# A POSSIBLE RESOLUTION TO HILBERT'S FIRST PROBLEM BY APPLYING CANTOR'S DIAGONAL ARGUMENT WITH CONDITIONED SUBSETS OF $\mathbb{R}$, WITH THAT OF $(\mathbb{N}, \mathbb{R})$.

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## ABSTRACT

*We present herein a new approach to the Continuum hypothesis **CH**. We will employ a string conditioning, a technique that limits the range of a string over some of its sub-domains for forming subsets K of $\mathbb{R}$. We will prove that these are well defined and in fact proper subsets of $\mathbb{R}$ by making use of Cantor's Diagonal argument in its original form to establish the cardinality of K between that of $(\mathbb{N}, \mathbb{R})$ respectively.*

## KEYWORDS

*Diagonal Argument, Continuum Hypothesis CH, Resolution to CH*

## 1 PREFACE

The continuum hypothesis (CH) is one of and if not the most important open problems in set theory, one that is important for both mathematical and philosophical reasons.
Philosophically and perhaps practically, mathematicians are divided on the matter of a resolution to CH. The uncanny persistence of the problem has led to several mainstream views surrounding its resolution. Discussions on the possibility of a resolution, notably one from the Institute of Advanced study at Princeton gives a vast account of the thoughts on the problem and a detailed summary of the progress made this far, along with what may constitute a solution see [1]. Most similar discussions express the current main sequence thoughts on the matter, and the division that exists amongst mathematicians in the views they hold with regard to a resolution, the nature of the resolution and what a resolution to CH may mean. Some of the mainstream views can be summarized as follows: Finitist-mathematicians believe that we only ever deal with the finite and as such and simply put, we can't really say much about the infinite. Pluralists naturally believe in the plurality of things, that any one of the outcomes of CH is possible. Though the efforts of both Cohen and Gödel showed the consistency of ZFC + /CH and ZFC + CH respectively, Cohen held a strong pluralist view that his demonstration that CH cannot be decided from ZFC alone, essentially resolved the matter. For Cohen's independence results, See [2]. Contrary to this however, Gödel believed that a well justified extension to ZFC was all that was necessary in the way of deciding CH. Gödels program seemingly the promising option forward aims to find an extension of ZFC axioms capable of deciding CH. Gödels Program: *De-*

*cide mathematically interesting questions independent of ZFC in well justified extensions of ZFC.* Gödel himself proposed the large cardinal axioms as a candidate.
Shortly thereafter however, this extension still proved to be insufficient for the task of deciding CH, as was demonstrated by the results of Levy and Solovay. The forcing techniques employed by Cohen have since paved the way toward establishing many consistency results such as the insufficiency of the large cardinal extension of ZFC in the way of deciding CH, also by Levy and Solovay. See [3]. This however, was the starting point for the work of W.H. Woodin demonstrating, (on the premise of large cardinals), the effective failure of CH via use of a canonical model in which CH fails [4]. It is well known that *forcing* cannot be used to decide CH, and for this primary reason, we explore here some new ideas outside of the scope of *forcing*.

We must admit that these Ideas arose from *Bit Conditioning*, a form of conditioning digits available to bit strings.

## 2 INTRODUCTION

Let us pause at this juncture to take account of the intent behind the next few paragraphs, which is firstly to formulate a language capable of dealing with forming well defined subsets of $F := \bigcup_{\forall i} f_i : \omega_< \to \{0,1\}$, ($\omega_<$ is to mean a finite set of ordinals/ordered-numbers), by restricting the range of the functions $f_i$ to 0 over arbitrary domain values of $f_i$ in $dom(f_i) := \omega_< := \{0,1,2,..,n\}$. Such functions ultimately constitute elements of the sets we are interested in. Trivial but important to note is that, set of all sequences of length $n$ having 'conditioned segments' is an example of a subset of $F$. Secondly, we aim to formulate a means of establishing cardinality of such *conditioned sequence sets*, now however of non-finite length, with that of $SEQ := \bigcup_{\forall i} f_i : \mathbb{N} \to \{0,1\}$ as a whole.

To achieve this result, we will make use of inductive arguments.

The fuzziness in the first point is best made clear with an example. Suppose we wished to form a finite set of elements of the form
$\{120005, 340004, 710004, ...\}$, here suggestive that each element is a 6-digit-sequence$\{0,..,5\} \to \{0,..,9\}$ with $s_2$ to $s_4$ included, being always 0. Such sequences are a subset of say all sequences of length 6.
A simplistic language, in the mathematical sense, capable of forming precisely such conditioned sequence sets will be valuable for establishing cardinality.

Returning to point two, the core diagonal argument works when one can show that for each element in some set $S$, there are infinitely many more in say $\mathbb{R}_2$ by the argument $Diag(S,\mathbb{R}_2)$ ($\mathbb{N}_2,\mathbb{R}_2$, naturals and reals in base two). A subtle nuance is that $\mathbb{N}_2 \to S$ does not need to be onto for the *Diag* argument to work. As trivial as this sounds, this is important, for it tells us that if we are to search for a set $S$ that exists cardinally between $\mathbb{N}_2,\mathbb{R}_2$ we can be certain that $S \subset \mathbb{R}_2$ and need not be onto $\mathbb{N}_2$, for the *Diag* argument to work.
If we consider the nature of how the diagonal argument is conventionally used, one observes that $\mathbb{R}$ is treated as comprising of all infinite sequences of the form $\mathbb{N} \to \{0,1\}$. In fact, every element of $\mathbb{R}$ is representable as some infinite binary sequence $\mathbb{R}_2$. This almost

begs the question, can one form subsets of SEQ?

With some imagination, a few ideas come to mind, along the lines of those described at the beginning of this section. Perhaps by restricting *Range* values of $f_i$ to 0 over segments of its domain. Surely, all elements of the form $S := \bigcup M_1 0 M_2 00 M_3 000 M_4 0000...$ with $M_i := \{f_i : \{0,..,e\} \to \{0,1\} | e \in \mathbb{N}\}$, forms a subset of SEQ.

This is interesting, as what then is the cardinality of $S$ with regards to $\mathbb{N}, \mathbb{R}$ respectively? What of the diagonal argument can we augment and or reuse to felicitate this comparison? These are some of the questions that we aim to answer in this letter for consideration by the experts.

Important point for consideration is the containment of $S$ strictly outside of $\mathbb{N}, \mathbb{Q}$, but we will get to this later.
With our conditioned sets $S$, as we term temporarily, we wish to achieve the following: $Diag(\mathbb{N}_2, S)$ and $Diag(S, \mathbb{R}_2)$, using *Diag* in short-form to depict Cantor's diagonal argument between the sets within brackets (Such as for the well established one between $Diag(\mathbb{N}, \mathbb{R})$). One would then have to make a case for using the diagonal argument interchangeably in the following sentences (Why this is so will become clear later on, and is the main focus of this article).

**A)** *Given* $0, 1, 2, ...., 12, ..., 1000, ..$, *(i.e., all naturals) there are infinitely many more Reals*

**B)** *Given some arbitrary collection* $e_1, ...., e_n, .... \subset \mathbb{R}$ *there are infinitely many more Reals*

**C)** *Given some arbitrary collection* $1, 2, ...., 12, ..., 1000, ..$, *(i.e., all naturals), there are infinitely many more* $e_1, ...., e_n, .... \in S \subset \mathbb{R}$

All of which should be establish-able via $Diag(\mathbb{N}, S)$ and $Diag(S, \mathbb{R})$.

The challenge would then be to pick such a collection, i.e. one possessing the characteristics necessary to enable $Diag(\mathbb{N}, S)$ and $Diag(S, \mathbb{R})$.
Fortunately, this can be any collection $S$.

## 3 Conceptual Ideas and Formalization

The aim of this section is to provide some definitions used for forming well defined subsets of $\bigcup\limits_{\forall i} f_i : \omega_< \to \{0,1\}$ having the properties expressed previously. We want to ultimately make an inductive argument involving finite sequences that will necessarily have to extend to infinite ones.

**Definition** (Sequence-Function)

We define a **finite** sequence-function to be a function $f : \omega_< \to \{0,1\}$, having $\omega_<$ the ordered set of numbers starting 0, up to some arbitrary $n \in \mathbb{N}, \{0,..,n\}$, as its domain, and $range(f) := \{1,0\}$.

Such functions, and sets thereof, are denoted by the symbols $f,s$, $S,S'$ respectively throughout this article unless otherwise specified.

**Definition** (Sequence-Function Sets)

We define any set of the form $S := \bigcup_{\forall i} f_i : \omega_< \to \{0,1\}$, to be a sequence-function set, typically denoted $S,S'$.

**Definition** (Sum)

Given an arbitrary sequence-function $f$, the binary operation $+_A$ on $f \in S$, is defined as $f +_A f = Bnum^{-1}(Bnum(f) + Bnum(f))$, where $Bnum(f)$ is the function $Bnum(f) \mapsto b \in \mathbb{B}$ with $b$ being the binary number equivalent of the image of $f$, $\mathbb{B}$ the binary number set as we have labelled it, $\{+\}$ being the standard arithmetic addition operation, and $Bnum^{-1}$ being the function $Bnum^{-1}(b \in \mathbb{B}) \mapsto (f)$ with $b$ being the binary equivalent of the ordered $Image(f)$.

*Remarks*

$Bnum(f) + Bnum(f)$ is written $A_2$, and in general $A_j$ for many such sums.
As an example: $A_2(1001) = 10010$. It is to be clearly mentioned that the domain of the function(singular) remains unaffected after *Summing* which strictly affects the image of $f$ alone.

**Definition** (Conditioned sub-sequence)

Given an arbitrary sequence-function $f$, a segment $I_K$ of the $Image(f)$ over which $Range(f) := \{0\}$, for two or more domain values, is defined to be a *conditioned sub-sequence*.

*Remarks*

These are strictly partial functions $f_K$ associated with $f$. Both have identical domain values over $I_K$. $D(I_i), D(/I_i)$ respectively denote the domain sets associated with these image intervals. Here the set $D(/I_i)$ of domain values, are associated with all domain intervals not associated with $f_K$.

**Definition** (Length)

Given an arbitrary sequence-function, a segment-length (just length where there is no confusion) of a random image segment $s_k := s[n_1,n_2] \in s \in S$ of $s$, with $Dom(s_K) = [n_1,n_2]$, the domain interval of the the segment $s_K$, of $Image(f_k)$, denoted by $\mathscr{M}(s_K[n_1,n_2])$, ($\mathscr{M}(s_k)$ where there is no confusion) is defined to mean $|n_2 - n_1|$ for $n_i \in \mathbb{N}$.

*Remarks*

Where the *Length* of the entire function is concerned, we simply write $\mathscr{M}(f)$

To enable the choosing of such elements for $S \subset \mathbb{R}$, we focus our attention here on sequences that are incapable of being reduced to a natural number via finite Sum. This is tricky, as we will see shortly.

It is a well known fact that certain rational numbers such as 1/3 = 0.3333... are associated with fractional portions having sequence-like characteristics, i.e., non terminating fractionals. So naturally, if we are to talk solely about the *irrational-sequences*, we need some means of removing these from the power set $2^{\aleph_0}$. Intuitively one way of attempting this is to have some idea of what may constitute an *'irrational sequence'*. It turns out that binary sequences following a certain schema (SH) belong to a subset SS of the irrational numbers (Or comparable to.). Numbers such as $(Y.11111111...), (Y.333333...)$ having a sequence-like fractional portion,i.e., an infinite sequence of numbers, can be transformed into to a natural number via finite Sum, thus In searching for such a schema, it should be noted that such periodically repetitive sequences are to be discounted.
Formulation of the schema (SH)(i.e., A generalization of sorts of types of arrangements of $0's$ and $1's$ possible, forming binary sequences), holds a strong relationship with the effect addition has on binary numbers. Consider the simple case of 010010 + 010010 = 100100
The alignment of (1) symbols in such summations, when added, results in the position of (1) shifting a position to the left. $0's$ in alignment have no bearing on the result aside from its sum resulting in 0. The formulation of *SH* thus requires that the binary sequences associated with *irrational-sequences* are so arranged, that it is impossible to form, via finite additions of some $s$ to itself, $A_j(s) = X.111....$ for $j, X \in \mathbb{N}$.

(SH) begs an alternate schema to those that are periodically recursive, such as for instance :$(110001100011000..)$. Such recursive schemas hold the property that via finite Sum result in 111111...
As an example: $(110001100011000..) +_A ... +_A ... +_A (111001110011100..) = (111111111111111..)$
It is the above consideration that sparked the idea that targeting the number of $0's$ between pairs of $1's$ forming a binary sequence is what holds the key to forming (SH) i.e., a schema not having this property). Noteworthy is the observation that a means of forming a non-recursive binary sequence is by increasing the number $0's$ between pairs of $1's$, and as a natural extension to this is having arbitrary finite-length sequences $M_i$ spaced suchlike forming the sequence.
It is almost arbitrary why such sequences would form part of the *irrational sequences*, as, conditioned sub-sequences larger in length so to say, require more Summing in the way of resulting in 1111.. spanning its length. If there is **always** in existence one such conditioned sub-sequence greater in length than all preceding conditioned sub-sequences, then no amount of Sum on such a sequence is sufficient in the way of resulting in 111... .

**Definition** (Unconditioned sub-sequence)

Given an arbitrary sequence-function $f$ of arbitrary length, an *unconditioned sub-sequence*

$P_i$ of $f$, is defined to mean a segment of $Image(f)$ where $Range(f) := \{0,1\}$.

*Remarks*

A formidable task in set theory is precision in defining sets. *Whereof we do not know, thereof we must remain silent.* Present theory struggles in the ability to form precisely a collection of $\bigcup f_i : \mathbb{N} \to \{0,1\}$, primarily because it requires the existence of a choice function and the acceptance of ZFC-axioms of arithmetic. As its existence is highly debated we need to be precise and relay caution. What we wish to do is condition the functions both inductively and in a precisely defined manner, allowing for set formation. In order to achieve this, we will need to make use of these definitions.

**Definition** (Imposed Sequence Sets)

Let $I = \bigcup_{\forall i} D(I_i)$ be a union/collection of non-overlapping conditioned sub-sequence domain-intervals(intervals) of conditioned sub-sequences, of an arbitrary sequence-function $f$ of sufficient length. An *imposed* sequence-function is defined to be any arbitrary sequence-function $f : \omega_< \to \{0,1\}$ with the conditioning:

$$s = \begin{cases} Range(f) := \{0,1\} & \text{if } dom(f) \in I_i \in \bigcup_{\forall i} \mathrm{D}(\backslash I_i) \\ Range(f) := \{0\} & \text{if } dom(f) \in I_i \in \bigcup_{\forall i} D(I_i)_{Zero-Segments} \end{cases}$$

written $f \leftarrow (I_i)$. Should the entire set $S$ abide by the conditioning, we then write $S \leftarrow (I_i)$ and $S$ is said to be imposed by $I_i$, so long as there is no confusion in the meaning.

*Remarks*

We only ever require conditioned sub-sequences to define an *imposition*, however we do provide additional information in certain circumstances to effect clarity by including the non-segment-interval domain values in the arguments that follow.

**Definition** (Unconditioned sub-sequence Combination $C_i(g)$)

The set $Ci(g)$ is defined to be the set of functions $\bigcup_{\forall i} f_i : \{1,..,g\} \to \{0,1\}$, $g \in \mathbb{N}$.

**Definition** (Span Sets, we denote as $span(S \leftarrow (I_i,P_i))$, (Simply $span(I_i,P_i)$))

Given an arbitrary sequence-function $f$ of arbitrary length, $span(f)$ for $f \leftarrow (I_i,P_i)\ \mathscr{M}(P_i), \mathscr{M}(I_i) \neq 0 \forall i$, of arbitrary Conditioned and Unconditioned sub-sequences forming $f$, is defined to be the set of all functions $\bigcup f_j \leftarrow (I_i,P_i)$ .

**Definition** (Sequence-Function Set Product)

Let $S, S', U$ be arbitrary sequence-function sets such that $\Pi := S \cup S' \cup U$, and let $\forall i, j, f_i \in S$

and $f'_j \in S'$. The multiplication of $(S,S')$ is defined to be the binary operation $\Pi \times \Pi \to \Pi$ : $S \otimes S' : \bigcup\limits_{\forall i,j} \{f_i +_A f'_j\} \in U$.
Such sets are comparable to a direct product set.
Written $S \otimes S'$, again, where $S \otimes S' = U := \{f_1 +_A f'_1, f_1 +_A f'_2, .., f_1 +_A f'_j, f_2 +_A f'_1, f_2 +_A f'_2, .., f_2 +_A f'_j \text{ ... ... } f_i +_A f'_1, f_i +_A f'_2, .., f_i +_A f'_j\}$

*Remarks*

A natural implication of span multiplication is a change in the set of conditioned sub-sequences and unconditioned sub-sequences imposed on the resulting set.

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Figure 1: The above figure illustrates a series of arbitrary elements belonging to $span(S)$ where $S \leftarrow (I_i, P_i)$. Units within the grey area are indicative of the unconditioned sub-sequences associated with each sequence depicted.

**Definition** (Set of Irrational Sequences)

Any set $SR$ of all non-duplicate sequence-functions $s \leftarrow (I_i, P_i)$ having the property $\mathscr{M}(I_i) > \mathscr{M}(I_{i-1}), \forall i \in \mathbb{N}$, is defined to be a set of irrational sequences.

**Definition** ($1_{q_1,t_1}$)

Given an arbitrary sequence-function $f \in S$, $1_{q_1,t_1}$ is defined to be an unconditioned-segment of $f$, $P(f)[q_1,t_1]$, with $1's$ spanning its length.

**Lemma 0**

$\forall s \in SR$, no $j \in \mathbb{N}$ exists such that $A_j(s) = 111....$

proof

Let $I_{q_1}[q_1,t_1], I_{q_2}[q_2,t_2]$ be arbitrary consecutive conditioned sub-sequences of $s \in SR$, if $A_j(S)$ results in $P_{q_1}[q_1,t_1] = 1_{q_1,t_1}$, then $A_g(S)|g > j$ is required in order for $P_{q_2}[q_2,t_2]$ to result in $1_{q_2,t_2}$. Since the chosen conditioned sub-sequences in concern are arbitrary, for infinite sequence-functions imposed in this manner, no finite set of Sums exists such that

$A_g(S) = 11111....$

We can be certain that any sequence following an *irrational* progression will not be reducible to 0.1111. , and this is the exact property we require to keep such sequences from exhibiting properties of $\mathbb{Q}$ when unrestricted in length, in a manner of speaking.

**Definition** (Conditioned sub-sequence Removed Sequence Set)

Given an arbitrary sequence-function $s \in S$ of some sequence-function set $S$ having conditioned sub-sequences and unconditioned sub-sequences $(I_i, P_i)|\forall i$ respectively. The sequence-function $s'$ having the ordered arrangement of $P_i|\forall i$ as its image is defined to be a conditioned sub-sequence removed sequence-function.

The set of all such sequence-functions of S is defined to be the conditioned sub-sequence removed sequence set associated with $S$, written $S_{/I}$.

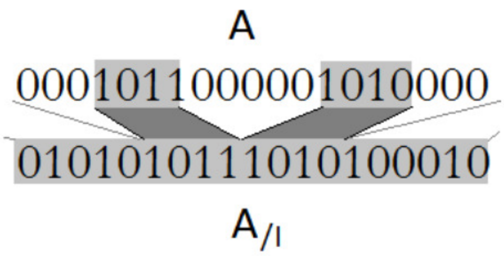


Figure 2: Conditioned sub-sequence Removed sequences.

**Definition** (Half Paired Elements)

Given an arbitrary sequence function $s \leftarrow (I_i, P_i)$. The resulting function formed by conditioning the first half of each unconditioned sub-sequence $P_i|\forall i$ of $s$ with a conditioned sub-sequence (sub-sequence conditioning) of length $\mathscr{M}(P_i)(Mod_L)2$, is defined to be a half paired function element $/s$ of $s$.

**Definition** (Half Conditioned sub-sequence Extended Sequence Set)

A half conditioned sub-sequence extended sequence set is defined to be the set $/S$ of all non-duplicate half-paired function elements associated with a set $S$ of sequences.

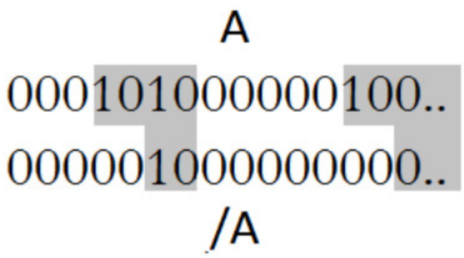


Figure 3: Conditioned sub-sequence Extended sequences.

## 4 The Theory

We will devote the second part of this article toward establishing the cardinality of $\mathbb{S}_{\mathbb{R}}$ with respect to $\mathbb{N}$ and $\mathbb{R}$ respectively. We will achieve this by forming a ratio of the form: $|(\mathbb{N})| : |(\mathbb{R})|$ as $|(\mathbb{N})| : |(H)|$, with $H \subset \mathbb{S}_{\mathbb{R}} \subset \mathbb{R}$ and for $H \subset \mathbb{S}_{\mathbb{R}}$, $|(H)| : |(\mathbb{R})|$ as $|(\mathbb{N})| : |(\mathbb{R})|$. ($\mathbb{S}_{\mathbb{R}}$ here refers to the collection of all Irrational-Sequences of indefinite length). The difficulty in establishing this result surrounds $H$ being a subset of $\mathbb{S}_{\mathbb{R}}$ and as such any attempt at pairing elements of $H$ with those of $\mathbb{R}$ becomes challenging for obvious reasons. We will however formulate a means of overcoming this challenge by implementing a technique of pairing and *shrinking* via denotation, that which is in one-to-one correspondence with the sequence set $H$. Finally we will argue that $\mathbb{R}$ has a cardinally larger spanning-set than $H$ by demonstrating that the unpaired remainder $K \subset \mathbb{S}_{\mathbb{R}}$ is such that any attempt at pairing elements of $K \to \mathbb{R}$ respectively is much the same as attempting the pairing $(\mathbb{N}, \mathbb{R})$.

Before continuing, we prove the following important lemma.

**Lemma 1.0**

If a sequence-function $s$ is formed having an image mapped to the diagonal values $d$ of a set of sequence-functions $s_1 \ldots s_n \in D \leftarrow (I_i, P_i)$, then $s \leftarrow (I_i, P_i)$ and as such constitutes an element of $D$.

*Proof*
Any conditioned sub-sequence of length $k$, makes use of $k$ elements and thus $k$-diagonal entries in the way of producing a sequence of length $k$. For any such set of conditioned sub-sequences in alignment, the result is clearly a conditioned sub-sequence of same length, and the same is thus true of any arbitrary series of unconditioned sub-sequences in alignment as well.

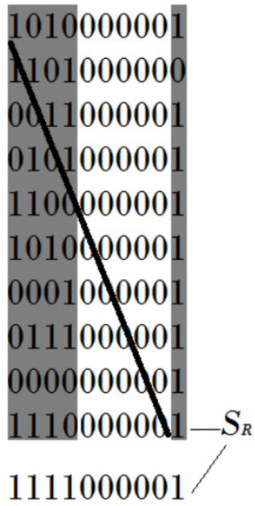


Figure 4: Diagonal argument as applied to induced sets.

# 5 Main Argument

Let all $s \in SR$ be such that $s \leftarrow (I_i, P_i)$, where $\forall i, \mathscr{M}(I_{i+1}) = \mathscr{M}(I_i) + 1$. $\forall i$, and $\mathscr{M}(P_i) = g | g \in 2n | n \in \mathbb{N}$. Let $I_i', P_i^2$ be the conditioned $Hs$ and unconditioned sub-sequences respectively of all sequences $s' \in S'$ belonging to $s' \in hs(SR) = span(I_i', P_i^2)$. Let in addition $\mathscr{M}(P_i^1) + \mathscr{M}(P_i^2) = g$, with $\mathscr{M}(P_i^2) = \frac{g}{2}$ , then

$$SR = span(I_i, P_i^1) \otimes span(I_i', P_i^2) \tag{1}$$

Rewriting elements of $(\Gamma_1 := span(I_i, P_i^1), \Gamma_2 := span(I_i', P_i^2))$ as $\alpha_i \in \Gamma_1, \alpha_i' \in \Gamma_2$ respectively, we can see from (2) that $SR := \Gamma_2 \otimes span(I_i, P_i^1)$.
Now, if we try and pair $\{\bigcup\limits_{\forall i} \alpha_i\}$ with $\{\bigcup\limits_{\forall i} \alpha_i\} \otimes span(I_i', P_i^2) := \{\bigcup\limits_{\forall i} \alpha_i\} \otimes \{\bigcup\limits_{\forall i} \alpha_i'\}$, we note that the elements of both sequence-sets in one-to-one correspondence have already been *shrunk* (To shrink via denotation is to mean the re-representation of a sub-sequence via the use of a variable.), any attempt at pairing $\bigcup\limits_{\forall i} \alpha_i$ with $span(I_i', P_i^2)$ and $\bigcup\limits_{\forall i} \alpha_i$, is the same as attempting to pair elements of $(\mathbb{N}, \mathbb{R})$ respectively. This can be seen if one uses $span(I_i', P_i^2)$ in the diagonal argument instead of $\mathbb{R}$ and takes into account Lemma 1.0 in the following way:
For any arbitrary set of paired-elements $(\alpha_1 : \alpha_1), (\alpha_2 : \alpha_2), ..., (\alpha_n : \alpha_n)$, attempting the mapping of elements of $s_1, s_2, ... \in span(I_i', P_i^2)$ alongside such a pairing, having in mind that $\bigcup\limits_{\forall i} \alpha_i \rightarrow \bigcup\limits_{\forall i} \alpha_i$ is obviously onto, quickly shows that one can easily find an unpaired element $s_j \in span(I_i', P_i^2)$ by employing the diagonal argument using $s_1, s_2, ... \in span(I_i', P_i^2)$.

$$\alpha_1 : (\alpha_1, 100100101111...),$$
$$\alpha_2 : (\alpha_2, 001010100001...),$$
$$...,$$

$$\alpha_n : (\alpha_n, 010101011110...)$$

(Example) An attempt at pairing $\Gamma_1 \rightarrow SR$.

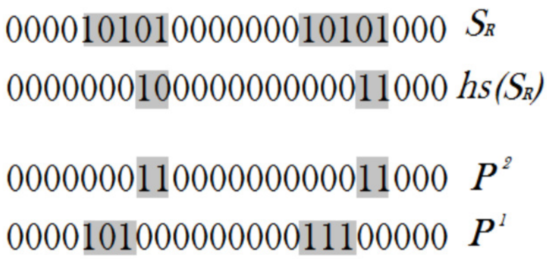


Figure 5: Elements and Half Paired Elements.

# 6 CONCLUSION

As $\omega_<$ is arbitrary, the previous arguments made, can be applied inductively for all $\omega_j$. The results as such, follow in general as outlined in the introduction to the previous section.

One can apply the diagonal argument involving $(\mathbb{N}_2, SR)$. Since $SR \subset \mathbb{R}_2$ as already established. This is all that needs proving in the way of establishing the cardinality of $SR$ between that of $(\mathbb{N}_2, \mathbb{R}_2)$

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