of the above definitions.

2.11 Theorem [13]

Let A and B are any two IMFS's of dimension k drawn from a set X. Then $A \subseteq B$ if and only if $[B]_{(\alpha ,\beta)} \subseteq [A]_{(\alpha ,\beta)}$ for every $\alpha, \beta\in [0,1]^k$ with $0 \le \alpha_i + \beta_i \le 1$ for all i.

2.12 Definition

An intuitionistic multi-fuzzy set (In short IMFS) $A = \{ < x , \mu_A(x), \nu_A(x) > : x\in G \}$ of a group G is said to be an **intuitionistic multi-anti fuzzy subgroup** of G (In short IMAFSG) if it satisfies the following : For all $x,y\in G$,

(i) $\mu_A(xy) \le \max \{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(x^{-1}) = \mu_A(x)$
(iii) $\nu_A(xy) \ge \min \{\nu_A(x), \nu_A(y)\}$
(iv) $\nu_A(x^{-1}) = \nu_A(x)$

2.13 Definition

An intuitionistic multi-fuzzy set (In short IMFS) $A = \{ < x, \mu_A(x), \nu_A(x) > : x\in G \}$ of a group G is said to be an **intuitionistic multi-anti fuzzy subgroup** of G (In short IMAFSG) if it satisfies :

(i) $\mu_A(xy^{-1}) \le \max\{\mu_A(x), \mu_A(y)\}$ and
(ii) $\nu_A(xy^{-1}) \ge \min\{\nu_A(x), \nu_A(y)\}$,$\forall x,y\in G$

2.13.1 Remark

(i) If A is an IFS of a group G, then we can not say about the complement of A, because it is not an IFS of G.
(ii) If A is an IAFSG of a group G, then A^c is need not be an IFS of G.
(iii) A is an IMAFSG of a group G $\Leftrightarrow$ each IFS $\{ < x, \mu_{iA}(x), \nu_{iA}(x) : x\in G > \}_{i=1}^k$ is an IAFSG of G.
(iv) If A is an IMAFSG of a group G, then in general, we can not say A^c is an IMFSG of the group G.

2.14 Definition

An IMAFSG $A = \{ < x, \mu_A(x), \nu_A(x) > : x\in G \}$ of a group G is said to be an **intuitionistic multi-anti fuzzy normal subgroup** (In short IMAFNSG) of G if it satisfies :

(i) $\mu_A(xy) = \mu_A(yx)$ and
(ii) $\nu_A(xy) = \nu_A(yx)$, for all $x,y\in G$

2.15 Theorem

An intuitionistic multi-anti fuzzy subgroup (IMAFSG) A of a group G is said to be normal if it satisfies :

(i) $\mu_A(g^{-1}xg) = \mu_A(x)$ and

(ii) $\nu_A(g^{-1}xg) = \nu_A(x)$, for all $x \in A$ and $g \in G$

Proof Let $x \in A$ and $g \in G$ be any element.

Then $\mu_A(g^{-1}xg) = \mu_A(g^{-1}(xg)) = \mu_A((xg)g^{-1})$, since A is normal.

$= \mu_A(x(gg^{-1})) = \mu_A(xe) = \mu_A(x)$. Hence the proof (i).

Now, $\nu_A(g^{-1}xg) = \nu_A(g^{-1}(xg)) = \nu_A((xg)g^{-1})$, since A is normal.

$= \nu_A(x(gg^{-1})) = \nu_A(xe) = \nu_A(x)$. Hence the proof (ii).

2.16 Definition

Let (G, .) be a groupoid and A, B be any two IMFS's having same dimension k of G. Then the **product** of A and B is denoted by $A \circ B$ and it is defined as :

$\forall x \in G$, $A \circ B(x) = (\mu_{A \circ B}(x), \nu_{A \circ B}(x))$ where

$$\mu_{A \circ B}(x) = \begin{cases} \max\,[\min\{\mu_A(y), \mu_B(z)\} : yz=x\ ,\ \forall y,z \in G] \\ 0_k=(0, 0, \ldots , k \text{ times})\ , \text{ if } x \text{ is not expressible as } x=yz \qquad \text{and} \\ \min\,[\max\{\nu_A(y), \nu_B(z)\} : yz=x\ ,\ \forall y,z \in G] \end{cases}$$

$$\nu_{A \circ B}(x) = \begin{cases} 1_k=(1, 1, \ldots , k \text{ times})\ , \text{ if } x \text{ is not expressible as } x=yz \end{cases}$$

That is, $\forall x \in G$,

$$A \circ B(x) = \begin{cases} (\ \max[\min\{\mu_A(y), \mu_B(z)\}:yz=x,\forall y,z \in G],\ \min[\max\{\nu_A(y), \nu_B(z)\}:yz=x,\forall y,z \in G]\ ) \\ (0_k,1_k), \text{ if } x \text{ is not expressible as } x=yz \end{cases}$$

That is, $\forall x \in G$,

$$A \circ B(x) = \begin{cases} (\max[\min\{\mu_{iA}(y),\mu_{iB}(z)\}:yz=x,\forall y,z \in G],\min[\max\{\nu_{iA}(y),\nu_{iB}(z)\}:yz=x,\forall y,z \in G])^{k}_{i=1} \\ (0,1)_k\ , \text{if } x \text{ is not expressible as } x=yz \quad \text{where } (0,1)_k=((0,1), (0,1), \ldots , k \text{ times}) \end{cases}$$

2.17 Definition

Let X and Y be any two non-empty sets and $f : X \to Y$ be a mapping. Let A and B be any two IMFS's having same dimension k, of X and Y respectively. Then the **image** of $A(\subseteq X)$ under the map f is denoted by f(A) and it is defined as :

$\forall y \in Y$, $f(A)(y) = (\mu_{f(A)}(y), \nu_{f(A)}(y))$ where

$$\mu_{f(A)}(y) = \begin{cases} \max\{\mu_A(x) : x \in f^{-1}(y)\} \\ 0_k \text{ , otherwise} \end{cases}$$

and

$$\nu_{f(A)}(y) = \begin{cases} \min\{\nu_A(x) : x \in f^{-1}(y)\} \\ 1_k \text{ , otherwise} \end{cases}$$

That is,
$$f(A)(y) = \begin{cases} (\max\{\mu_{iA}(x) : x \in f^{-1}(y)\},\ \min\{\nu_{iA}(x) : x \in f^{-1}(y)\})^{k}_{i=1} \\ (0,1)_k \text{ , otherwise where } (0,1)_k = ((0,1), (0,1), \dots , k \text{ times}) \end{cases}$$

Also, the **pre-image** of $B(\subseteq Y)$ under the map f is denoted by $f^{-1}(B)$ and it is defined as :
$\forall x \in X$, $f^{-1}(B)(x) = (\mu_B(f(x)), \nu_B(f(x)))$

3. PROPERTIES OF (α, β) –LOWER CUT OF INTUITIONISTIC MULTI-FUZZY SET

In this section we shall prove some theorems on intuitionistic multi-anti fuzzy subgroups of a group G with the help of their (α, β) –lower cuts.

3.1 Proposition

If A and B are any two IMFS's of a universal set X, then their (α, β) –lower cuts satisfies the following :

(i) $[A]_{(\alpha, \beta)} \subseteq [A]_{(\delta, \theta)}$ if $\alpha \le \delta$ and $\beta \ge \theta$
(ii) $A \subseteq B$ implies $[B]_{(\alpha, \beta)} \subseteq [A]_{(\alpha, \beta)}$
(iii) $[A \cap B]_{(\alpha, \beta)} = [A]_{(\alpha, \beta)} \cap [B]_{(\alpha, \beta)}$
(iv) $[A \cup B]_{(\alpha, \beta)} \subseteq [A]_{(\alpha, \beta)} \cup [B]_{(\alpha, \beta)}$ (Here equality holds if $\alpha_i + \beta_i = 1, \forall i$)
(v) $[\cap A_i]_{(\alpha, \beta)} = \cap [A_i]_{(\alpha, \beta)}$,where $\alpha, \beta, \delta, \theta \in [0,1]^k$

3.2 Proposition

Let (G, .) be a groupoid and A, B be any two IMFS's of G. Then we have $[A \circ B]_{(\alpha, \beta)} = [A]_{(\alpha, \beta)} [B]_{(\alpha, \beta)}$ where $\alpha, \beta \in [0,1]^k$.

3.3 Theorem

If A is an intuitionistic multi-anti fuzzy subgroup of a group G and $\alpha, \beta \in [0,1]^k$, then the (α, β)-lower cut of A, $[A]_{(\alpha, \beta)}$ is a subgroup of G, where $\mu_A(e) \le \alpha$, $\nu_A(e) \ge \beta$ and 'e' is the identity element of G.

Proof since $\mu_A(e) \le \alpha$ and $\nu_A(e) \ge \beta$, $e \in [A]_{(\alpha, \beta)}$. Therefore, $[A]_{(\alpha, \beta)} \ne \phi$.
Let $x,y \in [A]_{(\alpha, \beta)}$. Then $\mu_A(x) \le \alpha$, $\nu_A(x) \ge \beta$ and $\mu_A(y) \le \alpha$, $\nu_A(y) \ge \beta$.
Then $\forall i$, $\mu_{iA}(x) \le \alpha_i$, $\nu_{iA}(x) \ge \beta_i$ and $\mu_{iA}(y) \le \alpha_i$, $\nu_{iA}(y) \ge \beta_i$.
$\Rightarrow \max\{\mu_{iA}(x), \mu_{iA}(y)\} \le \alpha_i$ and $\min\{\nu_{iA}(x), \nu_{iA}(y)\} \ge \beta_i$,$\forall i$……………(1)
$\Rightarrow \mu_{iA}(xy^{-1}) \le \max\{\mu_{iA}(x), \mu_{iA}(y)\} \le \alpha_i$ and $\nu_{iA}(xy^{-1}) \ge \min\{\nu_{iA}(x), \nu_{iA}(y)\} \ge \beta_i$,$\forall i$, since A is an intuitionistic multi-anti fuzzy subgroup of a group G and by (1).
$\Rightarrow \mu_{iA}(xy^{-1}) \le \alpha_i$ and $\nu_{iA}(xy^{-1}) \ge \beta_i$,$\forall i$.
$\Rightarrow \mu_A(xy^{-1}) \le \alpha$ and $\nu_A(xy^{-1}) \ge \beta$
$\Rightarrow xy^{-1} \in [A]_{(\alpha, \beta)}$
$\Rightarrow [A]_{(\alpha, \beta)}$ is a subgroup of G.
Hence the Theorem.

3.4 Theorem

The IMFS A is an intuitionistic multi-anti fuzzy subgroup of a group G $\Leftrightarrow$ each (α,β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G, $\forall$ $\alpha, \beta \in [0, 1]^k$.

Proof From the above Theorem 3.3, it is clear.

3.5 Theorem

If A is an intuitionistic multi-anti fuzzy normal subgroup of a group G and $\alpha, \beta \in [0, 1]^k$, then (α,β)-lower cut $[A]_{(\alpha, \beta)}$ is a normal subgroup of G, where $\mu_A(e) \le \alpha$, $\nu_A(e) \ge \beta$ and 'e' is the identity element of G.

Proof Let $x \in [A]_{(\alpha, \beta)}$ and $g \in G$. Then $\mu_A(x) \le \alpha$ and $\nu_A(x) \ge \beta$.

That is, $\mu_{iA}(x) \le \alpha_i$ and $\nu_{iA}(x) \ge \beta_i$ $\forall i$ ……….(1)

Since A is an intuitionistic multi-anti fuzzy normal subgroup of G,

$\mu_{iA}(g^{-1}xg) = \mu_{iA}(x)$ and $\nu_{iA}(g^{-1}xg) = \nu_{iA}(x)$, $\forall i$.
$\Rightarrow \mu_{iA}(g^{-1}xg) = \mu_{iA}(x) \le \alpha_i$ and $\nu_{iA}(g^{-1}xg) = \nu_{iA}(x) \ge \beta_i$,$\forall i$, by using(1).
$\Rightarrow \mu_{iA}(g^{-1}xg) \le \alpha_i$ and $\nu_{iA}(g^{-1}xg) \ge \beta_i$,$\forall i$.
$\Rightarrow \mu_A(g^{-1}xg) \le \alpha$ and $\nu_A(g^{-1}xg) \ge \beta$
$\Rightarrow g^{-1}xg \in [A]_{(\alpha, \beta)}$
$\Rightarrow [A]_{(\alpha, \beta)}$ is a normal subgroup of G.

Hence the Theorem.

3.6 Theorem

If A is an intuitionistic multi-fuzzy subset of a group G, then A is an intuitionistic multi-anti fuzzy subgroup of G $\Leftrightarrow$ each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G ,for all $\alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \le 1$, $\forall i$.

Proof $\Rightarrow$ Let A be an intuitionistic multi-anti fuzzy subgroup of a group G. Then by the Theorem 3.4, each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G for all $\alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1, \forall i$.

$\Leftarrow$ Conversely, let A be an intuitionistic multi-fuzzy subset of a group G such that each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G for all $\alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1, \forall i$.

To prove that A is an intuitionistic multi-anti fuzzy subgroup of G, we prove :

(i) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(xy) \geq \min\{\nu_A(x), \nu_A(y)\}$ for all $x,y \in G$
(ii) $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$

For proof (i): Let $x,y \in G$ and for all i,

$$\text{let } \alpha_i = \max\{\mu_{iA}(x), \mu_{iA}(y)\} \text{ and } \beta_i = \min\{\nu_{iA}(x), \nu_{iA}(y)\}.$$

Then $\forall i$, we have $\mu_{iA}(x) \leq \alpha_i$, $\mu_{iA}(y) \leq \alpha_i$ and $\nu_{iA}(x) \geq \beta_i$, $\nu_{iA}(y) \geq \beta_i$

That is, $\forall i$, we have $\mu_{iA}(x) \leq \alpha_i$, $\nu_{iA}(x) \geq \beta_i$ and $\mu_{iA}(y) \leq \alpha_i$, $\nu_{iA}(y) \geq \beta_i$

Then we have $\mu_A(x) \leq \alpha$, $\nu_A(x) \geq \beta$ and $\mu_A(y) \leq \alpha$, $\nu_A(y) \geq \beta$

That is, $x \in [A]_{(\alpha, \beta)}$ and $y \in [A]_{(\alpha, \beta)}$

Therefore, $xy \in [A]_{(\alpha, \beta)}$, since each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup by hypothesis.

Therefore, $\forall i$, we have $\mu_{iA}(xy) \leq \alpha_i = \max\{\mu_{iA}(x), \mu_{iA}(y)\}$ and
$\nu_{iA}(xy) \geq \beta_i = \min\{\nu_{iA}(x), \nu_{iA}(y)\}$.

That is, $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(xy) \geq \min\{\nu_A(x), \nu_A(y)\}$ and hence (i).

For proof (ii): Let $x \in G$ and $\forall i$, let $\mu_{iA}(x) = \alpha_i$ and $\nu_{iA}(x) = \beta_i$.

Then $\mu_{iA}(x) \leq \alpha_i$ and $\nu_{iA}(x) \geq \beta_i$ is true $\forall i$.

Therefore, $\mu_A(x) \leq \alpha$ and $\nu_A(x) \geq \beta$

Therefore, $x \in [A]_{(\alpha, \beta)}$.

Since each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G for all $\alpha, \beta \in [0,1]^k$ and $x \in [A]_{(\alpha, \beta)}$,we have

$x^{-1} \in [A]_{(\alpha, \beta)}$ which implies that $\mu_{iA}(x^{-1}) \leq \alpha_i$ and $\nu_{iA}(x^{-1}) \geq \beta_i$ is true $\forall i$.

$\Rightarrow \mu_{iA}(x^{-1}) \leq \mu_{iA}(x)$ and $\nu_{iA}(x^{-1}) \geq \nu_{iA}(x)$ is true $\forall i$.

Thus, $\forall i$, $\mu_{iA}(x) = \mu_{iA}((x^{-1})^{-1}) \leq \mu_{iA}(x^{-1}) \leq \mu_{iA}(x)$ which implies that $\mu_{iA}(x^{-1}) = \mu_{iA}(x)$,$\forall i$ and hence $\mu_A(x^{-1}) = \mu_A(x)$.

And $\forall i$, $\nu_{iA}(x)= \nu_{iA}((x^{-1})^{-1}) \geq \nu_{iA}(x^{-1}) \geq \nu_{iA}(x)$ which implies that $\nu_{iA}(x^{-1}) = \nu_{iA}(x)$,$\forall i$ and hence $\nu_A(x^{-1}) = \nu_A(x)$.

Hence A is an intuitionistic multi-anti fuzzy subgroup of G and hence the Theorem.

3.7 Theorem

If A and B are any two intuitionistic multi-anti fuzzy subgroups (IMAFSG's) of a group G, then $(A\cup B)$ is an intuitionistic multi-anti fuzzy subgroup of G.
Proof since A and B are IMAFSG's of G, we have $\forall x,y\in G$,

(i) $\mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$ and $\nu_A(xy^{-1}) \geq \min\{\nu_A(x), \nu_A(y)\}$
(ii) $\mu_B(xy^{-1}) \leq \max\{\mu_B(x), \mu_B(y)\}$ and $\nu_B(xy^{-1}) \geq \min\{\nu_B(x), \nu_B(y)\}$ …….(1)

Now $A\cup B = \{ < x, \mu_{A\cup B}(x), \nu_{A\cup B}(x) > : x\in G \}$ where $\mu_{A\cup B}(x) = \max\{ \mu_A(x), \mu_B(x) \}$ and $\nu_{A\cup B}(x) = \min \{ \nu_A(x), \nu_B(x) \}$.

$$\begin{aligned}\text{Then } \mu_{A\cup B}(xy^{-1}) &= \max\{ \mu_A(xy^{-1}), \mu_B(xy^{-1}) \} \\ &\leq \max\{ \max\{\mu_A(x), \mu_A(y)\}, \max\{\mu_B(x), \mu_B(y)\} \}, \text{ by using (1)} \\ &= \max\{ \max\{\mu_A(x), \mu_B(x)\}, \max\{\mu_A(y), \mu_B(y)\} \} \\ &= \max\{ \mu_{A\cup B}(x), \mu_{A\cup B}(y) \}\end{aligned}$$

$$\begin{aligned}\text{and } \nu_{A\cup B}(xy^{-1}) &= \min\{ \nu_A(xy^{-1}), \nu_B(xy^{-1}) \} \\ &\geq \min\{ \min\{\nu_A(x), \nu_A(y)\}, \min\{\nu_B(x), \nu_B(y)\} \}, \text{ by using (1)} \\ &= \min\{ \min\{\nu_A(x), \nu_B(x)\}, \min\{\nu_A(y), \nu_B(y)\} \} \\ &= \min\{ \nu_{A\cup B}(x), \nu_{A\cup B}(y) \}\end{aligned}$$

That is, $\mu_{A\cup B}(xy^{-1}) \leq \max\{ \mu_{A\cup B}(x), \mu_{A\cup B}(y) \}$ and $\nu_{A\cup B}(xy^{-1}) \geq \min\{ \nu_{A\cup B}(x), \nu_{A\cup B}(y) \}$, $\forall x,y\in G$.

Hence $(A\cup B)$ is an intuitionistic multi-anti fuzzy subgroup of G.

Hence the Theorem.

3.8 Theorem

The intersection of any two IMAFSG's of a group G need not be an IMAFSG of G.

Proof Consider the abelian group G = { e, a, b, ab } with usual multiplication such that $a^2 = e = b^2$ and ab = ba. Let A = { < e, (0.2, 0.2), (0.7, 0.8) >, < a, (0.5, 0.5), (0.4, 0.4) >, < b, (0.5, 0.5), (0.2, 0.4) >, < ab, (0.4, 0.5), (0.2, 0.4) > } and B = { < e, (0.3, 0.1), (0.7, 0.8) >, < a, (0.8, 0.4), (0.2, 0.6) >, < b, (0.6, 0.4), (0.4, 0.5) >, < ab, (0.8, 0.4), (0.2, 0.5) > } be two IMFS's having dimension two of the group G. Clearly A and B are IMAFSG's of G.

Then $A\cap B$ = { < e, (0.2, 0.1), (0.7, 0.8) >, < a, (0.5, 0.4), (0.4, 0.6) >, < b, (0.5, 0.4), (0.4, 0.5) >, < ab, (0.4, 0.4), (0.2, 0.5) > }. Here, it is easily verify that $A\cap B$ is not an IMAFSG of G.
Hence the Theorem.

3.9 Theorem

Let A and B be an IMAFSG's of a group G. But it is an uncertain to verify that A∩B is an IMAFSG of G.

Proof This proof is done by the following two examples that are discussed in two cases : case(i) and case(ii).

Case (i) : A and B are IMAFSG's of a group G ⇒ A∪B is an IMAFSG of G but A∩B is not an IMAFSG of the group G.

Consider the abelian group G = { e, a, b, ab } with usual multiplication such that $a^2 = e = b^2$ and ab = ba. Let A = { < e, (0.2, 0.2), (0.7, 0.8) >, < a, (0.5, 0.5), (0.4, 0.4) >, < b, (0.5, 0.5), (0.2, 0.4) >, < ab, (0.4, 0.5), (0.2, 0.4) > } and B = { < e, (0.3, 0.1), (0.7, 0.8) >, < a, (0.8, 0.4), (0.2, 0.6) >, < b, (0.6, 0.4), (0.4, 0.5) >, < ab, (0.8, 0.4), (0.2, 0.5) > } be two IMFS's having dimension two of the group G. Clearly A and B are IMAFSG's of G.

Then A∪B = { < e, (0.3, 0.2), (0.7, 0.8) >, < a, (0.8, 0.5), (0.2, 0.4) >, < b, (0.6, 0.5), (0.2, 0.4) >, < ab, (0.8, 0.5), (0.2, 0.4) > } and A∩B = { < e, (0.2, 0.1), (0.7, 0.8) >, < a, (0.5, 0.4), (0.4, 0.6) >, < b, (0.5, 0.4), (0.4, 0.5) >, < ab, (0.4, 0.4), (0.2, 0.5) > }.

Here, it is easily verify that A∪B is an IMAFSG of G but A∩B is not an IMAFSG of G. Hence case (i).

Case (ii) : A and B are IMAFSG's of a group G ⇒ both A∪B and A∩B are IMAFSG's of the group G.

Consider the abelian group G = { e, a, b, ab } with usual multiplication such that $a^2 = e = b^2$ and ab = ba. Let A = { < e, (0.2, 0.1), (0.8, 0.8) >, < a, (0.5, 0.4), (0.4, 0.6) >, < b, (0.5, 0.4), (0.4, 0.5) >, < ab, (0.5, 0.4), (0.4, 0.5) > } and B = { < e, (0.3, 0.2), (0.7, 0.7) >, < a, (0.8, 0.5), (0.2, 0.4) >, < b, (0.6, 0.5), (0.4, 0.2) >, < ab, (0.8, 0.4), (0.2, 0.2) > } be two IMFS's having dimension two of the group G. Clearly A and B are IMAFSG's of G.

Then A∪B = { < e, (0.3, 0.2), (0.7, 0.7) >, < a, (0.8, 0.5), (0.2, 0.4) >, < b, (0.6, 0.5), (0.4, 0.2) >, < ab, (0.8, 0.4), (0.2, 0.2) > } and A∩B = { < e, (0.2, 0.1), (0.8, 0.8) >, < a, (0.5, 0.4), (0.4, 0.6) >, < b, (0.5, 0.4), (0.4, 0.5) >, < ab, (0.5, 0.4), (0.4, 0.5) > }.

Here, it is easily to verify that both A∪B and A∩B are IMAFSG's of G. Hence case (ii).

From case (i) and case (ii), clearly it is an uncertain to verify that A∩B is an IMAFSG of G.

Hence the Theorem.

3.10 Theorem

Let A and B be any two IMAFSG's of a group G. Then A∘B is an IMAFSG of G ⇔ A∘B=B∘A

Proof Since A and B are IMAFSG's of G, each (α, β)-lower cuts $[A]_{(\alpha, \beta)}$ and $[B]_{(\alpha, \beta)}$ are subgroups of G, $\forall \alpha,\beta \in [0,1]^k$ with $\alpha_i + \beta_i \le 1, \forall i$ ………………(1)

Suppose A∘B is an IMAFSG of G.

$\Leftrightarrow$ each (α, β)-lower cuts $[A\circ B]_{(\alpha, \beta)}$ are subgroups of G, $\forall \alpha,\beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

Now, from (1), $[A]_{(\alpha, \beta)} [B]_{(\alpha, \beta)}$ is a subgroup of G $\Leftrightarrow$ $[A]_{(\alpha, \beta)} [B]_{(\alpha, \beta)} = [B]_{(\alpha, \beta)} [A]_{(\alpha, \beta)}$,since if H and K are any two subgroups of G, then HK is a subgroup of G $\Leftrightarrow$ HK=KH.

$\Leftrightarrow [A\circ B]_{(\alpha, \beta)} = [B\circ A]_{(\alpha, \beta)}$, $\forall \alpha,\beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.
$\Leftrightarrow A\circ B = B\circ A$

Hence the Theorem.

3.11 Theorem

If A is any IMAFSG of a group G, then $A\circ A = A$.

Proof Since A is an IMAFSG of a group G,

each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G, $\forall \alpha,\beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

$\Rightarrow [A]_{(\alpha, \beta)}[A]_{(\alpha, \beta)} = [A]_{(\alpha, \beta)}$, since H is a subgroup of G $\Rightarrow$ HH = H.

$\Rightarrow [A\circ A]_{(\alpha, \beta)} = [A]_{(\alpha, \beta)}$, $\forall \alpha,\beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

$\Rightarrow A\circ A = A$

Hence the Theorem.

4. INTUITIONISTIC MULTI-FUZZY COSETS

In this section we shall prove some theorems on intuitionistic multi-fuzzy cosets of a group G.

4.1 Definition

Let G be a group and A be an IMAFSG of G. Let $x\in G$ be a fixed element. Then the set $xA = \{(g, \mu_{xA}(g), \nu_{xA}(g)) : g\in G \}$ where $\mu_{xA}(g) = \mu_A(x^{-1}g)$ and $\nu_{xA}(g) = \nu_A(x^{-1}g)$, $\forall g\in G$ is called the intuitionistic multi-fuzzy left coset of G determined by A and x.

Similarly, the set $Ax = \{ (g, \mu_{Ax}(g), \nu_{Ax}(g)) : g\in G \}$ where $\mu_{Ax}(g) = \mu_A(gx^{-1})$ and $\nu_{Ax}(g) = \nu_A(gx^{-1})$,$\forall g\in G$ is called the intuitionistic multi-fuzzy right coset of G determined by A and x.

4.2 Remark

It is clear that if A is an intuitionistic multi-anti fuzzy normal subgroup of G, then the intuitionistic multi-fuzzy left coset and the intuitionistic multi-fuzzy right coset of A on G coincides and in this case, we simply call it as intuitionistic multi-fuzzy coset.

4.3 Example

Let G be a group. Then $A = \{ < x, \mu_A(x), \nu_A(x) >, x\in G / \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e) \}$ is an intuitionistic multi-anti fuzzy normal subgroup of G.

Proof It is easy to verify.

4.4 Theorem

Let A be an intuitionistic multi-anti fuzzy subgroup of a group G and x be any fixed element of G. Then the following are holds :

(i) $x[A]_{(\alpha,\beta)} = [xA]_{(\alpha,\beta)}$
(ii) $[A]_{(\alpha,\beta)}x = [Ax]_{(\alpha,\beta)}$, $\forall\alpha, \beta\in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

Proof For proof (i),

Now $[xA]_{(\alpha,\beta)} = \{ g\in G : \mu_{xA}(g) \leq \alpha$ and $\nu_{xA}(g) \geq \beta \}$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

Also $x[A]_{(\alpha,\beta)} = x\{ y\in G : \mu_A(y) \leq \alpha$ and $\nu_A(y) \geq \beta \}$
$= \{ xy\in G : \mu_A(y) \leq \alpha$ and $\nu_A(y) \geq \beta \}$ ………………(1)

put $xy = g \Rightarrow y = x^{-1}g$. Then (1) becomes as,

$$x[A]_{(\alpha,\beta)} = \{ g\in G : \mu_A(x^{-1}g) \leq \alpha \text{ and } \nu_A(x^{-1}g) \geq \beta \}$$
$$= \{ g\in G : \mu_{xA}(g) \leq \alpha \text{ and } \nu_{xA}(g) \geq \beta \}$$
$$= [xA]_{(\alpha,\beta)}$$

Therefore, $x[A]_{(\alpha,\beta)} = [xA]_{(\alpha,\beta)}$, $\forall\alpha, \beta\in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

Hence the proof (i).

For proof (ii),

Now $[Ax]_{(\alpha,\beta)} = \{ g\in G : \mu_{Ax}(g) \leq \alpha$ and $\nu_{Ax}(g) \geq \beta \}$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

Also $[A]_{(\alpha,\beta)}x = \{ y\in G : \mu_A(y) \leq \alpha$ and $\nu_A(y) \geq \beta \}x$
$= \{ yx\in G : \mu_A(y) \leq \alpha$ and $\nu_A(y) \geq \beta \}$ ………………(2)

put $yx = g \Rightarrow y = gx^{-1}$. Then (2) becomes as,

$$[A]_{(\alpha,\beta)}x = \{ g\in G : \mu_A(gx^{-1}) \leq \alpha \text{ and } \nu_A(gx^{-1}) \geq \beta \}$$
$$= \{ g\in G : \mu_{Ax}(g) \leq \alpha \text{ and } \nu_{Ax}(g) \geq \beta \}$$
$$= [Ax]_{(\alpha,\beta)}$$

Therefore, $[A]_{(\alpha,\beta)}x = [Ax]_{(\alpha,\beta)}$, $\forall\alpha, \beta\in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall$i.

Hence the proof (ii) and hence the Theorem.

4.5 Theorem

Let A be an intuitionistic multi-anti fuzzy subgroup of a group G. Let x,y be any two elements of G such that $\alpha = \max\{ \mu_A(x), \mu_A(y) \}$ and $\beta = \min\{ \nu_A(x), \nu_A(y) \}$. Then the following are holds :

(i) $xA = yA \Leftrightarrow x^{-1}y \in [A]_{(\alpha, \beta)}$
(ii) $Ax = Ay \Leftrightarrow xy^{-1} \in [A]_{(\alpha, \beta)}$
(iii)

Proof For (i), Now $xA = yA \Leftrightarrow [xA]_{(\alpha, \beta)} = [yA]_{(\alpha, \beta)}$, $\forall \alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall i$.

$\Leftrightarrow x[A]_{(\alpha, \beta)} = y[A]_{(\alpha, \beta)}$,by Theorem 4.4(i).
$\Leftrightarrow x^{-1}y \in [A]_{(\alpha, \beta)}$,since each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G. Hence the proof (i).

For (ii), Now $Ax = Ay \Leftrightarrow [Ax]_{(\alpha, \beta)} = [Ay]_{(\alpha, \beta)}$, $\forall \alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall i$.
$\Leftrightarrow [A]_{(\alpha, \beta)}x = [A]_{(\alpha, \beta)}y$,by Theorem 4.4(ii).
$\Leftrightarrow xy^{-1} \in [A]_{(\alpha, \beta)}$,since each (α, β)-lower cut $[A]_{(\alpha, \beta)}$ is a subgroup of G. Hence the proof (ii) and hence the Theorem.

5. HOMOMORPHISM OF INTUITIONISTIC MULTI-ANTI FUZZY SUBGROUPS

In this section we shall prove some theorems on intuitionistic multi-anti fuzzy subgroups of a group with the help of a homomorphism.

5.1 Proposition

Let $f : X \rightarrow Y$ be an onto map. If A and B are intuitionistic multi-fuzzy sets having the dimension k of X and Y respectively, then for each (α, β)-lower cuts $[A]_{(\alpha, \beta)}$ and $[B]_{(\alpha, \beta)}$, the following are holds :

(i) $f([A]_{(\alpha, \beta)}) \subseteq [f(A)]_{(\alpha, \beta)}$
(ii) $f^{-1}([B]_{(\alpha, \beta)}) = [f^{-1}(B)]_{(\alpha, \beta)}$, $\forall \alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1$, $\forall i$.

Proof For (i), Let $y \in f([A]_{(\alpha, \beta)})$.

Then there exists an element $x \in [A]_{(\alpha, \beta)}$ such that $f(x) = y$.

Then we have $\mu_A(x) \leq \alpha$ and $\nu_A(x) \geq \beta$,since $x \in [A]_{(\alpha, \beta)}$.

$\Rightarrow \mu_{iA}(x) \leq \alpha_i$ and $\nu_{iA}(x) \geq \beta_i$, $\forall i$.
$\Rightarrow \min\{\mu_{iA}(x) : x \in f^{-1}(y)\} \leq \alpha_i$ and $\max\{\nu_{iA}(x) : x \in f^{-1}(y) \} \geq \beta_i$, $\forall i$.
$\Rightarrow \min\{\mu_A(x) : x \in f^{-1}(y)\} \leq \alpha$ and $\max\{\nu_A(x) : x \in f^{-1}(y) \} \geq \beta$
$\Rightarrow \mu_{f(A)}(y) \leq \alpha$ and $\nu_{f(A)}(y) \geq \beta$
$\Rightarrow y \in [f(A)]_{(\alpha, \beta)}$

Therefore, $f([A]_{(\alpha, \beta)}) \subseteq [f(A)]_{(\alpha, \beta)}$, $\forall A \in IMFS(X)$. Hence the proof (i).
For the proof (ii),

Let $x \in [f^{-1}(B)]_{(\alpha, \beta)} \Leftrightarrow \{x \in X : \mu_{f^{-1}(B)}(x) \leq \alpha , \ \nu_{f^{-1}(B)}(x) \geq \beta \}$
$\Leftrightarrow \{x \in X : \mu_{if^{-1}(B)}(x) \leq \alpha_i , \ \nu_{if^{-1}(B)}(x) \geq \beta_i \}, \forall i$.
$\Leftrightarrow \{x \in X : \mu_{iB}(f(x)) \leq \alpha_i , \ \nu_{iB}(f(x)) \geq \beta_i \}, \forall i$.

$\Leftrightarrow \{x\in X : \mu_B(f(x)) \le \alpha\ ,\ \nu_B(f(x)) \ge \beta\ \}$
$\Leftrightarrow \{\ x\in X : f(x)\in [B]_{(\alpha,\ \beta)}\ \}$
$\Leftrightarrow \{\ x\in X : x\in f^{-1}(\ [B]_{(\alpha,\ \beta)}\ )\ \}$
$\Leftrightarrow f^{-1}(\ [B]_{(\alpha,\ \beta)}\ )$

Hence the proof (ii).

5.2 Theorem

Let $f\ : G_1 \rightarrow G_2$ be an onto homomorphism and if A is an IMAFSG of group G_1, then f(A) is an IMAFSG of group G_2.

Proof By Theorem 3.6, it is enough to prove that each $(\alpha,\ \beta)$-lower cuts $[f(A)]_{(\alpha,\ \beta)}$ is a subgroup of G_2 for all $\alpha,\ \beta\in [0,1]^k$ with $\alpha_i + \beta_i \le 1,\ \forall i$.

Let $y_1, y_2\in [f(A)]_{(\alpha,\ \beta)}$. Then $\mu_{f(A)}(y_1) \le \alpha\ ,\ \nu_{f(A)}(y_1) \ge\ \beta$ and $\mu_{f(A)}(y_2) \le \alpha\ ,\ \nu_{f(A)}(y_2) \ge \beta$
$\Rightarrow \mu_{if(A)}(y_1) \le \alpha_i\ ,\ \nu_{if(A)}(y_1) \ge \beta_i$ and $\mu_{if(A)}(y_2) \le \alpha_i\ ,\ \nu_{if(A)}(y_2) \ge \beta_i\ ,\forall i.$ …………..(1)

By the proposition 5.1(i), we have $f(\ [A]_{(\alpha,\ \beta)}\ ) \subseteq [f(A)]_{(\alpha,\ \beta)}\ ,\forall A\in IMFS(G_1)$

Since f is onto, there exists some x_1 and x_2 in G_1 such that $f(x_1) = y_1$ and $f(x_2) = y_2$.
Therefore, (1) becomes as,

$\mu_{if(A)}(\ f(x_1)\ ) \le \alpha_i\ ,\ \nu_{if(A)}(\ f(x_1)\ ) \ge \beta_i$ and $\mu_{if(A)}(\ f(x_2)\ ) \le \alpha_i\ ,\ \nu_{if(A)}(\ f(x_2)\ ) \ge \beta_i\ ,\forall i.$

$\Rightarrow \mu_{iA}(x_1) \le \mu_{if(A)}(\ f(x_1)\ ) \le \alpha_i\ ,\ \nu_{iA}(x_1) \ge\ \nu_{if(A)}(\ f(x_1)\ ) \ge \beta_i$ and
$\mu_{iA}(x_2) \le \mu_{if(A)}(\ f(x_2)\ ) \le \alpha_i\ ,\ \nu_{iA}(x_2) \ge \nu_{if(A)}(\ f(x_2)\ ) \ge \beta_i\ ,\forall i.$

$\Rightarrow \mu_{iA}(x_1) \le \alpha_i\ ,\ \nu_{iA}(x_1) \ge \beta_i$ and $\mu_{iA}(x_2) \le \alpha_i\ ,\ \nu_{iA}(x_2) \ge \beta_i\ ,\forall i.$

$\Rightarrow \mu_A(x_1) \le \alpha\ ,\ \nu_A(x_1) \ge \beta$ and $\mu_A(x_2) \le \alpha\ ,\ \nu_A(x_2) \ge \beta$

$\Rightarrow \max\{\mu_A(x_1), \mu_A(x_2)\} \le \alpha$ and $\min\{\nu_A(x_1), \nu_A(x_2)\} \ge \beta$

$\Rightarrow \mu_A(x_1x_2^{-1}) \le \max\{\mu_A(x_1), \mu_A(x_2)\} \le \alpha$ and $\nu_A(x_1x_2^{-1}) \ge \min\{\nu_A(x_1), \nu_A(x_2)\} \ge \beta$,since $A\in IMAFSG(G_1)$.

$\Rightarrow \mu_A(x_1x_2^{-1}) \le \alpha$ and $\nu_A(x_1x_2^{-1}) \ge \beta$

$\Rightarrow x_1x_2^{-1}\in [A]_{(\alpha,\ \beta)}$

$\Rightarrow f(x_1x_2^{-1})\in f(\ [A]_{(\alpha,\ \beta)}\ ) \subseteq [f(A)]_{(\alpha,\ \beta)}$

$\Rightarrow f(x_1)f(x_2^{-1})\in [f(A)]_{(\alpha,\ \beta)}$

$\Rightarrow f(x_1)f(x_2)^{-1}\in [f(A)]_{(\alpha,\ \beta)}$

$\Rightarrow y_1y_2^{-1}\in [f(A)]_{(\alpha,\ \beta)}$

$\Rightarrow [f(A)]_{(\alpha,\ \beta)}$ is a subgroup of G_2 , $\forall \alpha,\ \beta\in [0,1]^k$.

$\Rightarrow f(A) \in IMAFSG(G_2)$

Hence the Theorem.

5.3 Corollary

If $f : G_1 \rightarrow G_2$ be a homomorphism of a group G_1 onto a group G_2 and $\{ A_i : i \in I \}$ be a family of IMAFSG's of group G_1, then $f(\cup A_i)$ is an IMAFSG of group G_2.

5.4 Theorem

Let $f : G_1 \rightarrow G_2$ be a homomorphism of a group G_1 into a group G_2. If B is an IMAFSG of G_2, then $f^{-1}(B)$ is also an IMAFSG of G_1.

Proof By Theorem 3.6, it is enough to prove that each (α, β)-lower cuts $[f^{-1}(B)]_{(\alpha, \beta)}$ is a subgroup of $G_1, \forall \alpha, \beta \in [0,1]^k$ with $\alpha_i + \beta_i \leq 1, \forall i$.

Let $x_1, x_2 \in [f^{-1}(B)]_{(\alpha, \beta)}$. Then it implies that

$\mu_{f^{-1}(B)}(x_1) \leq \alpha$, $\nu_{f^{-1}(B)}(x_1) \geq \beta$ and $\mu_{f^{-1}(B)}(x_2) \leq \alpha$, $\nu_{f^{-1}(B)}(x_2) \geq \beta$.

$\Rightarrow \mu_B(f(x_1)) \leq \alpha$, $\nu_B(f(x_1)) \geq \beta$ and $\mu_B(f(x_2)) \leq \alpha$, $\nu_B(f(x_2)) \geq \beta$.

$\Rightarrow \max\{ \mu_B(f(x_1)), \mu_B(f(x_2)) \} \leq \alpha$ and $\min\{ \nu_B(f(x_1)), \nu_B(f(x_2)) \} \geq \beta$.

$\Rightarrow \mu_B(f(x_1)f(x_2)^{-1}) \leq \max\{ \mu_B(f(x_1)), \mu_B(f(x_2)) \} \leq \alpha$ and
$\nu_B(f(x_1)f(x_2)^{-1}) \geq \min\{ \nu_B(f(x_1)), \nu_B(f(x_2)) \} \geq \beta$,since $B \in IMAFSG(G_2)$.

$\Rightarrow f(x_1)f(x_2)^{-1} \in [B]_{(\alpha, \beta)}$

$\Rightarrow f(x_1x_2^{-1}) \in [B]_{(\alpha, \beta)}$,since f is a homomorphism.

$\Rightarrow x_1x_2^{-1} \in f^{-1}([B]_{(\alpha, \beta)}) = [f^{-1}(B)]_{(\alpha, \beta)}$,by the proposition 5.1(ii).

$\Rightarrow x_1x_2^{-1} \in [f^{-1}(B)]_{(\alpha, \beta)}$

$\Rightarrow [f^{-1}(B)]_{(\alpha, \beta)}$ is a subgroup of G_1

$\Rightarrow f^{-1}(B)$ is an IMAFSG of G_1.

Hence the Theorem.

5.5 Theorem

Let $f : G_1 \rightarrow G_2$ be a surjective homomorphism and if A is an IMAFNSG of group G_1, then f(A) is also an IMAFNSG of group G_2 .

Proof Let $g_2 \in G_2$ and $y \in f(A)$.

Since f is surjective,

there exists $g_1 \in G_1$ and $x \in A$ such that $f(x) = y$ and $f(g_1) = g_2$.

Since A is an IMAFNSG of G_1,

$\mu_A(g_1^{-1}xg_1) = \mu_A(x)$ and $\nu_A(g_1^{-1}xg_1) = \nu_A(x)$,$\forall x \in A$ and $g_1 \in G_1$.

Now consider, $\mu_{f(A)}(g_2^{-1}yg_2) = \mu_{f(A)}(f(g_1^{-1}xg_1))$,since f is a homomorphism.

$$
\begin{aligned}
&= \mu_{f(A)}(y') \text{ ,where } y' = f(g_1^{-1}xg_1) = g_2^{-1}yg_2 \\
&= \min\{ \mu_A(x') : f(x') = y' \text{ for } x' \in G_1 \} \\
&= \min\{ \mu_A(x') : f(x') = f(g_1^{-1}xg_1) \text{ for } x' \in G_1 \} \\
&= \min\{ \mu_A(g_1^{-1}xg_1) : f(g_1^{-1}xg_1)=y' = g_2^{-1}yg_2 \text{ for } x \in A,\ g_1 \in G_1\} \\
&= \min\{ \mu_A(x) : f(g_1^{-1}xg_1) = g_2^{-1}yg_2 \text{ for } x \in A,\ g_1 \in G_1\} \\
&= \min\{ \mu_A(x) : f(g_1)^{-1}f(x)f(g_1) = g_2^{-1}yg_2 \text{ for } x \in A,\ g_1 \in G_1\} \\
&= \min\{ \mu_A(x) : g_2^{-1}f(x)g_2 = g_2^{-1}yg_2 \text{ for } x \in G_1\} \\
&= \min\{ \mu_A(x) : f(x) = y \text{ for } x \in G_1\} \\
&= \mu_{f(A)}(y)
\end{aligned}
$$

Similarly, we can easily prove that $\nu_{f(A)}(g_2^{-1}yg_2) = \nu_{f(A)}(y)$.

Hence f(A) is an IMAFNSG of G_2 and hence the Theorem.

5.6 Theorem

If A is an IMAFNSG of a group G, then there exists a natural homomorphism $f : G \to G/A$ is defined by $f(x) = xA$, $\forall x \in G$.

Proof Let $f : G \to G/A$ be defined by $f(x) = xA$, $\forall x \in G$.

Claim1: f is a homomorphism

That is, to prove: $f(xy) = f(x)f(y)$,$\forall x,y \in G$.

That is, to prove: $(xy)A = (xA)(yA)$,$\forall x,y \in G$.

Since A is an IMAFNSG of G, we have $\mu_A(g^{-1}xg) = \mu_A(x)$ and
$\nu_A(g^{-1}xg) = \nu_A(x)$,$\forall x \in A$ and $g \in G$.

Or, equivalently, $\mu_A(xy) = \mu_A(yx)$ and $\nu_A(xy) = \nu_A(yx)$,$\forall x,y \in G$.

Also, $\forall g \in G$, we have

$(xA)(g) = (\mu_{xA}(g), \nu_{xA}(g)) = (\mu_A(x^{-1}g), \nu_A(x^{-1}g))$

$(yA)(g) = (\mu_{yA}(g), \nu_{yA}(g)) = (\mu_A(y^{-1}g), \nu_A(y^{-1}g))$

$[(xy)A](g) = (\mu_{(xy)A}(g), \nu_{(xy)A}(g)) = (\mu_A[(xy)^{-1}g], \nu_A[(xy)^{-1}g])$
$\forall g \in G$, by definition 2.16, we have

$[(xA)(yA)](g) = (\max[\min\{\mu_{xA}(r), \mu_{yA}(s)\} : g = rs], \min[\max\{\nu_{xA}(r), \nu_{yA}(s)\} : g = rs])$
$= (\max[\min\{\mu_A(x^{-1}r), \mu_A(y^{-1}s)\} : g = rs], \min[\max\{\nu_A(x^{-1}r), \nu_A(y^{-1}s)\} : g = rs]$
)

Claim2: $\mu_A[(xy)^{-1}g] = \max[\min\{\mu_A(x^{-1}r), \mu_A(y^{-1}s)\} : g = rs]$ and
$\nu_A[(xy)^{-1}g] = \min[\max\{\nu_A(x^{-1}r), \nu_A(y^{-1}s)\} : g = rs], \forall g \in G$.

Now consider $\mu_A[(xy)^{-1}g] = \mu_A[y^{-1}x^{-1}g]$
$= \mu_A[y^{-1}x^{-1}rs]$,since g = rs.
$= \mu_A[y^{-1}(x^{-1}rsy^{-1})y]$
$= \mu_A[x^{-1}rsy^{-1}]$,since A is normal.
$\leq \max\{ \mu_A(x^{-1}r), \mu_A(sy^{-1}) \}$,since A is IMAFSG of G.
$= \max\{ \mu_A(x^{-1}r), \mu_A(y^{-1}s) \}$, $\forall g = rs \in G$, since A is normal

Therefore, $\mu_A[(xy)^{-1}g] = \min[\max\{ \mu_A(x^{-1}r), \mu_A(y^{-1}s) \} : g = rs]$, $\forall g \in G$.
$= \max[\min\{ \mu_A(x^{-1}r), \mu_A(y^{-1}s) \} : g = rs]$, $\forall g \in G$.

Similarly,we can easily prove $\nu_A[(xy)^{-1}g] = \min[\max\{ \nu_A(x^{-1}r), \nu_A(y^{-1}s) \} : g = rs]$, $\forall g \in G$.
Hence the claim2.

Thus, $[(xy)A](g) = [(xA)(yA)](g)$,$\forall g \in G$.

$\Rightarrow (xy)A = (xA)(yA)$
$\Rightarrow f(xy) = f(x)f(y)$

$\Rightarrow$ f is a homomorphism

Hence the claim1 and hence the Theorem.

6. CONCLUSION

In the theory of fuzzy sets, the level subsets are vital role for its development. Similarly, the (α, β)-lower cut of an intuitionistic multi-fuzzy sets are very important role for the development of the theory of intuitionistic multi-fuzzy sets. In this paper an attempt has been made to study some algebraic natures of intuitionistic multi-anti fuzzy subgroups and their properties with the help of their (α, β)–lower cut sets.

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