ON OPTIMIZATION OF MANUFACTURING OF FIELD-EFFECT HETEROTRANSISTORS A THREE-STAGE AMPLIFIER CIRCUIT, TO INCREASE THEIR DENSITY

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ABSTRACT

In this paper we introduce an approach to increase density of field-effect heterotransistors framework a three-stage amplifier circuit. At the same time one can obtain decreasing of dimensions of the above transistors. Dimensions of the elements will be decreased due to manufacture heterostructure with specific structure, doping of required areas of the heterostructure by diffusion or ion implantation and optimization of annealing of dopant and/or radiation defects.

Keywords:
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field-effect heterotransistors; three-stage amplifier circuit; optimization of technological process; analytical approach for modeling.

1. INTRODUCTION

Currently density of elements of integrated circuits and their performance intensively increasing. Simultaneously with increasing of the density of the elements of integrated circuit their dimensions decreases. One way to decrease dimensions of these elements of these integrated circuit is manufacturing of these elements in thin-film heterostructures [1-4]. An alternative approach to decrease dimensions of the elements of integrated circuits is using laser and microwave types annealing [5-7]. Using these types of annealing leads to generation inhomogeneous distribution of temperature. Due to Arrhenius law the inhomogeneity of the diffusion coefficient and other parameters of process. The in homogeneity gives us possibility to decrease dimensions of elements of integrated circuits. Changing of properties of electronic materials could be obtain by using radiation processing of these materials [8,9].

In this paper we consider a three-stage amplifier circuit described in Ref. [10] (see Fig.1). We assume, that element of the considered circuit has been manufactured in heterostructure from Fig. 1. The heterostructure consist of a substrate and an epitaxial layer. The epitaxial layer includes into itself several sections manufactured by using another materials. The sections have been doped by diffusion or ion implantation to generation into these sections required type of conductivity (*n* or *p*). Framework this paper we analyzed redistribution of dopant during annealing of dopant and/or radiation defects to formulate conditions for decreasing of dimensions of the considered circuit.
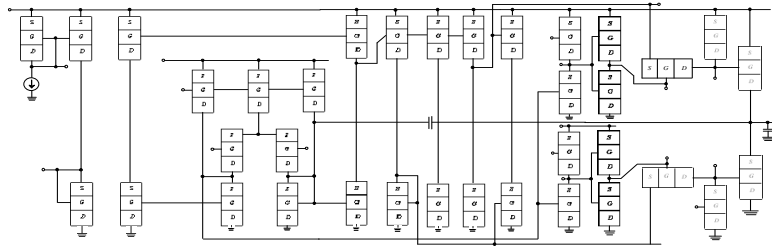
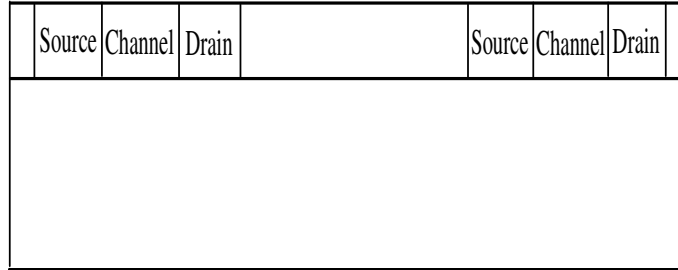Fig. 1*a*. Circuit of the three-stage amplifier circuit [10]

Source	Channel	Drain		Source	Channel	Drain

Fig. 1*b*. Heterostructure with two layers and sections in the epitaxial layer

2. METHOD OF SOLUTION

We determine spatio-temporal distribution of concentration of dopant by solving the following boundary problem

$$\frac{\partial C(x,y,z,t)}{\partial t} == \frac{\partial}{\partial x}\left[D_C \frac{\partial C(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_C \frac{\partial C(x,y,z,t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D_C \frac{\partial C(x,y,z,t)}{\partial z}\right] \quad (1)$$

with boundary and initial conditions

$$\frac{\partial C(x,y,z,t)}{\partial x}\bigg|_{x=0} = 0, \ \frac{\partial C(x,y,z,t)}{\partial x}\bigg|_{x=L_x} = 0, \ \frac{\partial C(x,y,z,t)}{\partial y}\bigg|_{y=0} = 0, \quad (2)$$

$$\frac{\partial C(x,y,z,t)}{\partial y}\bigg|_{x=L_y} = 0, \ \frac{\partial C(x,y,z,t)}{\partial z}\bigg|_{z=0} = 0, \ \frac{\partial C(x,y,z,t)}{\partial z}\bigg|_{x=L_z} = 0,$$

$$C(x,y,z,0)=f(x,y,z).$$

Here $C(x,y,z,t)$ is the spatio-temporal distribution of concentration of dopant; T is the temperature of annealing; D_C is the dopant diffusion coefficient. Value of dopant diffusion coefficient depends on properties of materials, speed of heating and cooling of heterostructure (with account Arrhenius law). Dependences of dopant diffusion coefficients could be approximated by the following function [9,11,12]

$$D_C = D_L(x,y,z,T) \left[1 + \xi \frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right] \left[1 + \varsigma_1 \frac{V(x,y,z,t)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,t)}{(V^*)^2}\right], \quad (3)$$

where $D_L(x,y,z,T)$ is the spatial (due to existing several layers wit different properties in hetero-structure) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficient; $P(x,y,z,T)$ is the limit of solubility of dopant; parameter γ could be integer framework the following interval $\gamma \in [1,3]$ [9]; $V(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation vacancies; V^* is the equilibrium distribution of concentration of vacancies. Concentration dependence of dopant diffusion coefficient have been discussed in details in [9]. It should be noted, that using diffusion type of doping did not leads to generation radiation defects and $\zeta_1 = \zeta_2 = 0$. We determine spatio-temporal distributions of concentrations of point defects have been determine by solving the following system of equations [11,12]

$$\frac{\partial I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right] - k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t) -$$

$$- k_{I,I}(x,y,z,T)I^2(x,y,z,t) \quad (4)$$

$$\frac{\partial V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right] - k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t) -$$

$$- k_{V,V}(x,y,z,T)V^2(x,y,z,t)$$

with boundary and initial conditions

$$\frac{\partial \rho(x,y,z,t)}{\partial x}\bigg|_{x=0} = 0, \frac{\partial \rho(x,y,z,t)}{\partial x}\bigg|_{x=L_x} = 0, \frac{\partial \rho(x,y,z,t)}{\partial y}\bigg|_{y=0} = 0,$$

$$\frac{\partial \rho(x,y,z,t)}{\partial y}\bigg|_{y=L_y} = 0,$$

$$\frac{\partial \rho(x,y,z,t)}{\partial z}\bigg|_{z=0} = 0, \frac{\partial \rho(x,y,z,t)}{\partial z}\bigg|_{z=L_z} = 0, \rho(x,y,z,0) = f_\rho(x,y,z). \quad (5)$$

Here $\rho = I, V$; $I(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation interstitials; $D_\rho(x,y,z,T)$ is the diffusion coefficients of radiation interstitials and vacancies; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and diinterstitials; $k_{I,V}(x,y,z,T)$ is the parameter of recombination of point radiation defects; $k_{\rho,\rho}(x,y,z,T)$ are the parameters of generation of simplest complexes of point radiation defects.

We determine spatio-temporal distributions of concentrations of divacancies $\Phi_V(x, y, z, t)$ and diinterstitials $\Phi_I(x,y,z,t)$ by solving the following system of equations [11,12]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right] +$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t) \quad (6)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right] +$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=L_z}=0, \quad (7)$$

$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z).$$

Here $D_{\Phi\rho}(x,y,z,T)$ are the diffusion coefficients of complexes of radiation defects; $k_\rho(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

We determine spatio-temporal distributions of concentrations of dopant and radiation defects by using method of averaging of function corrections [13] with decreased quantity of iteration steps [14]. Framework the approach we used solutions of Eqs. (1), (4) and (6) in linear form and with averaged values of diffusion coefficients D_{0L}, D_{0I}, D_{0V}, $D_{0\Phi I}$, $D_{0\Phi V}$ as initial-order approximations of the required concentrations. The solutions could be written as

$$C_1(x,y,z,t)=\frac{F_{0C}}{L_x L_y L_z}+\frac{2}{L_x L_y L_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t),$$

$$I_1(x,y,z,t)=\frac{F_{0I}}{L_x L_y L_z}+\frac{2}{L_x L_y L_z}\sum_{n=1}^{\infty}F_{nI}c_n(x)c_n(y)c_n(z)e_{nI}(t),$$

$$V_1(x,y,z,t)=\frac{F_{0C}}{L_x L_y L_z}+\frac{2}{L_x L_y L_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nV}(t),$$

$$\Phi_{I1}(x,y,z,t) = \frac{F_{0\Phi_I}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_I} c_n(x) c_n(y) c_n(z) e_{n\Phi_I}(t),$$

$$\Phi_{V1}(x,y,z,t) = \frac{F_{0\Phi_V}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_V} c_n(x) c_n(y) c_n(z) e_{n\Phi_V}(t),$$

Where

$$e_{n\rho}(t) = \exp\left[-\pi^2 n^2 D_{0\rho} t \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)\right],$$

$$F_{n\rho} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(v) f_\rho(u,v,w) \, dw \, dv \, du, \quad c_n(\chi) = \cos(\pi n \chi / L_\chi).$$

The second-order approximations and approximations with higher orders of concentrations of dopant and radiation defects we determine framework standard iterative procedure [13,14]. Framework this procedure to calculate approximations with the n-order one shall replace the functions $C(x,y,z,t)$, $I(x,y,z,t)$, $V(x,y,z,t)$, $\Phi_I(x,y,z,t)$, $\Phi_V(x,y,z,t)$ in the right sides of the Eqs. (1), (4) and (6) on the following sums $\alpha_{n\rho} + \rho_{n-1}(x,y,z,t)$. As an example we present equations for the second-order approximations of the considered concentrations

$$\frac{\partial C_2(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left(\left[1 + \varsigma_1 \frac{V(x,y,z,t)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1 + \xi \frac{[\alpha_{2C} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\} \times$$

$$\times D_L(x,y,z,T) \frac{\partial C_1(x,y,z,t)}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_L(x,y,z,T)\left[1 + \varsigma_1 \frac{V(x,y,z,t)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,t)}{(V^*)^2}\right] \times$$

$$\times \left\{1 + \xi \frac{[\alpha_{2C} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\} \frac{\partial C_1(x,y,z,t)}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_L(x,y,z,T) \frac{\partial C_1(x,y,z,t)}{\partial z} \times\right.$$

$$\left.\times \left[1 + \varsigma_1 \frac{V(x,y,z,t)}{V^*} + \varsigma_2 \frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1 + \xi \frac{[\alpha_{2C} + C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right) \qquad (8)$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_I(x,y,z,T) \frac{\partial I_1(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_I(x,y,z,T) \frac{\partial I_1(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[D_I(x,y,z,T) \frac{\partial I_1(x,y,z,t)}{\partial z}\right] - [\alpha_{2I} + I_1(x,y,z,t)][\alpha_{2V} + V_1(x,y,z,t)] \times$$

$$\times k_{I,V}(x,y,z,T) - k_{I,I}(x,y,z,T)[\alpha_{2I} + I_1(x,y,z,t)]^2 \qquad (9)$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_V(x,y,z,T) \frac{\partial V_1(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_V(x,y,z,T) \frac{\partial V_1(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right] - [\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)] \times$$

$$\times k_{I,V}(x,y,z,T) - k_{V,V}(x,y,z,T)[\alpha_{2V}+V_1(x,y,z,t)]^2$$

$$\frac{\partial \Phi_{I2}(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T) \times \right.$$

$$\left. \times \frac{\partial \Phi_{I1}(x,y,z,t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,t)}{\partial z}\right] + k_{I,I}(x,y,z,T) \times$$

$$\times I^2(x,y,z,t) - k_I(x,y,z,T)I(x,y,z,t) \qquad (10)$$

$$\frac{\partial \Phi_{V2}(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T) \times \right.$$

$$\left. \times \frac{\partial \Phi_{I1}(x,y,z,t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,t)}{\partial z}\right] + k_{V,V}(x,y,z,T) \times$$

$$\times V^2(x,y,z,t) - k_V(x,y,z,T)V(x,y,z,t).$$

Integration of the left and right sides of Eqs.(8)-(10) gives us possibility to obtain relations for the second-order approximations of concentrations of dopant and radiation defects in final form

$$C_2(x,y,z,t) = \frac{\partial}{\partial x}\left(\int_0^t \left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{ 1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\} \times \right.$$

$$\left. \times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial x}\right)d\tau + \frac{\partial}{\partial y}\left(\int_0^t \left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right] \times \right.$$

$$\left. \times D_L(x,y,z,T)\left\{ 1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial y}d\tau\right) + \frac{\partial}{\partial z}\left(\int_0^t D_L(x,y,z,T) \times \right.$$

$$\left. \times \left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{ 1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial z}d\tau\right) +$$

$$+ f_C(x,y,z) \qquad (8a)$$

$$I_2(x,y,z,t) = \frac{\partial}{\partial x}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau\right] + \frac{\partial}{\partial y}\left[\int_0^t D_I(x,y,z,T) \times \right.$$

$$\left. \times \frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau\right] + \frac{\partial}{\partial z}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau\right] - \int_0^t k_{I,I}(x,y,z,T) \times$$

$$\times [\alpha_{2I} + I_1(x,y,z,\tau)]^2 d\tau + f_I(x,y,z) - \int_0^t k_{I,V}(x,y,z,T)[\alpha_{2I} + I_1(x,y,z,\tau)] \times$$

$$\times [\alpha_{2V} + V_1(x,y,z,\tau)]d\tau \quad (9a)$$

$$V_2(x,y,z,t) = \frac{\partial}{\partial x}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau\right] + \frac{\partial}{\partial y}\left[\int_0^t D_V(x,y,z,T) \times\right.$$

$$\left.\times\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\right] + \frac{\partial}{\partial z}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau\right] - \int_0^t k_{V,V}(x,y,z,T) \times$$

$$\times [\alpha_{2I} + V_1(x,y,z,\tau)]^2 d\tau + f_V(x,y,z) - \int_0^t k_{I,V}(x,y,z,T)[\alpha_{2I} + I_1(x,y,z,\tau)] \times$$

$$\times [\alpha_{2V} + V_1(x,y,z,\tau)]d\tau$$

$$\Phi_{I2}(x,y,z,t) = \frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial x}d\tau\right] + \frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_I}(x,y,z,T) \times\right.$$

$$\left.\times\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial y}d\tau\right] + \frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial z}d\tau\right] + \int_0^t k_{I,I}(x,y,z,T) \times$$

$$\times I^2(x,y,z,\tau)d\tau - \int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau + f_{\Phi_I}(x,y,z) \quad (10a)$$

$$\Phi_{V2}(x,y,z,t) = \frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial x}d\tau\right] + \frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_V}(x,y,z,T) \times\right.$$

$$\left.\times\frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial y}d\tau\right] + \frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial z}d\tau\right] + \int_0^t k_{V,V}(x,y,z,T) \times$$

$$\times V^2(x,y,z,\tau)d\tau - \int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau + f_{\Phi_V}(x,y,z)$$

We determine average values of the second-orders approximations of the considered concentrations by using the following standard relations [13,14]

$$\alpha_{2\rho} = \frac{1}{\Theta L_x L_y L_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}[\rho_2(x,y,z,t) - \rho_1(x,y,z,t)]dz\,dy\,dx\,dt. \qquad (11)$$

Substitution of relations (8a)-(10a) into relation (11) gives us possibility to obtain relations for the required average values $\alpha_{2\rho}$

$$\alpha_{2C} = \frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)dz\,dy\,dx, \qquad (12)$$

$$\alpha_{2I} = \frac{1}{2A_{II00}}\left\{(1 + A_{IV01} + A_{II10} + \alpha_{2V}A_{IV00})^2 - 4A_{II00}[\alpha_{2V}A_{IV10} - A_{II20} + A_{IV11} -\right.$$

$$-\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,dz\,dy\,dx\Bigg]\Bigg\}^{\frac{1}{2}}-\frac{1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00}}{2A_{II00}} \quad (13\text{a})$$

$$\alpha_{2V}=\frac{1}{2B_4}\sqrt{\frac{(B_3+A)^2}{4}-4B_4\left(y+\frac{B_3 y-B_1}{A}\right)}-\frac{B_3+A}{4B_4}, \quad (13\text{b})$$

where $A_{abij}=\dfrac{1}{\Theta L_x L_y L_z}\displaystyle\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{a,b}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)\,dz\,dy\,dx\,dt,$

$$B_4=A_{IV00}^2 A_{IV00}^2-2\left(A_{IV00}^2-A_{II00}A_{VV00}\right)^2,$$

$$B_3=A_{IV00}A_{IV00}^2+A_{IV01}A_{IV00}^3+A_{IV00}A_{II10}A_{IV00}^2-$$
$$-4\left(A_{IV00}^2-A_{II00}A_{VV00}\right)\left[2A_{IV01}A_{IV00}+2A_{IV00}(1+A_{IV01}+A_{II10})-2A_{II00}(A_{IV10}+A_{VV10}+1)\right]-$$
$$-4A_{IV10}A_{II00}A_{IV00}^2+2A_{IV00}A_{IV01}A_{IV00}^2,$$

$$B_2=A_{IV00}^2\Big\{(1+A_{IV01}+A_{II10})^2+A_{IV00}^2 A_{IV01}^2+A_{IV00}\times$$
$$\times 2A_{IV00}(A_{IV00}+A_{IV00}A_{IV01}+A_{IV00}A_{II10}-4A_{IV10}A_{II00})-4A_{II00}\Bigg[A_{IV11}-A_{II20}-\frac{1}{L_x L_y L_z}\times$$
$$\times\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,dz\,dy\,dx\Bigg]\Big\{[2A_{IV01}A_{IV00}+2A_{IV00}(1+A_{IV01}+A_{II10})-2A_{II00}(A_{IV10}+1+$$
$$+A_{VV10})]^2+2\Bigg[A_{IV01}(1+A_{IV01}+A_{II10})+\frac{2}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_V(x,y,z)\,dz\,dy\,dx-2A_{II00}(A_{VV20}-$$
$$-A_{IV11})+A_{IV01}(1+A_{IV01}+A_{II10})][2A_{IV00}(1+A_{IV01}+A_{II10})+2A_{IV01}A_{IV00}-2A_{II00}(A_{IV10}+1+$$
$$+A_{VV10})]\Big\}, \quad B_1=2A_{IV00}A_{IV01}(1+A_{IV01}+A_{II10})^2-8\Bigg[A_{IV11}-\frac{1}{L_x}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,dz\,dy\,dx\times$$

$$\times\frac{1}{L_y L_z}-A_{II20}\Bigg]+A_{IV00}A_{IV01}A_{II00}+A_{IV01}^2(A_{IV00}+A_{IV00}A_{IV01}+A_{IV00}A_{II10}-4A_{IV10}A_{II00})-$$
$$-2\Bigg[\frac{2A_{II00}}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,dz\,dy\,dx+A_{IV01}(1+A_{IV01}+A_{II10})-2A_{II00}(A_{VV20}-A_{IV11})+$$
$$+A_{IV01}(1+A_{IV01}+A_{II10})][2A_{IV00}(1+A_{IV01}+A_{II10})-2(A_{IV10}+A_{VV10}+1)A_{II00}+2A_{IV01}A_{IV00}],$$

$$B_0=4A_{II00}A_{IV01}^2\Bigg[A_{II20}+\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,dz\,dy\,dx-A_{IV11}\Bigg]+A_{IV01}^2(A_{IV01}+A_{II10}+$$
$$+1)^2-\Bigg[\frac{2A_{II00}}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_V(x,y,z)\,dz\,dy\,dx+A_{IV01}(1+A_{IV01}+A_{II10})-2A_{II00}(A_{VV20}-A_{IV11})+$$

$$+ A_{IV01}(1 + A_{IV01} + A_{II10})]^2, \; y = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} + \frac{B_2}{6},$$

$$q = (2B_1 B_3 - 8B_0) \times$$

$$\times \frac{B_2}{48} + \frac{B_2^3}{216} + \frac{B_0(4B_2 - B_3^2) - B_1^2}{8}, \; p = \left[3(2B_1 B_3 - 8B_0) - 2B_2^2\right]/72,$$

$$A = \sqrt{8y + B_3^2 - 4B_2},$$

$$\alpha_{2\Phi_I} = A_{II20} - \frac{1}{\Theta L_x L_y L_z} \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_I(x,y,z,T) I(x,y,z,t) \, dz \, dy \, dx \, dt +$$

$$+ \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_{\Phi I}(x,y,z) \, dz \, dy \, dx \; (14)$$

$$\alpha_{2\Phi_V} = A_{VV20} - \frac{1}{\Theta L_x L_y L_z} \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_V(x,y,z,T) V(x,y,z,t) \, dz \, dy \, dx \, dt +$$

$$+ \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_{\Phi V}(x,y,z) \, dz \, dy \, dx.$$

The considered substitution gives us possibility to obtain equation for parameter α_{2C}. Solution of the equation depends on value of parameter γ. Analysis of spatio-temporal distributions of concentrations of dopant and radiation defects has been done by using their second-order approximations framework the method of averaged of function corrections with decreased quantity of iterative steps. The second-order approximation is usually enough good approximation to make qualitative analysis and obtain some quantitative results. Results of analytical calculation have been checked by comparison with results of numerical simulation.

3. DISCUSSION

In this section we analyzed the spatio-temporal distribution of concentration of dopant in the considered heterostructure during annealing. Figs. 2 shows spatial distributions of concentrations of dopants infused (Fig. 2a) or implanted (Fig. 2b) in epitaxial layer. Value of annealing time is equal for all distributions framework every figure 2a and 2b. Increasing of number of curves corresponds to increasing of difference between values of dopant diffusion coefficients in layers of heterostructure. The figures show that presents of interface between layers of heterostructure gives us possibility to increase absolute value of gradient of concentration of dopant in direction, which is perpendicular to the interface. We obtain increasing of absolute value of the gradient in neighborhood of the interface. Due to the increasing one can obtain decreasing dimensions of transistors, which have been used in the pass transistor multiplexer circuit. At the same time with increasing of the gradient homogeneity of concentration of dopant in enriched area increases.
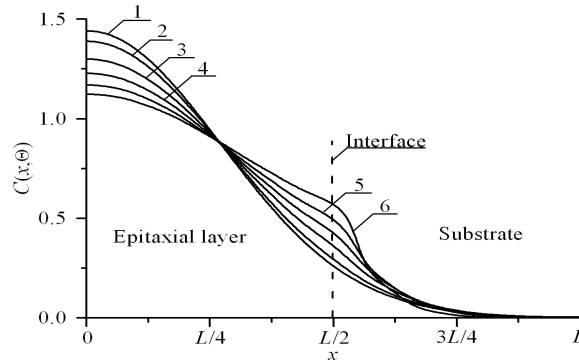
Fig.2*a*. Distributions of concentration of infused dopant in heterostructure

from Figs. 1 and 2 in direction, which is perpendicular to interface between epitaxial layer substrate. Increasing of number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate
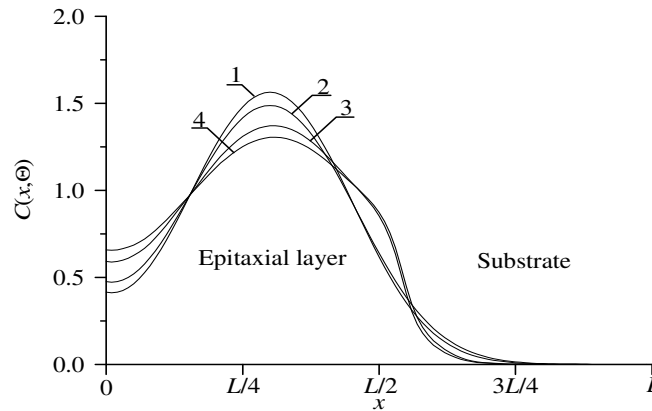

Fig.2*b*. Distributions of concentration of implanted dopant in heterostructure

from Figs. 1 and 2 in direction, which is perpendicular to interface between epitaxial layer substrate. Curves 1 and 3 corresponds to annealing time $\Theta = 0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 corresponds to annealing time $\Theta = 0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 corresponds to homogenous sample. Curves 3 and 4 corresponds to heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate

To choose annealing time it should be accounted decreasing of absolute value of gradient of concentration of dopant in neighborhood of interface between substrate and epitaxial layer with increasing of annealing time. Decreasing of value of annealing time leads to decreasing of homogeneity of concentration of dopant in enriched area (see Fig. 3*a* for diffusion doping of materials and Fig. 3*b* for ion doping of materials). Let us determine compromise value of annealing time framework recently introduced criteria [15-20]. Framework the criteria we approximate real distributions of concentration of dopant by ideal rectangle distribution $\psi (x,y,z)$. Farther we determine compromise value of annealing time by minimization of the mean-squared error
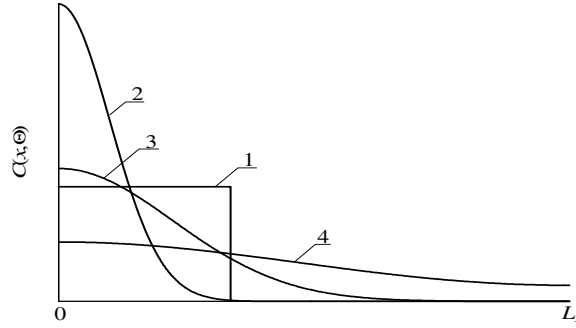
Fig. 3*a*. Spatial distributions of dopant in heterostructure after dopant infusion.

Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

Fig.3*b*. Spatial distributions of dopant in heterostructure after ion implantation.

Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time
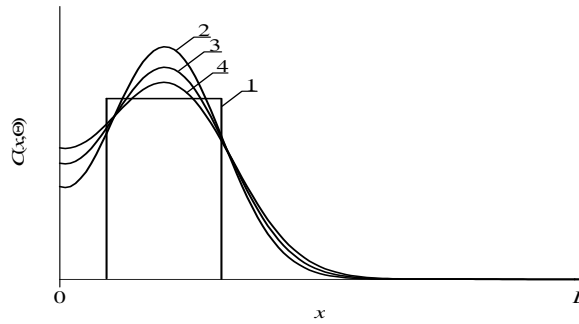
$$U = \frac{1}{L_x L_y L_z} \int\limits_{0}^{L_x} \int\limits_{0}^{L_y} \int\limits_{0}^{L_z} \left[C(x,y,z,\Theta) - \psi(x,y,z) \right] dz\, dy\, dx. \qquad (15)$$
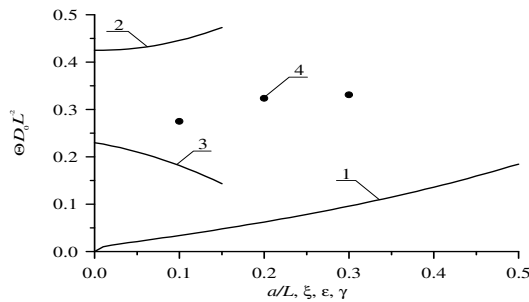

Fig.4*a*. Dependences of dimensionless optimal annealing time for doping by diffusion, which have been obtained by minimization of mean-squared error, on several parameters.

Curve 1 is the dependence of dimensionless optimal annealing time on the relation *a/L* and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for *a/L*=1/2 and $\xi = \gamma = 0$. Curve 3 is the dependence of dimensionless optimal annealing time on value

of parameter ξ for a/L =1/2 and $\varepsilon=\gamma=0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon=\xi=0$
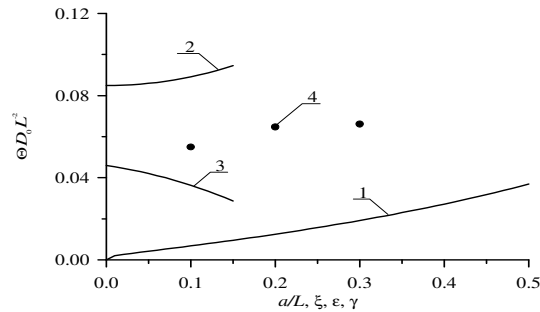

Fig.4*b*. Dependences of dimensionless optimal annealing time for doping by ion implantation, which have been obtained by minimization of mean-squared error, on several parameters.

Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi=\gamma=0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi=\gamma=0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L =1/2 and $\varepsilon=\gamma=0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon=\xi=0$

Dependences of optimal annealing time are presented on Figs. 4 for diffusion and ion types of doping, respectively. It should be noted, that it is necessary to anneal radiation defects after ion implantation. One could find spreading of concentration of distribution of dopant during this annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant did not achieves any interfaces during annealing of radiation defects, it is practically to additionally anneal the dopant. In this situation optimal value of additional annealing time of implanted dopant is smaller, than annealing time of infused dopant. At the same time ion type of doping gives us possibility to decrease mismatch-induced stress in heterostructure [21].

4. CONCLUSION

In this paper we model redistribution of infused and implanted dopants during manufacture the three-stage amplifier circuit on field-effect heterotransistors. Several recommendations to optimize manufacture the heterotransistors have been formulated. Analytical approach to model diffusion and ion types of doping with account concurrent changing of parameters in space and time has been introduced. At the same time the approach gives us possibility to take into account nonlinearity of doping processes.

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