ANALYSIS AND CONTROL OF AN INFORMATION DISSEMINATION MODEL

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ABSTRACT

When emergencies like a pandemic occur, public opinion goes berserk, and a lot of uncertain and unreliable information is generated and spreads uncontrollably. This paper presents a mathematical framework for understanding and controlling the dissemination mechanism ofuncertain information on online social platforms.Bifurcation analysis and Multiobjective nonlinear model predictive control calculations were performed on a dynamic information dissemination model triggered after major emergencies. Bifurcation analysis is a powerful mathematical tool used to deal with the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives must be met simultaneously. The MATLAB program MATCONT was used to perform the bifurcation analysis. The MNLMPC calculations were performed using the optimization language PYOMO in conjunction with the state-of-the-art global optimization solvers IPOPT and BARON.The bifurcation analysis revealed the existence of a branch point. The branch point is beneficial because itenables the multiobjective nonlinear model predictive control calculations to converge to the Utopia point, which is the most beneficial solution. A combination of bifurcation analysis and multiobjective nonlinear model predictive control for an information dissemination model is the main contribution of this paper.

KEYWORDS

information, dissemination, bifurcation, optimization, control

1. BACKGROUND

Trpevski et al (2010)[1] developed a model for rumor spreading over networks. Centola(2010)[2] discussed the spread of behavior in an online social network experiment. Crokidakis et al (2012)[3] studied the effects of mass media on opinion spreading in the Sznajd sociophysics model. Zhao et al (2012)[4] investigated the impact of authorities' media and rumor dissemination on the evolution of emergencies. He et al (2017)[5] developed cost-efficient strategies for restraining rumor spreading in mobile social networks. Litou et al (2015)[6]developed efficient techniques for time-constrained information dissemination using location-based social networks. Hong et al (2017)[7] investigated food safety internet public opinion transmission simulation and management countermeasures considering information authenticity. Hu et al (2018)[8] developed a rumor spreading model considering the proportion of wisemen in the crowd. Rui et al (2018)[9] provide SPIR: the potential spreaders involved SIR model for information diffusion in social networks. Zan(2018)[10] extended this work and provided DSIR double-rumors spreading model in complex networks. Tian et al (2019)[11] developed a rumor spreading model with considering debunking behavior in emergencies. Wang et al (2019)[12] discussed the dissemination and control model of public opinion in online social networks based on users' relative weight. Haihong et al (2019)[13] developed a theme and sentiment analysis model of public opinion dissemination based on generative adversarial network. Gao et al (2019) [14] provided an evaluation of governmental safety regulatory functions in preventing major accidents. Yin et al (2019) [15] studied near casting forwarding behaviors and information propagation. Zhang et al (2020)[16] discussed emergency management planning for major epidemic disease prevention and control. De Las Heras-Pedrosa et al (2020)[17] researched sentiment analysis and emotion understanding during the COVID-19 pandemic in Spain and its impact on digital ecosystems. Int J Environ Res Public Health (2020) 17(15):5542. doi:10.3390/ ijerph17155542Yin et al (2020)[18] studied the information propagation dynamics in the Chinese Sina-microblog. Li et al (2020)[19] investigated the temporal and spatial evolution of online public sentiment on emergencies. AtehortuaNA et al (2021)[20] regarded fake news messaging as a pandemic. Cheng et al (2021)[21] showed how major public health emergencies affect changes in international oil prices. Yao et al (2021)[22] provide an intelligent response to public health emergencies. Zhang et al (2021)[23] conducted research on the situational awareness of a major emergency under incomplete information. Allington et al (2021)[24] studied health-protective behaviour, social media usage and conspiracy belief during the COVID-19 public health emergency. Lv et al (2021)[25] developed a panic spreading model with different emotions under emergency. Yang et al (2022)[26] investigated the public sentiment in major public emergencies through the complex networks method. Mo et al (2022) [27] studied the transmission effect of extreme risks in China's financial sectors at major emergencies. Jalan et al (2022)[28] investigated the burden of mental distress in the US associated with trust in media for COVID-19 information. Zhang et al (2022) [29] studied the internet public opinion dissemination mechanism of COVID-19: evidence from the Shuanghuanglian event. Li et al (2022)[[30] developed a simulation model on the network public opinion communication model of major public health emergencies and management system design. We(2022)[31] developed a network public opinion propagation control model of major emergencies based on heat conduction theory. Kang et al (2022)[32] provided a dynamic analysis and performed optimal control considering cross transmission and variation of information. Tan et al (2023)[33] developed an optimization model and algorithm of the logistics vehicle routing problem under a major emergency. Li et al (2024)[34] provided a dynamic analysis and optimal control study of an uncertain information dissemination model triggered after major emergencies.

This paper aims to perform bifurcation analysis in conjunction with multiobjective nonlinear model predictive control (MNLMPC) for the uncertain information dissemination model described in Li et al (2024)[34]. This paper is organized as follows. First, the model equations are presented. The numerical procedures (bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) are then described. This is followed by the results and discussion, and conclusions.

2. Equations of the Uncertain Information Dissemination Model Li et al (2024)

Based on different decision-making behaviors, netizens are categorized into eight groups: unknowns, sval; thinkers eval, uncertain information publishers fval, clarifiers of uncertain information tval, internet users who believe uncertain information fb, internet users who do not believe any online information mval, internet users who only believe true information tb, and information immunizers rval.

The differential equations representing the model are

$$\begin{aligned}
\frac{d(sval)}{dt} &= B-\delta(fval)sval-\left(g(sval)\right)\\
\frac{d(eval)}{dt} &= \alpha_3\left(\delta(fval)sval\right)-eval\left(\beta_1+\beta_2+g\right)\\
\frac{d(fval)}{dt} &= \alpha_1\left(\delta(fval)sval\right)+\beta_1 eval-fval(\omega+\mu_1+\mu_2+g)\\
\frac{d(tval)}{dt} &= \alpha_2\left(\delta(fval)sval\right)+\beta_2 eval+fval(\omega)-tval(\eta_1+\eta_2+g)\\
\frac{d(fb)}{dt} &= \mu_1(fval)-fb\left(\gamma_1+g\right)\\
\frac{d(mval)}{dt} &= \mu_2(fval)+tval(\eta_2)-mval\left(\gamma_2+g\right)\\
\frac{d(tb)}{dt} &= tval(\eta_1)-tb\left(\gamma_3+g\right)\\
\frac{d(rval)}{dt} &= \gamma_1 fb+\gamma_2 mval+\gamma_3 tb+\alpha_4\delta fval(sval)-\left(g*rval\right)
\end{aligned} \tag{1}$$

The base parameter values are

B=1;g=0.2; α_1 =0.4; α_2 =0.15; α_3=0.4; α_4 =0.05; β_1=0.2; β_2 =0.2; μ_1 =0.2; μ_2 =0.2;
η_1=0.3; η_2 =0.3; ω=0.3; γ_1=0.5; γ_2=0.5; γ_3=0.5; δ =0.5;

For the bifurcation analysis δ is used as the bifurcation parameter and for the MNLMPC calculations α_2 ; α_3 ; β_2 and ω are chosen as the control parameters.

3. BIFURCATION ANALYSIS

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles . A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge Govearts, and Kuznetsov, 2003[35]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[36]). This program detects Limit points(LP), branch points(BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x,\alpha) \tag{2}$$

$x \in R^n$ Let the bifurcation parameter be α Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w=[w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw=0 \tag{3}$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \tag{4}$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector w_{n+1} = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \tag{5}$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998[37]; 2009[38]) and Govaerts [2000] [39]

4. MULTIOBJECTIVE NONLINEAR MODEL PREDICTIVE CONTROL (MNLMPC)

Flores Tlacuahuaz et al (2012)[40] developed a multiobjective nonlinear model predictive control (MNLMPC) method that is rigorous and does not involve weighting functions or additional constraints. This procedure is used for performing the MNLMPC calculations Here $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ (j=1, 2..n) represents the variables that need to be minimized/maximized simultaneously for a problem involving a set of ODE

$$\frac{dx}{dt} = F(x,u) \tag{6}$$

t_f being the final time value, and n the total number of objective variables and . u the control parameter. This MNLMPC procedure first solves the single objective optimal control problem independently optimizing each of the variables $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ individually. The minimization/maximization of $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^* . Then the optimization problem that will be solved is

$$\begin{aligned} &\min\left(\sum_{j=1}^{n}\left(\sum_{t_i=0}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2 \\ &\textit{subject to } \frac{dx}{dt} = F(x,u); \end{aligned} \tag{7}$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained.

Pyomo (Hart et al, 2017)[41] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006)[42]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005)[43].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^* at various time intervals t_i. The subscript i is the index for each time step.
2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$ and get the control values for various times.
3. Implement the first obtained control values
4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j.

Sridhar (2024)[44] proved that the MNLMPC calculations to converge to the Utopia solution when the bifurcation analysis revealed the presence of limit and branch points . This was done by imposing the singularity condition on the co-state equation (Upreti, 2013)[45]. If the minimization of q_1 lead to the value q_1^* and the minimization of q_2 lead to the value q_2^* The MNLPMC calculations will minimize the function $(q_1 - q_1^*)^2 + (q_2 - q_2^*)^2$. The multi objective optimal control problem is

$$\min \quad (q_1 - q_1^*)^2 + (q_2 - q_2^*)^2 \quad \textit{subject to} \ \frac{dx}{dt} = F(x,u) \tag{8}$$

Differentiating the objective function results in

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 2(q_1 - q_1^*)\frac{d}{dx_i}(q_1 - q_1^*) + 2(q_2 - q_2^*)\frac{d}{dx_i}(q_2 - q_2^*) \tag{9}$$

The Utopia point requires that both $(q_1 - q_1^*)$ and $(q_2 - q_2^*)$ are zero. Hence

$$\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) = 0 \tag{10}$$

the optimal control co-state equation (Upreti; 2013)[45] is

$$\frac{d}{dt}(\lambda_i) = -\frac{d}{dx_i}((q_1 - q_1^*)^2 + (q_2 - q_2^*)^2) - f_x \lambda_i; \quad \lambda_i(t_f) = 0 \tag{11}$$

λ_i is the Lagrangian multiplier. t_f is the final time. The first term in this equation is 0 and hence

$$\frac{d}{dt}(\lambda_i) = -f_x \lambda_i; \lambda_i(t_f) = 0 \tag{12}$$

At a limit or a branch point, for the set of ODE $\frac{dx}{dt} = f(x,u)$ f_x is singular. Hence there are two different vectors-values for $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) > 0$ and $\frac{d}{dt}(\lambda_i) < 0$. In between there is a vector $[\lambda_i]$ where $\frac{d}{dt}(\lambda_i) = 0$. This, coupled with the boundary condition $\lambda_i(t_f) = 0$ will lead to $[\lambda_i] = 0$ This makes the problem an unconstrained optimization problem, and the only solution is the Utopia solution.

5. Results and Discussion

For the bifurcation analysis, δ is used as the bifurcation parameter, and a branch point occurred at $[sval, eval, fval, tval, fb, mval, tb, rval, \delta]$ values of (5.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.3375).

This is shown in Fig. 1.

For the MNLMPC calculation , $\sum_{t_{i=0}}^{t_i = t_f} eval(t_i), \sum_{t_{i=0}}^{t_i = t_f} tval(t_i)$ were minimized individually, and each led to a value of 0. The multiobjective optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i = t_f} eval(t_i) - 0)^2 + (\sum_{t_{i=0}}^{t_i = t_f} tval(t_i) - 0)^2$ subject to the equations governing the model. This led to a value of zero (the Utopia solution). The MNLMPC control values obtained for $\alpha_2; \alpha_3; \beta_2$ and ω were (1.271, 1.667, 0.6769,1.1667).

The various profiles for this MNLMPC calculation are shown in Figs. 2, 3, 4. The obtained control profile of $\alpha_3; \beta_2$ and ω exhibited noise (Fig. 5). This issue was addressed using the Savitzky-Golay Filter. The smoothed version of this profile is shown in Fig. 6 . The MNLMPC calculations converged to the Utopia solution, validating the analysis by Sridhar (2024)[44], which demonstrated that the presence of a limit point/branch point enables the MNLMPC calculations to reach the optimal (Utopia) solution.

6. CONCLUSIONS

Bifurcation analysis and Multiobjective nonlinear model predictive control calculations were performed on a dynamic information dissemination model triggered after major emergencies. The bifurcation analysis revealed the existence of a branch point. The branch point (which causes multiple steady-state solutions from a singular point) is very beneficial because it enables the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in this model. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control(MNLMPC) for a dynamic information dissemination model triggered after major emergencies is the main contribution of this paper.

DATA AVAILABILITY STATEMENT

All data used is presented in the paper

CONFLICT OF INTEREST

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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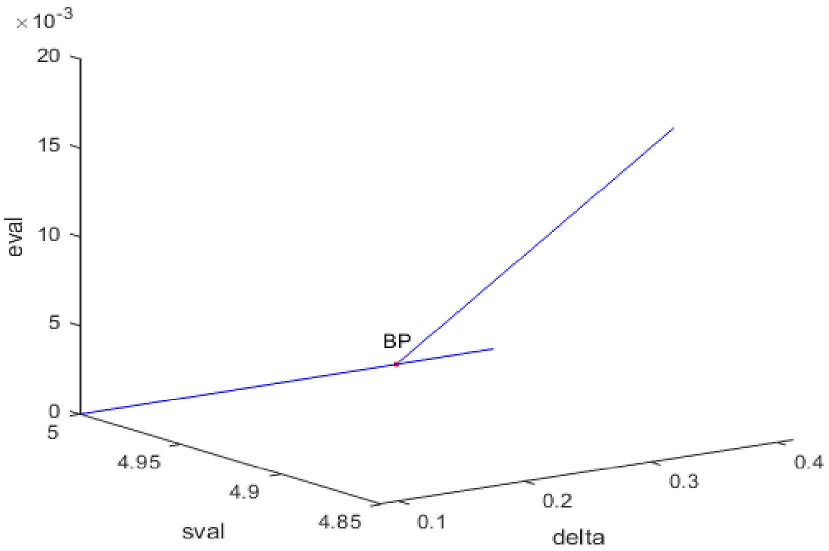

Fig. 1 Bifurcation Diagram indicating branch point

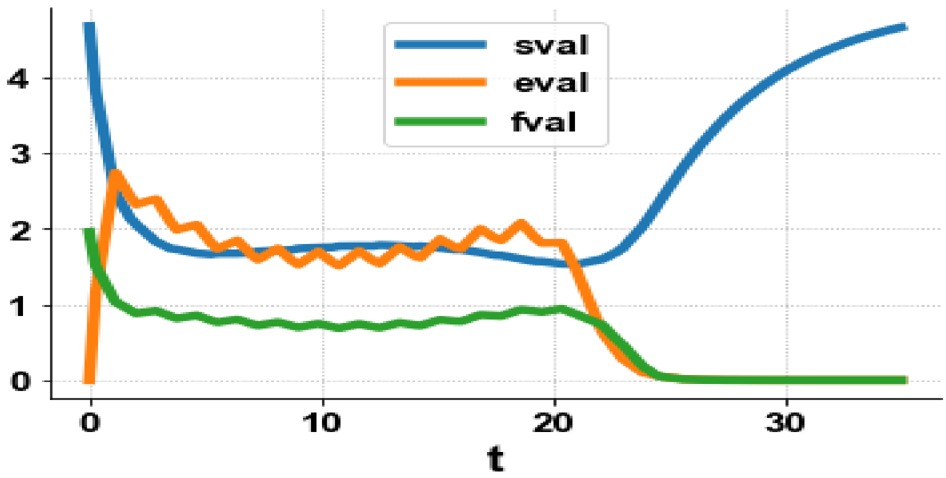

Fig. 2 sval, eval, fval profiles for MNLMPC calculations

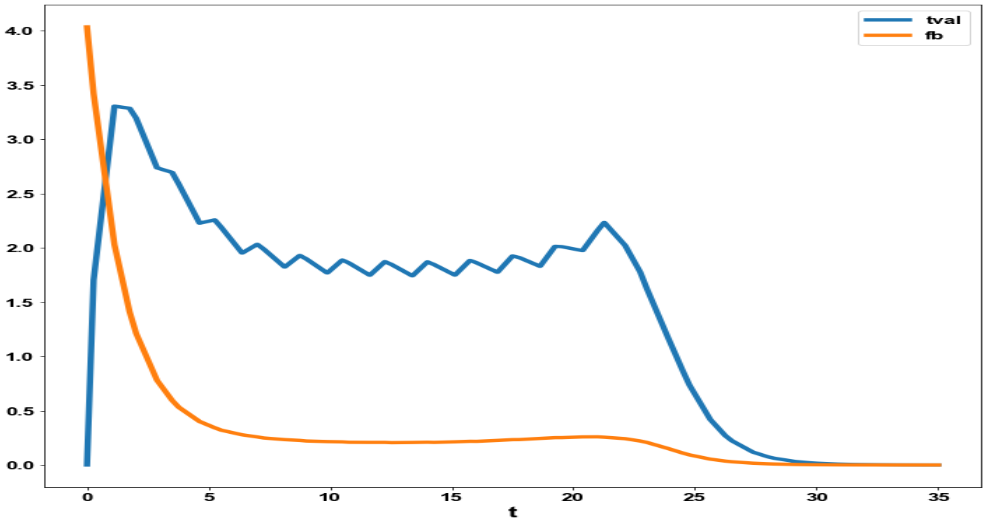

Fig. 3 tval, fb profiles for MNLMPC calculations

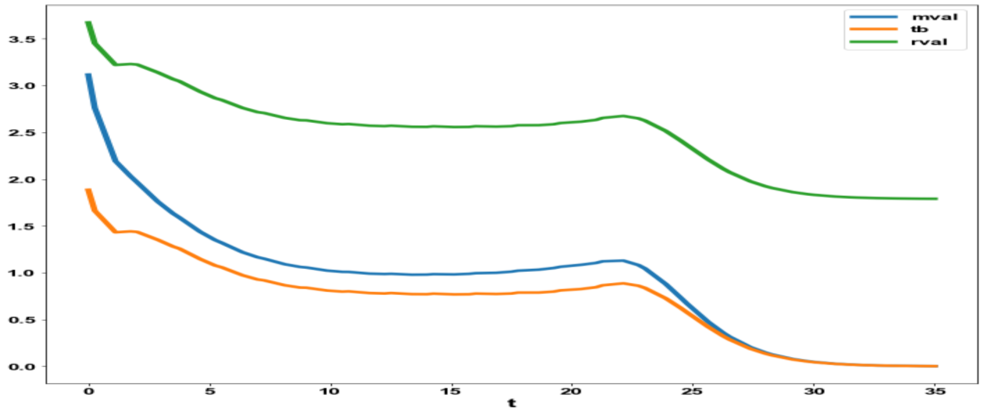

Fig. 4 mval, tb rval profiles for MNLMPC calculations

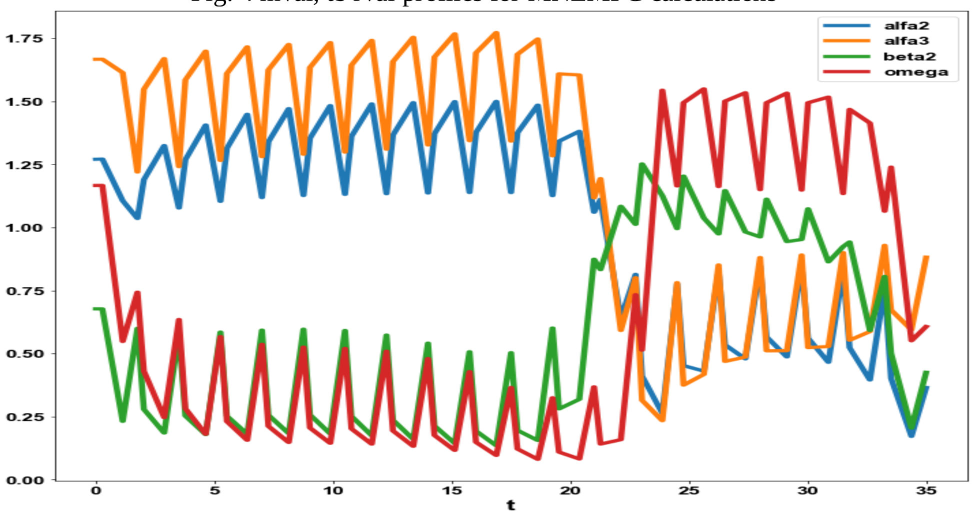

Fig. 5 Control profiles (showing noise) for MNLMPC calculations

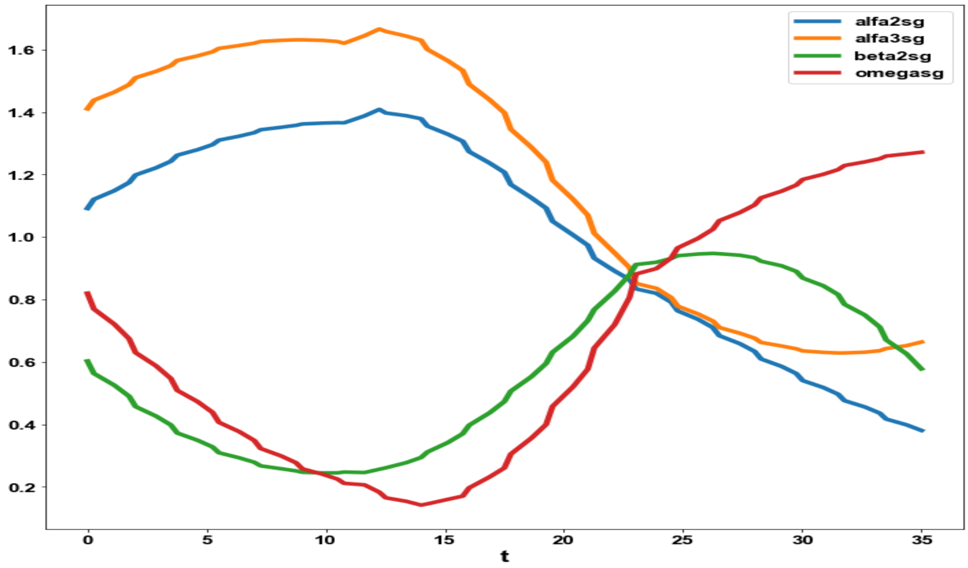

Fig. 6 Control profiles (noise eliminated with Savitzky Golay filter)