Moving Target Detection Using CA, SO and GO-CFAR detectors in Nonhomogeneous Environment

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Modernization of radar technology and improved signal processing techniques are necessary to improve detection systems in complex situations. A fundamental problem in radar systems is to automatically detect targets while maintaining a desired constant false alarm probability. This work studies two detection approaches, the first with a fixed threshold and the other with an adaptive one. In the latter, we have learned the three types of detectors CA, SO, and GO-CFAR. This research aims to apply intelligent techniques to improve detection performance in a nonhomogeneous environment using standard CFAR detectors. The objective is to maintain the false alarm probability and enhance target detection by combining intelligent techniques. With these objectives in mind, implementing standard CFAR detectors is applied to nonhomogeneous environment data. The primary focus is understanding the reason for the false detection when applying standard CFAR detectors in a nonhomogeneous environment and how to avoid it using intelligent approaches.

Keywords: CFAR detector, detection, adaptive threshold, non-homogeneous, false alarm probability.

I. Introduction

One of the primary radar concepts is to decide if the target is present or not. It can be done by looking at the display and waiting until a bright reflection returned from the target compared with the background brightness [1]. Modern radars perform target detection automatically. There are many detection techniques that have been developed in the literature [1][2][3]. Radar systems' primary detection technique applies a fixed threshold on the received signal. The fixed level of the threshold is decided considering the background (thermal noise, external noise, and clutter). The radar will declare that the target is present if any sample exceeds this threshold. Once the target is displayed, the radar processes this detection to extract target information based on the radar application.

A CFAR detector was designed to avoid false detection in performing fixed threshold. The CFAR detector estimates the noise background (or clutter) statistics to set a threshold that adaptively varies with different background conditions. Cell-Averaging CFAR (CA-CFAR) is one of the earliest CFAR introduced by Finn and Johnson in 1986 [4]. There are many other CFAR detectors, such as the Greatest Of CFAR (GO-CFAR) and Smallest Of CFAR (SO-CFAR), which were designed to either maintain false alarms or improve detection based on specific situations of the noise background.

The CFAR detectors perform best when the background is uniformly distributed, such as thermal noise [1]. In practice, however, the radar-received signal could contain unwanted background clutter such as land, rain, and sea clutter [5]. These clutters could appear as targets in some cases, which makes target detection in the presence of background clutter more difficult. Therefore, it is essential to understand background clutter characteristics. The statistical model is a characteristic required for performance predictions, simulations, and design of detection processing [5].

In this work, we propose the analysis of the three types of CFAR, as well as a comparison between CA, SO, and GO-CFAR systems in homogeneous and nonhomogeneous environments. A detailed study, including the analysis, the tests, and the interpretations of the results, was carried out for the three types of detectors. The comparison made it possible to position the system compared to the others. Also, the advantage brought by each type has been justified through the simulation results, whatever the mathematical solution. Our work is organized as follows: First, we approach the theory of decision and its criteria with some decision criteria while specifying their interests and their limits. Then, we analyze CA, SO, and GO-CFAR systems. We have the results obtained by MATLAB programming and the interpretation of the graphs obtained in different environmental situations according to the SNR variation.

II. Decision Theory and Criterion

A. Decision Theory

Statistical decision theory used in several fields such as radar, sonar, digital communication, and ultrasound imaging, attempts to distinguish between information carrying useful signals and noise. In the binary detection problem, observations can be classified into two mutually exclusive sets, the hypothesis nothing (H_0) which represents target absence. In this case, the received signal is composed of samples of noise plus clutter, and the hypothesis (H_1), which represents the target presence, where the target signal is included in the received signal.

Random nature of the signals received by the radar receiver requires processing before compared to a chosen detection threshold. To make the correct decision, four probabilities will be used to judge a decision criterion performance, and to choose between the two hypotheses, two types of error can occur. Type 1: H_1 is chosen when H_0 is true, which is called a false alarm, and type 2: H_0 is chosen when H_1 is true, referred to as a loss [3]: $P(D_0 / H_0)$ which corresponds to a non-detection probability; $P(D_1 / H_0) = 1 - P(D_0 / H_0)$ which corresponds to a false alarm probability (*Pfa*); $P(D_0 / H_1) = P_M$ which corresponds to a non-detection probability (P_m); $P(D_1 / H_1) = 1 - P(D_0 / H_1) = 1 - P_m$ which corresponds to a target detection probability.

We have: $$P(D_i/H_i) = \int_{Z_i} f_{X/H_j}(x/H_j)dx \quad (1)$$

Which is the conditional density function probability of the received signal y(t) under the hypothesis H_j [6]

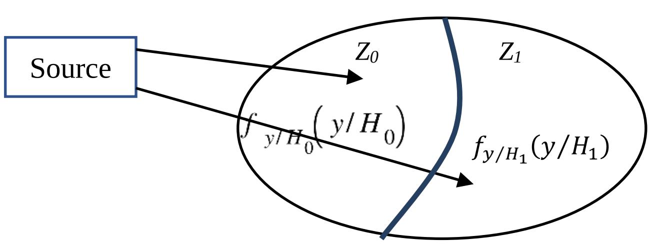

Fig 1: The region of decision

B. Decision Criterion

In the test problem for the binary hypotheses, there are two possible hypotheses at transmission and two possible decisions at the reception. The four different situations mentioned above will be: (1) H_0 true and we decide H_0; (2) H_0 true and we decide H_1; (3) H_1 true and we decide H_0; (4) H_1 true and we decide H_1.

Situations (1) and (4) correspond to good decisions, while the other two correspond to erroneous decisions. The decision rule that we are trying to design means making good decisions as often as possible. For this, we associate a criterion that measures the quality of the decision. Among these decision criteria, the two main ones are Bayes criterion and Neyman-Pearson criterion. Generally, radar systems use the Neyman-Pearson criterion, while in communication and for pattern recognition systems the Bayes criterion is used. [7]

1. Bayes Criterion

We will consider the problem of testing binary hypotheses. To formulate the Bayes criterion, we use two assumptions. In the first, we assume that the output probabilities of the source are known. These are called prior probabilities, where it takes one of the two values $P(H_0)$ and $P(H_1)$ plus detection costs (C_{ij}) to every possible behavior of the decision system where the cost is equivalent to a penalty or a reward. We have the conditions: [6]

$$P(H_0) + P(H_l) = 1 \qquad (2)$$

The detection costs C_{ij} are assigned to the couples $(D_i\, H_i)$ with the condition: $C_{ii} < C_{ii} \quad \forall i \neq j$

The purpose of the Bayes criterion is to determine the decision rule that leads to a minimum average cost. Bayes cost function (risk function) $R=E(C)$, is [3]:

$$R = E(C) = \sum_{j=0}^{1} \sum_{i=0}^{1} C_{ij} P(D_i, H_j) \qquad (3)$$

From Bayes' rule: $P(D_i, H_j) = P(D_i, H_j) * P(H_i)$ (4)

$$R = P_{00}C_{00}P(D_0/H_0) + P_l C_{01} P(D_0/H_l) \\ + P_0 C_{10} P(D_l/H_0) + P_1 C_{11} P(D_l/H_l) \qquad (5)$$

The conditional probabilities $P(D_i/H_j);\ i,j=0,1$ depending on the observation regions are:

$$P(D_i/H_j) = P\{\text{ Decide } (D_i/H_j) \text{ is true }\} \\ = \int_{Zi} f_{X/H_1}(x/H_j)dx \qquad (6)$$

$$R = P_0C_{10} + P_1C_{11} \\ + \int_{z0} \{P_1(C_{01} - C_{11}) f_{X/H_1}(x/H_j) - P_0(C_{10} - C_{00}) f(x/H_0)\} dx \qquad (7)$$

We observe that the quantity $P_0C_{10}+P_1C_{11}$ is constant, regardless of how we assign the points in the observation space. The risk is minimized here by choosing the decision region Z_0 in such a way that it includes only the points of Y for which the second boundary is larger [8].

$$\Lambda(X) = \frac{f_{X/H_1}(x/H_1)}{f_{X/H_0}(x/H_0)} \quad {>_1 \atop <_{H_0}} \frac{P_0(C_{10}-C_{00})}{P_1(C_{01}-C_{11})} \quad (8)$$

Where: $\Lambda(X)$ is the likelihood ratio and $\eta = \frac{P_0(C_{10}-C_{00})}{P_1(C_{01}-C_{11})}$ decision threshold.

In general, the costs and the probability of radar detection are worth the: $C_{ij} = 0, C_{ij} = 1, P_0 = P_1 = \frac{1}{2}$

The problem with this criterion is that it requires knowledge of the probabilities of $P(H_1)$ and $P(H_0)$ and the costs C_{ij}, in general, when this is not the case, other criteria are used, such as NEYMAN PEARSON.

2. Neyman Pearson Criterion

Unlike Bayes criterion, the Neyman-Pearson criterion does not require many assumptions; it corresponds to real situations. This criterion proposes to set the probability of false alarm Pfa at a level α. A specification from which an objective function J is built establishes this value [9]: $J(\lambda) = P_m + \lambda(Pfa - \alpha)$ (9)

Where: $\lambda(\lambda \geq 0)$ is the Lagrange multiplier. We note that for an observation space Z given, there are several decision regions Z_1 for which $Pfa=\alpha$.

The Lagrange multiplier is defined as maximizing the probability of detection or minimizing the probability of P_m [3].

$$J(\lambda) = \int_{Z_1} f_{X/H_1}(x/H_1)dx + \lambda\left[\int_{Z_1} f_{X/H_0}(x/H_0)dx - \alpha\right] \quad (10)$$

Since $Z = Z_0 \cup Z_1$. So, the equation (5) becomes: $J(\lambda) = \lambda(1-a) + \int_{z_0}\left[f_{X/H_1}(x/H_1) - \lambda f_{X/H_0}(x/H_0)\right]dx$ (11)

J is minimized when the values for which $f_{X/H_1}(x/H_1) > f_{X/H_0}(x/H_0)$ are assigned to the decision region Z_1 [8]. The solution to the inequality is: $\frac{f_{X/H_1}(x/H_1)}{f_{X/H_0}(x/H_0)} < \lambda$ (12)

Moreover, we can give the decision rule: $\Lambda(X) = \frac{f_{X/H_1}(x/H_1)}{f_{X/H_0}(x/H_0)} >^{H_1}_{H_0} \lambda$ (13)

$f_{X/H_0}(x/H_0)$ Represent the conditional probability of X under the hypothesis H_0; where λ is chosen in such a way as to satisfy the constraint [3]. $Pfa = \int_{\lambda}^{\infty} f_{X_0/H_0}(x/H_0)dx = \alpha$ (14)

The Neyman-Pearson criterion is applied in several fields because it does not require knowledge of the probabilities of the hypotheses H_1 and H_0 as well as the co cost C_{ij}, but its drawback is that it can only be used when the process is stationary, which is not always the case in practice. It caused the birth of adaptive sensing.

3. Minimax Criterion

The minimax test solves the problem of knowing a priori probabilities encountered previously. To obtain this test, we begin by analyzing the influence of the choice of the threshold on the Bayesian risk. Suppose that a certain value of the decision threshold η is fixed. Once the threshold is set, comparing the likelihood ratio with this threshold can obtain the performance of the test. The resulting decision rule of the minimax criterion is as follows:

$$\Lambda(X) \underset{H_0 H_1}{>>} \frac{(1-p_1)(C_{10}-C_{00})}{p_1(C_{01}-C_{11})} = \eta \quad (15)$$

To achieve this test, we must therefore choose a threshold leading to the values of Pm and *Pfa* satisfying the following equality: $C_{11} - C_{00} + (C_{01} - C_{11})P_m - (C_{10} - C_{00})Pfa = 0$ (16)

III. Generalities of CA SO GO-CFAR Detectors

There are several CFAR detection algorithms, the difference being in the method used to estimate the level of clutter depending on the type of environment. We detail in this section the calculation of false alarm probabilities and detection probabilities for CA, SO and GO-CFAR processors respectively in the presence of a Pearson distribution clutter.

Cell Averaging CFAR (CA-CFAR) proposed by Finn and Johnson [10] refers to an estimation method consisting of calculating the arithmetic sum of all the cells on either side of the test cell (Q=U+V). It is therefore necessary to modify the CA-CFAR. This is how Hansen and Sawyers proposed the GO-CFAR (Greatest Of CFAR), which is a detector comparing the arithmetic sums of the two windows U and V in order to choose the greatest value (Q=max (U, V)). Compared to CA-CFAR, it offers good results but does not solve the problem of the presence of interfering targets. It was then that Trunk proposed SO-CFAR (Smallest Of CFAR) which chose the smaller of the two windows U and V (Q=min (U, V)). If the two windows U and V contain at the same time interfering targets, then the performances of this detector as well as those of the GO-CFAR are considerably altered.

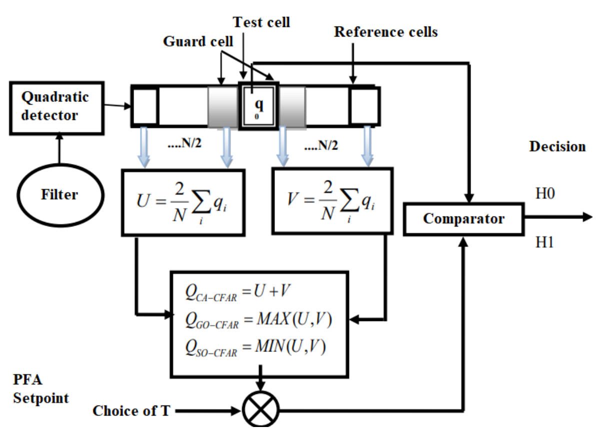

Fig 2: CA GO SO-CFAR Detectors

IV. Analysis of CA, SO, GO CFAR Detectors

We assume that the output of the quadratic detector for any range of cells is an exponential distribution for the purpose of analyzing the detection performance of the CFAR processor. The PDF probability density function is [2]:

$$f(x) = (1/2\lambda)\exp(-x/2\lambda)x \geq 0 \quad (17)$$

Under the null hypothesis H_0, the absence of the target, λ is the total noise power (clutter + thermal noise) which is denoted by μ

Under the alternative hypothesis H_1, presence of a target, μ is $\mu = (1+S)$, where s is the mean of the total signal to noise (*SNR*) of a target. The observations in *N+1* cells, including the test cell, are assumed to be statistically independent. So, for the latter, the value of λ in (1) is [2]:

$$\lambda = \begin{cases} \mu & \text{Under } H_1 \\ \mu(1+S) & \text{Under } H_2 \end{cases} \quad (18)$$

The optimal detector sets a threshold to determine the target presence under the assumption that the total noise power is known a priori. In this case the probability of false alarms is [3]:

$$Pfa = P[Y > Y_0/H_0] = \exp\,(-Y_0/2\mu) \quad (19)$$

Similarly, the optimal detection probability *Pd* is: $P^{opt} = P[Y > Y_0/H_1] = \exp\,(-Y_0/2\mu(1+S))$ (20)

By substituting (19) by (20), we will have: $P_d^{opt} = [Pfa]^{\frac{1}{1+S}}$ (21)

In the CFAR processor, the threshold varies according to the local information on the total power of the noise. The Z statistic is a random variable whose distribution depends on the CFAR regime chosen and the adjacent distribution of each of the samples in the reference range. Thus, the processor performance is the average detection probability and false alarm probability. *Pfa* is in general:

$$Pfa = E_Z\{P[Y \geq TZ/H_0]\} \quad (22)$$

we can rewrite it this way:

$$\begin{aligned} Pfa &= E_Z\left\{\int_{TZ}^{\infty} (1/2\mu)\exp\,(-y/2\mu)\mathrm{d}y\right\} \\ &= E_Z\{\exp\,(-TZ/2\mu)\} \\ &= M_Z(T/2\mu) \end{aligned} \quad (23)$$

Where M_Z represents the *MGF* of the random variable *Z*. The detection probability P_d is: $P_d = E_Z\{P[Y \geq TZ/H_1]\}$ (24)

Given that, according to hypothesis H_1 of signal presence $2\lambda = 2\mu(1+S)$, one can determine the probability of detection P_d by simply replacing μ by $\mu(1+S)$ in the equation (23): $P_d = M_Z[T/2\mu(1+S)]$ (25)

For a CFAR regime, *M Z (T / 2μ)* must be independent of μ. It is true for all the CFAR processors that we will study.

A. Analysis of the CA-CFAR Processor

The sum of *N* cells in the reference window estimates the total noise power [6]: $Z = U + V = \sum_{i=1}^{N} X_i$ (26)

Exponential density is a special case of gamma density with $\alpha = 1$ in the probability density function (*PDF*) [2]:

$$f(y) = \beta^{-\alpha} y^{\alpha-1} \exp\,(-y/\beta)/\Gamma(\alpha) y \geq 0, \alpha \geq 0, \beta \geq 0 \quad (27)$$

With $\Gamma(\alpha)$ denoting the usual gamma function, which has the value $(\alpha-1)!$ The *Cdf* corresponding to this PDF is: $G \sim (\alpha,\beta)$ [2]. We write $y \sim G(\alpha,\beta)$, to mean that y is a random variable with a probability density function given in the previous equation (27).

The moment generating function corresponding to the distribution of $G(\alpha,\beta)$ is: $M_{\gamma}(\mu) = (1+\beta t)^{-\alpha}$ (28)

Using the notation above, we have $X \sim G(1,2\mu)$ and $Z \sim G(N, 2\mu)$.

The detection probability for the CA-CFAR processor is obtained by substituting (28) in (25) with β=2, which gives [2]:

$$Pd = [1 + T/(1 + S)]^{-N} \quad (29)$$

The false alarm probability is calculated from equation (29) by setting *S=0*: $Pfa = [1 + t]^{-N}$ (30)

The constant scale factor *T* is calculated from equation (30): $T = (Pfa)^{-\frac{1}{N}} - 1$ (31)

We notice from equations (29) and (30) that the detection and false alarm probabilities Pd and Pfa are independent of μ When the number of reference cells becomes large $(N \to \infty)$, the CFAR detector approaches the optimal detector [3].

$$P_d = \lim_{N\to\infty} [1 + T/(1 + S)]^{-N} = \exp(-T/(1 + S)) \quad (32)$$

And the false alarm probability:

$$\text{Pfa} = \lim_{N\to\infty} [1 + T]^{-N} = \exp(-T) \quad (33)$$

The table below represents the relationship between the *T* factor, the *Pfa* and *N*.

Table 1: Relation between *T* and *Pfa* for a CA-CFAR detector

N	*T*		
	Pfa=10^{-4}	*Pfa=10*$^{-6}$	*Pfa=10*$^{-8}$
8	2.182	4.623	9.00
16	0.778	1.371	2.162
24	0.468	0.778	1.154
32	0.344	0.540	0.778

1. Presence of Clutter in the Reference Window for a CA-CFAR

In the case of the clutter, the hypothesis of the statistical independence of the reference cells is maintained in the context of the analysis of the performance of the CA-CFAR detector when the reference window no longer contains radar echoes from the homogeneous environment.

Suppose the reference window contains r clutter cells with noise power $\mu_0(1 + C)$ and (N - r) cells from a homogeneous environment with noise μ_0. So, the noise power estimate is [2]: $Z = \sum_{i=0}^{r} X_i + \sum_{i=r+1}^{N} X_i = Z_1 + Z_2$ (34)

Where: $Z_1 - G(r, 2\mu_0(1 + C))$ and $Z_2 - G(N - r, 2\mu_0)$

Let assume that the test cell is from a clear environment, we have: $Pfa = [1 + (1 + C)T]^{-r}[1 + T]^{r-N}$ (35)

As the window sweeps around the range of cells, more cells from an inhomogeneous environment that contains clutter enter the reference window. In the end, the test cell comes from this environment we have: $Z_1 \sim G(r, 2\mu_0)$ and $Z_2 \sim G(N - r, 2\mu_0(1 + C))$

Then the equation (35) becomes: $Pfa = M_Z[T/2\mu_0(1 + C)] = (1 + C)^{-r}(1 + T/(1 + C))^{r-N}$ (36)

B. Analysis of GO and SO-CFAR Detectors

The CA-CFAR detector is modified in view of its adaptation to each of inhomogeneous clutter situations where the homogeneity hypothesis is no longer valid due to the edge effect and the multiple target situation. It's at that moment the use of GO-CFAR (Greatest Of Constant False Alarm Rate) detector has been proposed by Hansen and Sawyers to control the increase in the probability of false alarm Pfa. Here, the cells of the reference window are divided into two sub-windows U and V, one before the CUT (cell under test) and the other after the CUT. Their contents are summed and the larger of the two is used as the clutter power estimator.

$$Q = max(U,V) \tag{37}$$

Where U and V are defined in the following equation: $U = \sum_{i=1}^{n} q_i \qquad V = \sum_{i=n+1}^{N} q_i$ (38)

The windows *U* and *V* contain *n=N/2* identical cells, according to an exponential law, then the sum of *n* cells follows a gamma law *G(n, 1)* which is the probability density function of a gamma distribution with the parameter *n* and 1 such as:

$$p_U(q) = p_v(q) = \frac{1}{\Gamma(n)} q^{n-1} \exp(-q) \quad q \geq 0 \tag{39}$$

We then define the *Cdf* of windows U and V: $p_U(q) = p_V(q) = \int_0^a \frac{1}{\Gamma(n)} q^{n-1} \exp(-q) \mathrm{d}q$ (40)

Considering the properties of distributions, the cumulative distribution function of *Q* is [6]: $p_Q(q) = p_U(q) p_V(q)$ (41)

The probability density function of Q is equal to the derivative of the *Cdf* of Q:

$$\begin{aligned} p_Q(q) &= \frac{d}{\mathrm{d}q} p_Q(q) = 2 p_U(q) p_V(q) \\ &= 2 \frac{1}{\Gamma'(n)} q^{n-1} \exp(-q) \int_0^q \frac{q^{n-1}}{\Gamma(n)} \exp(-q) \mathrm{d}q \end{aligned} \tag{42}$$

The integral in the above expression can be expressed as a finite series: $\gamma(n,q) = \Gamma(n) \left[1 - \exp(-q) \sum_{i=0}^{n-1} \frac{q^i}{i!}\right]$ (43)

The substitution of equation (43) into equation (42), the probability density function of the test statistic *Q* can be given by:

$$p_Q(q) = \frac{2q^{n-1} \exp(-q)}{\Gamma(n)} \left[1 - \exp(-q) \sum_{i=0}^{n-1} \frac{q^i}{i!}\right] \tag{44}$$

The environment's moment generating function is: $M_Q(t) = \int_0^\infty p_Q(q) \exp(-tq) \mathrm{d}q$ (45)

In what follows, we rely on the following property: $\int_0^b t^m \exp(-tq) \mathrm{d}t = m!\, q^{-(m+1)} \times \left[1 - \sum (tq)^j \frac{\exp(-qt)}{j!}\right]$ (46)

Then the environment's moment generating function is determined using the above property:

$$M_Q(t) = 2(1+t)^{-n} - 2 \sum_{i=0}^{n-1} \binom{n+1-i}{i} (2+t)^{-(n+i)} \tag{47}$$

The GO-CFAR detection probability, by substituting equation (47) into equation (24) is:

$$P_D^{GO} = 2\left(1+\frac{T}{1+S}\right)^{-n} - 2\sum_{i=0}^{n-1}\binom{n+i-1}{i}\left(2+\frac{T}{1+S}\right)^{-(n+1)} \quad (48)$$

The probability of false alarm, by setting *S=0* in equation (48) is: $PGO = 2(1+T)^{-n} - 2\sum_{i=0}^{n-1}\binom{n+i-1}{i}(2+T) - (n+1)$ (49)

where *T* is the constant multiplier which depends on *Pfa* and *N*.

Modification of GO-CFAR introduces additional detection losses compared with the loss of the CA-CFAR processor in uniform noise; less than 0.3 dB is generally acceptable. But studies have shown that the presence of one or more interfering targets the reference cells increase the threshold, and seriously impair the detection of the primary target. Since we take the maximum of U et V, the window that contains the interfering targets is always selected. To overcome this problem, the SO-CFAR detector has been proposed in which the clutter level estimation is considered as the minimum.

With this detector, the noise power estimate is: $Q = min(U,V)$ (50)

where U and V are defined in equation (38). So, the probability density function of the Q test statistic [6] is:

$$\begin{aligned} p_Q(q) &= p_U(q)[1-p_V(q)] + p_V(q)[1-p_U(q)] \\ &= p_U(q) + p_V(q) - [p_U(q)p_V(q) + p_V(q)p_U(q)] \quad (51) \\ &= p_U(q) + p_V(q) - p_Q^{GO}(q) \end{aligned}$$

Consequently, by substituting the equation (51) in the equation (23) we have for S0-CFAR:

$$Pfa = M_U(T/\mu) + M_V(T/2\mu) - p_{fa}^{GO} \quad (52)$$

M_U and M_V are the moment generating function of *U* and *V*. They are calculated using the equation (28).

So, the SO-CFAR false alarm probability is: $p_{fa}^{SO} = 2(2+T)^{-n}\sum_{i=0}^{n-1}\binom{n+i-1}{i}(2+T)^{-i}$ (53)

The SO-CFAR detection probability, by setting *T=T/(1+S)* in equation (53) is:

$$p_D^{SO} = 2\left(2+\frac{T}{1+S}\right)^{-n}\sum_{i=0}^{n-1}\binom{n+i-1}{i}\left(2+\frac{T}{1+S}\right)^{-i} \quad (54)$$

1. Presence of Clutter in the Reference Window for GO-CFAR

The GO-CFAR processor works on controlling the false alarm rate in the clutter power transition regions. We have derived the exact expression of the *Pfa* in this case. Consider the special case where the lagging window has noise values from a bright environment and the main window has noise samples from the clutter region. In this case *U* and *V* defined in (38) are distributed as: $U \sim G(n, 2\mu_0)$ and $V \sim G(n, 2\mu_0(1+C))$

Then, if the test cell contains a sample of a clear environment, we have:

$$\begin{aligned} p_{fa}^{GO} &= (1+T)^{-n} + (1+(1+C)T)^{-n} \\ &-\sum(n+j_j-1)\left(1+T+\frac{1}{1+C}\right)^{-(n+j)} \quad (55) \\ &\times\{(1+C)^{-n} + (1+C)^{-J}\} \end{aligned}$$

If the test cell is surround by the clutter *U* and *V* are left as: $U \sim G(n, 2\mu_0(1+C))$ and $V \sim G(n, 2\mu_0)$

To obtain the expression of the *Pfa* we replace *T* by *T/(1+C)* in equation (55):

$$p_{fa}^{GO} = \left(1+\frac{T}{1+C}\right)^{-n} + (1+T)^{-n} - \sum(n+j)_j - 1)\left(1+\frac{T}{1+C}+\frac{1}{1+C}\right)^{-(n+j)} \times \{(1+C)^{-n} + (1+C)^{-j}\} \quad (56)$$

We substitute equation (56) into (52) to find the p_{fa}^{so} for SO-CFAR:

$$p_{fa}^{SO} = (1+T)^{-n} + (1+(1+C)T)^{-n} - \sum\left(n+j_j-1\right)\left(1+T+\frac{1}{1+C}\right)^{n-1} \times \{(1+C)^{-n} + (1+C)^{-j}\} \quad (57)$$

If the test cell is surrounded by the clutter, *U* and *V* defined in (38) are distributed as follows:

$U \sim G(n, 2\mu_0(1+C))$ and $V \sim G(n, 2\mu_0)$

To obtain the expression of the *Pfa* we replace *T* by *T/(1+C)* in equation (57):

$$p_{fa}^{SO} = \left(1+\frac{T}{1+C}\right)^{-n} + (1+T)^{-n} - \sum\left(n+jj_j-1\right)\left(1+\frac{T}{1+C}+\frac{1}{1+C}\right)^{-(n+j)} \times \{(1+C)^{-n} + (1+C)^{-j}\} \quad (58)$$

C. CFAR Loss

The method used to measure the relative performance of CFAR processors is called CFAR loss. Figure (3) shows the probability detection's graph as a function of *SNR*; it becomes higher with the increase of *SNR*. If knowledge of the average noise is known, then the Neyman-Pearson detector represents the best theoretical detection.

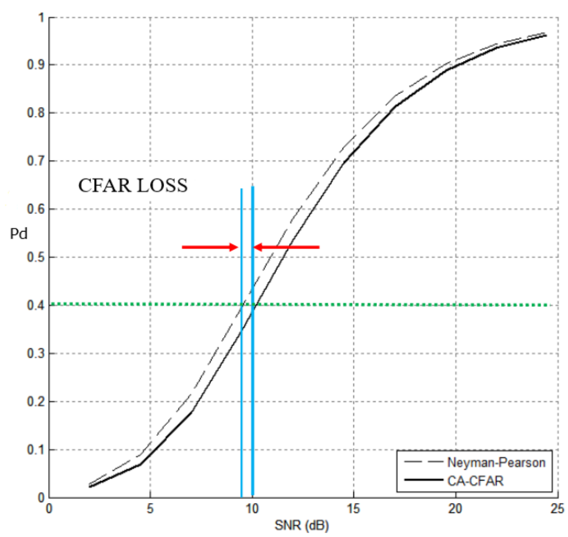

Fig 3: Probability detection graph illustrating CFAR loss.

CFAR loss here defined the quality of the CFAR detector: it is the difference in dB of the CFAR detector's SNR and that of a detector with a fixed threshold necessary to realize the identical detection probability. The fixed threshold detector is called in this case CFAR ideal; in figure (3), for a probability of detection 0.4, the CFAR loss and 1dB of *SNR*.

V. CA, SO and GO-CFAR Simulation and interpretation of the results.

A. Case of Homogeneous Environments

Here we present the variation of the detection probability for the CA, SO and GO-CFAR detectors as a function of the *SNR* by varying cells number *N*. The results are represented in figures 5.1 and 5.6 respectively. We have on the graphs of figures 5.1 - 5.6, the representations of the probabilities of detection according to the *SNR* ratio for a *Pfa* (with $Pfa=10^{-4}$ and $Pfa=10^{-6}$). These representations are made for the CA, SO and GO-CFAR in a homogeneous environment, for several reference cells of 8, 16 and 32 cells, and a curve represents the optimal ratio based on a fixed threshold and the CFAR loss for each process. to judge performance in this environment.

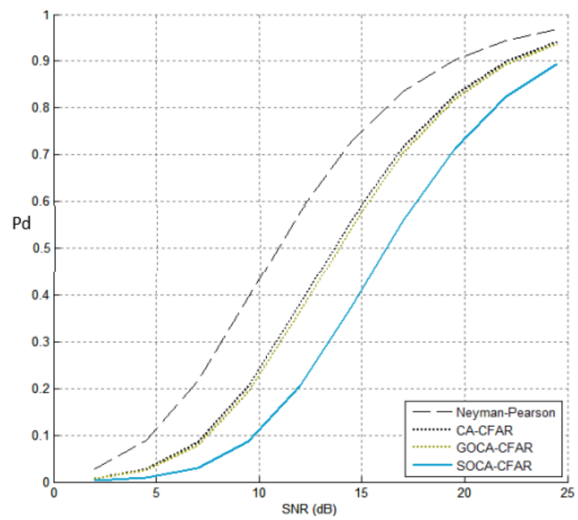

Fig 5.1: P_d as a function of *SNR* for CA GO SO-CFAR and optimum with *Pfa*=10^{-4} and *N*=8.

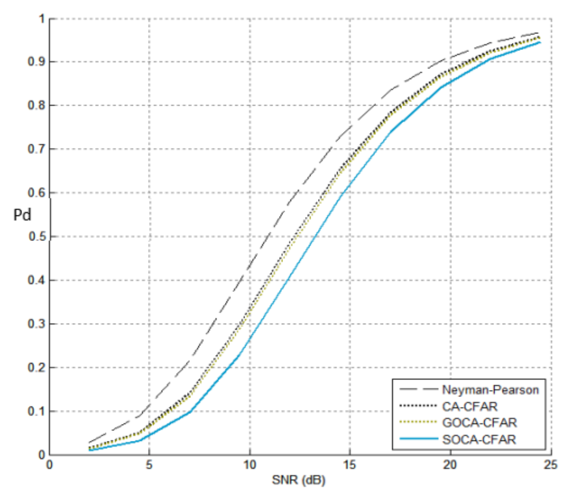

Fig 5.2: P_d as a function of SNR for CA GO SO-CFAR and optimum with *Pfa*=10^{-4} and *N*=16.

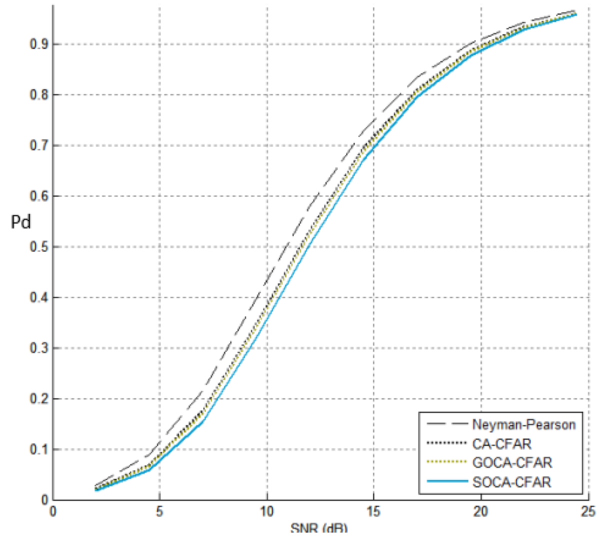

Fig 5.3: P_d as a function of *SNR* for CA GO SO-CFAR and optimum with *Pfa*=10^{-4} and *N*=32.

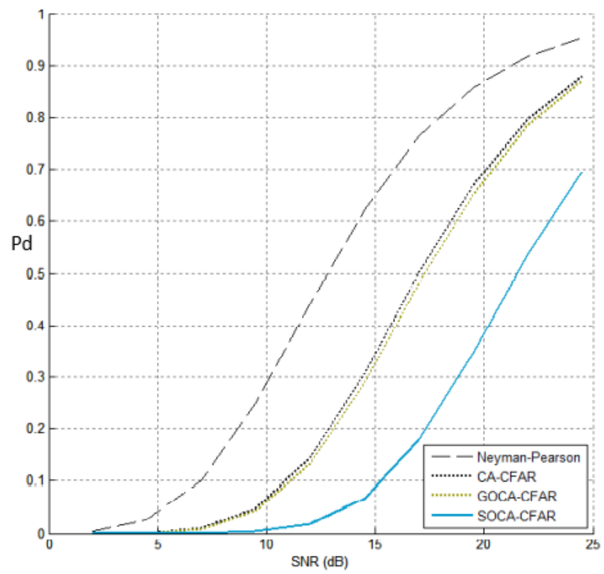

Fig 5.4: P_d as a function of *SNR* for CA GO SO-CFAR and optimum with *Pfa*=10^{-6} and *N*=8.

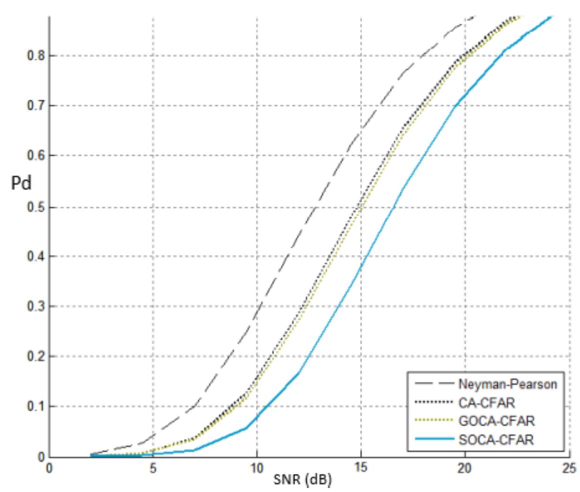

Fig 5.5: P_d as a function of *SNR* for CA GO SO-CFAR and optimum with *Pfa*=10^{-6} and *N*=16.

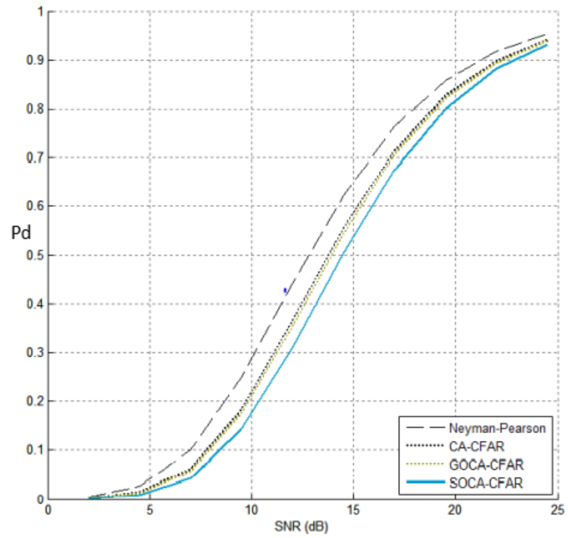

Fig 5.6: P_d as a function of SNR for CA GO SO-CFAR and optimum with $Pfa=10^{-6}$ and $N=32$.

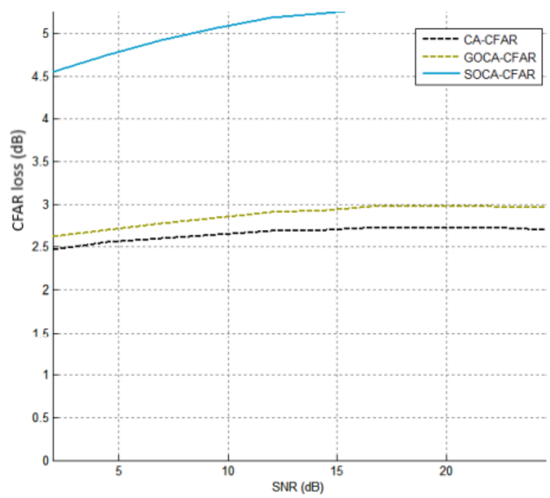

Fig 5.7: The CFAR loss in a homogeneous environment for $N=8$ and $Pfa=10^{-4}$.

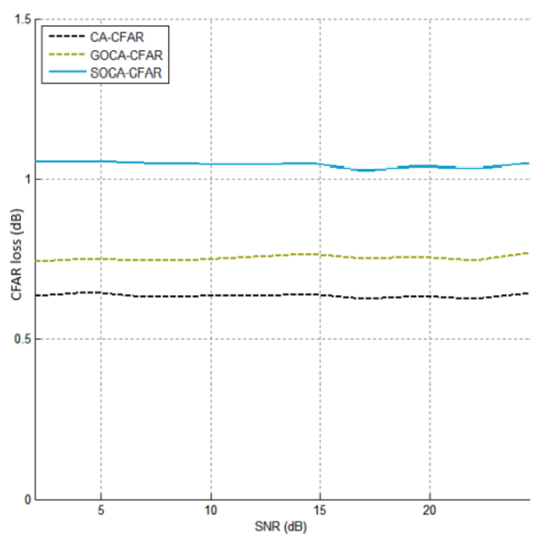

Fig 5.8: The CFAR loss in a homogeneous environment for $N=32$ and $Pfa=10^{-4}$.

Results and discussions: The performance of each system improves by SNR increase of the basic target, and we find that by increasing the size of the reference window, the probability of detection approaches that of the optimal detector. We also find that SO-CFAR's performance dependents on *N*, because for a small value of *N* the loss is high compared to other types of CFAR, but by increasing *N* it highly decreases. For the same *Pfa*, the CA-CFAR detector is slightly more efficient than SO and GO-CFAR in terms of detection probability. The CFAR loss of the SO and GO-CFAR processes can be explained by the fact that the SO and GO-CFAR using only half of its reference windows to estimate the average noise and clutter as opposed to the CA-CFAR which uses the set of cells in the reference window.

Conclusion: We conclude that the key point affecting the probability of detection remains in the first place the number of effective cells: the increase of the number of cells strictly goes with the increase of the detection probability for low values of *SNR*. In homogeneous medium, CA-CFAR represents higher performances compared to SO and GO-CFAR in terms of detection probability.

B. Case of Nonhomogeneous Environment

1. Situation of Clutter's presence

In this situation, two issues can be studied. The first issue is where the test cell is inside the clutter region while the second issue is where the test cell is not reached by the clutter. In this part, half of the reference cells are embedded in the clutter (*CNR=N/2*).

To illustrate the performances of CFAR detectors considered in the case of a non-homogeneous situation, we focus our results on the effect of the transition position R of the clutter and the *CNR* ratio on the *Pfa*. Figures (5.9 - 5.11) present the *Pfa* of the CA, SO, and GO-CFAR detectors as a function of the transition position in the clutter and for different values of the ratio (*CNR=5dB*, *CNR=10dB*, *CNR =15dB*) and a number of reference window cells *N*=32 and a *Pfa*=10^{-4}, the pink dotted line indicates the desired false alarm rate, and the blue line at 16 cells when half of the cells of reference are immersed in the clutter.

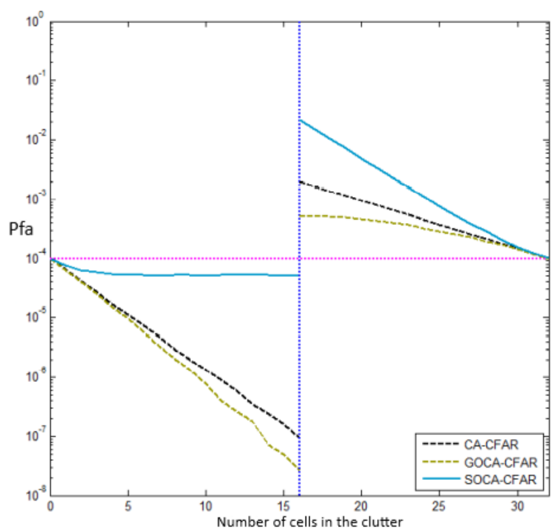

Fig 5.9: *Pfa* as a function of the clutter position transition R, CA SO and GO-CFAR detectors with desired *Pfa* =10^{-4}, *CNR=5dB*.

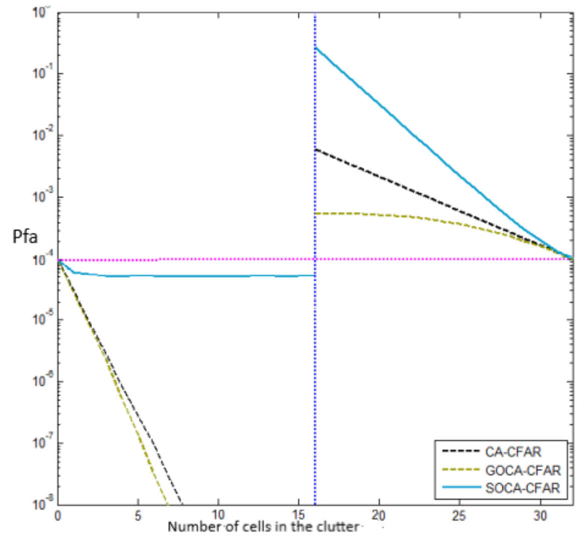

Fig 5.10: ***Pfa*** **as a function of the clutter position transition R, CA SO and GO-CFAR detectors with desired** ***Pfa*=10^{-4}, *CNR*=10dB.**

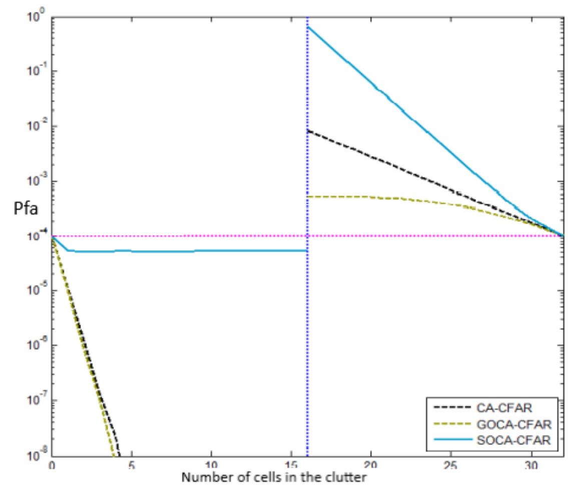

Fig 5.11: ***Pfa*** **as a function of the clutter position transition R, CA SO and GO-CFAR detectors with desired** ***Pfa*=10^{-4}, *CNR*=15dB.**

Results and discussions: In figures (5.9 - 5.11), we see a decrease in the *Pfa* when the position of the transition runs through cells 1 to 16, because it is due to the increase in the detection threshold although the cell under test is drowned in the region where the clutter power is low, because of the target's masking effect. On the other hand, there is an increase in the *Pfa* for the position of the transition from 16 to 32, this is due to a reduction in the detection threshold which thus leads to an increase in the *Pfa*. When the position of the transition runs from cells 1 to 16, the *Pfa* of the CA and GO-CFAR processes decreases dramatically (a little more for GO-CFAR), having a false alarm probability lower than the desired probabilities, the process SO-CFAR shows success in maintaining the false alarm rate which shows resistance to masked targets. For the position of the transition from 16 to 32 we find that the process of the CA-CFAR false alarm probability reaches 10^{-2}, that means a probability of false alarm 100 times higher than the desired probability *Pfa*=10^{-4}. Concerning the SO-CFAR process, the false alarm probability is 1000 times higher than the desired probability *Pfa*=10^{-4}, while the GO-CFAR process undergoes a 10^{-1} increase in the false alarm probability.

Conclusion: The best performance in a non-homogeneous environment to maintain the false alarm rate is obtained by the SO-

CFAR processor when the cell under test is drowned in low-power clutter, while the GO-CFAR processor presents the best results and performance when the test cell is drowned in high power clutter.

2. Situation of interference's presence

In this part, we will simulate all the three processors to study the problem of masking in a non-homogeneous medium and we will represent the performance of the studied CFAR processors with one or more interfering targets in the reference window. Figures (5.12 - 5.16) display the probability of detection in the presence of foreign targets as a function of the main target *SNR* for a $Pfa=10^{-4}$ and $N=32$, we will represent the CFAR loss for each case in figures (5.17 - 5.21).

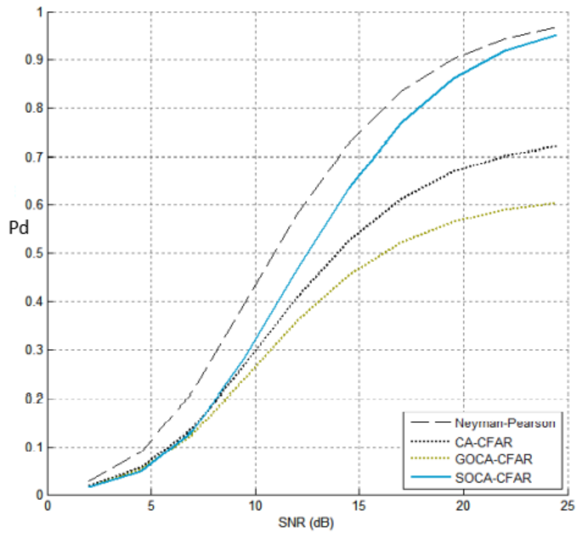

Fig 5.12: P_d as a function of *SNR* for CA GO SO-CFAR with a single interfering target located in a single window.

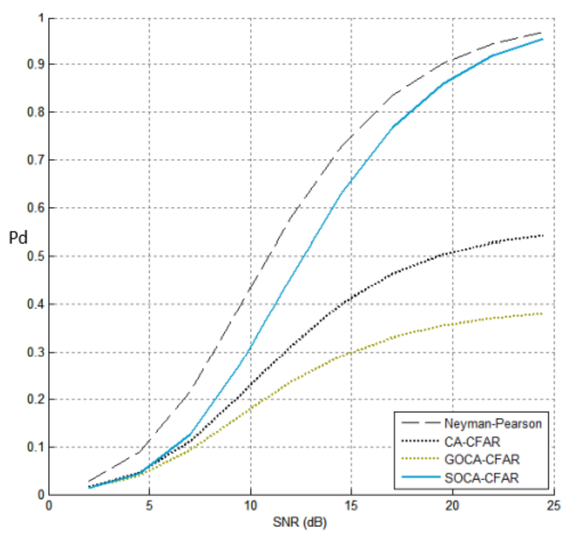

Fig 5.13: P_d as a function of *SNR* for CA GO and SO-CFAR with two interfering targets located in a single window.

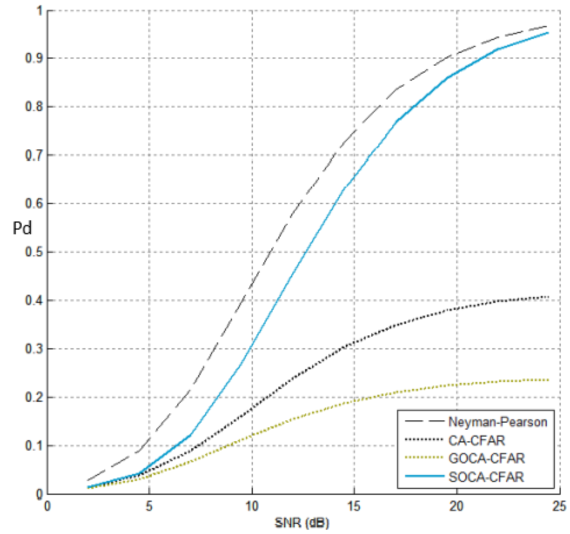

Fig 5.14: P_d as a function of *SNR* for CA GO SO-CFAR with three interfering targets located in a single window.

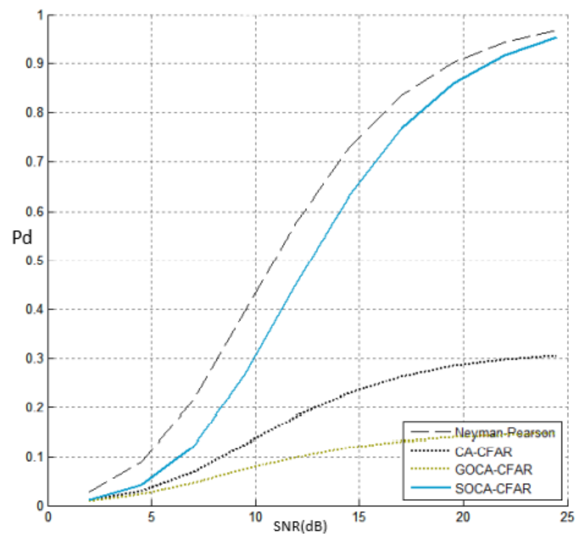

Fig 5.15: P_d as a function of *SNR* for CA GO SO-CFAR with four interfering targets located in a single window.

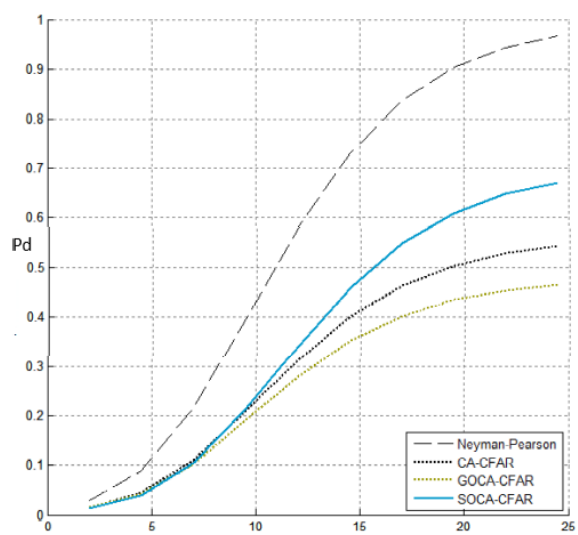

Fig 5.16: P_d as a function of *SNR* for CA GO SO-CFAR with a single target interfering in each of the two windows.

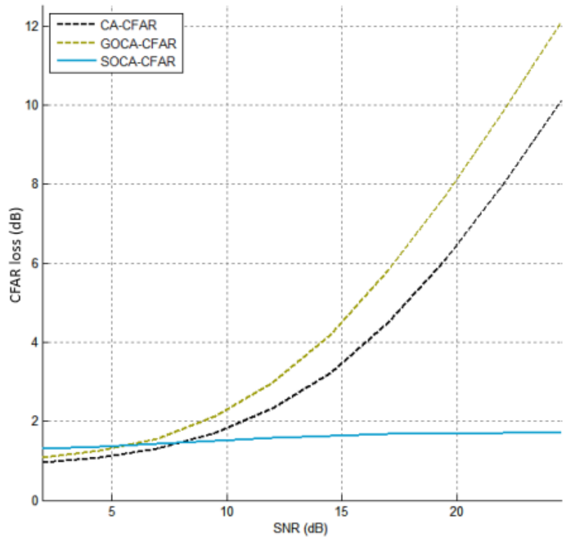

Fig 5.17: CFAR loss for a single interfering target located in a single window.

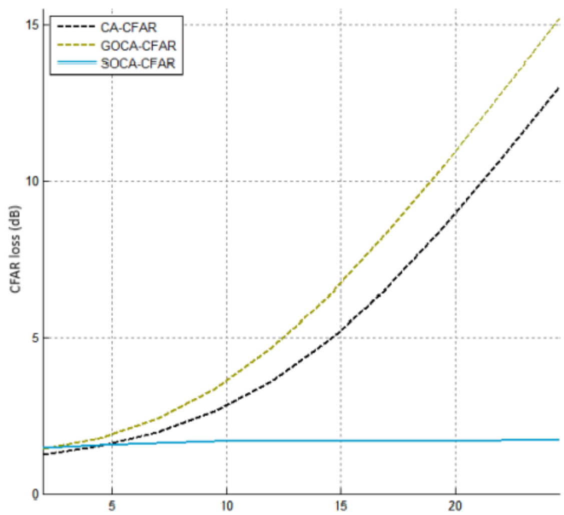

Fig 5.18: CFAR loss for two interfering targets located in a single window.

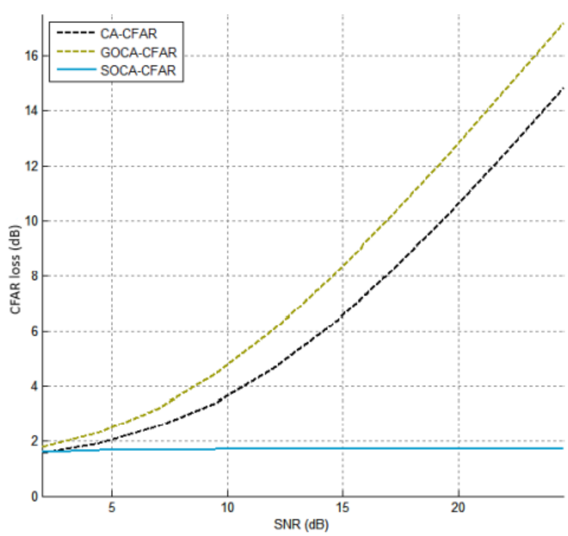

Fig 5.19: CFAR loss for three interfering targets located in a single window.

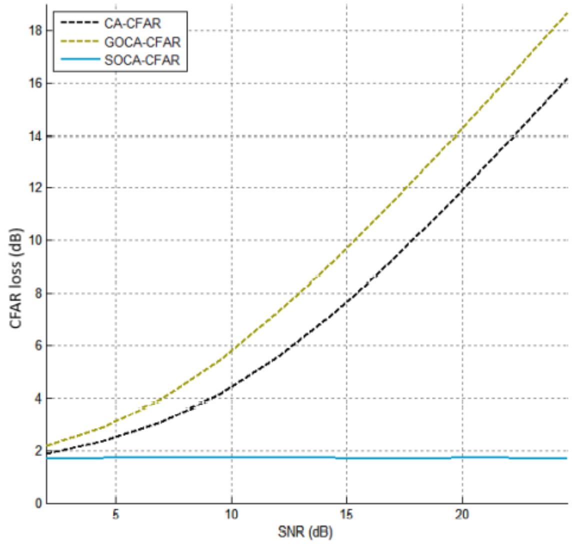

Fig 5.20: CFAR loss for four interfering targets located in a single window.

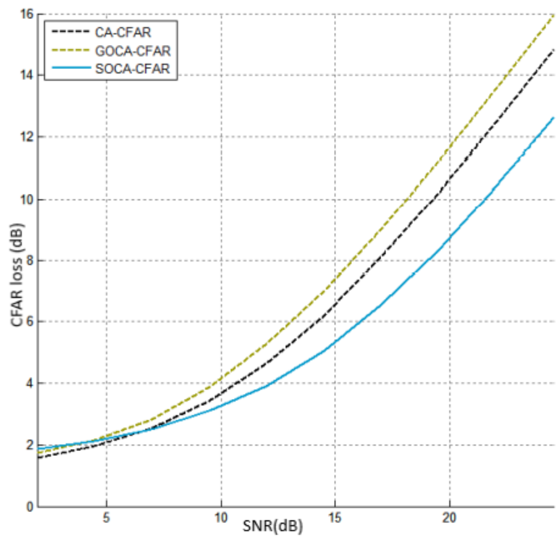

Fig 5.21: CFAR loss for a single target interfering in each of the two windows.

Results and discussions: Figures (5.12 - 5.16) represent the performance of CA SO and GO-CFAR processors with and without interfering targets respectively. We find that the detection probability of CA and GO-CFAR processors in this environment decreases dramatically with increasing the number of interfering targets. On the other hand, there is an increase in the loss of CA and GO-CFAR processes greater than 2dB in figures (5.17 - 5.21).

From these results, we can see that the SO-CFAR process is able to solve the problem of several targets in the reference window, either in the main window or the delay window. Although the SO-CFAR process is preferred in a multiple target environment, it has undesirable effects when the interfering targets are within two halves of the reference window.

Conclusion: We can say that GO and CA-CFAR performances or the presence of interfering targets in the reference cells causes a serious degradation in the detection probability. The SO-CFAR showed robustness in the presence of interfering targets in a single window, but this performance deteriorated in the presence of interfering targets in both windows.

VI. Conclusion

In this research work, we have studied the CFAR processors used mainly in radars to get a constant false alarm rate through an adaptive variable threshold. Here, we have studied and simulated three types of CFAR, namely CA, SO and GO-CFAR. The aim of these simulations was to plot the probabilities of detection as a function of the SNR for different numbers of cells and to compare the performances of these detectors. We have observed that the results of the simulations coincide perfectly with the theoretical results.

Regarding the performance comparison, we found that the CA-CFAR detector behaves well in a homogeneous environment, but when the medium presents transition regions or in front of multiple targets, its performance degrades rapidly. As for the SO-CFAR and GO-CFAR detectors, our simulations allowed us to observe that the detection probability is significantly improved compared to the CA-CFAR when multiple targets are present. The SO-CFAR processor is given the best performance on maintaining the false alarm rate in a non-homogeneous environment, whereas the GO-CFAR processor presents the best performance when the test cell is surrounded by high power clutter.

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