ON MODEL OF HEAT TRANSFER IN GASES AND LIQUIDS IN A TRANSPORTATION SYSTEM BY LOCAL COOLING OR HEATING

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ABSTRACT

In this paper we consider an approach of changing the temperature of liquids and gases in the pipeline by local heating and cooling. We introduce a model of heat transfer with account forced convection. Also we introduce an analytical approach for prognosis of heat transfer.

KEYWORDS

Heat transfer; controlling of temperature of gases and liquids; analytical approach for prognosis.

1. INTRODUCTION

In a several technical equipment it is necessary to conduct heat sink and stabilization of temperature of the liquid or gaseous heat carrier. Framework this paper for the transport of heat carrier, we consider a cylindrical transportation system (a frequent example of such a system is a pipe) with a circular section (see Fig. 1). This pipe has a metal section with known dimension, having a porous metal inside. This section maintains a stable temperature (for example, the section in question is in a stream of water with a given temperature). The main aim of this paper is estimation of distribution of temperature of the heat carrier. The accompanying aim of this work are the creation of a model that allows an assessment of a given temperature field and the formation of an analytical approach to analyze of the considered distribution of temperature.

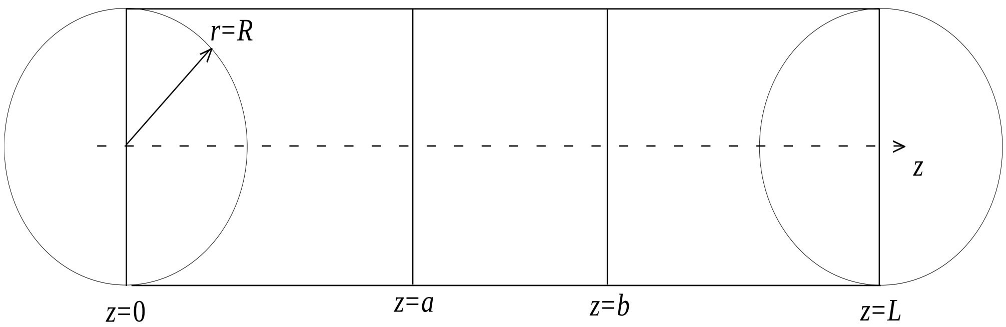

Fig. 1. Structure of the considered pipe is L (axial coordinate $z\in[0,L]$) with radius R. This tube has a section $z\in[a,b]$

2. METHOD OF SOLUTION

To solve our aims we calculate distribution of the considered temperature in space and time. The required distribution of temperature has been calculated by solving of the following boundary value problem [1]

$$c\frac{\partial T(r,\varphi,z,t)}{\partial t}=div\{\lambda\cdot grad[T(r,\varphi,z,t)]-\vec{v}(r,\varphi,z,t)\cdot c\cdot T(r,\varphi,z,t)\cdot C\}+p(r,z,t),\quad (1)$$

where r, φ, z and t are the current cylindrical coordinates and time ($r\in[0,R]$, $\varphi\in[0,2\pi]$, $z\in[0,L]$); function $T(r,\varphi,z,t)$ is the distribution of temperature in space and time; c is the system heat capacity; C is the concentration of the transported substance in the pipeline; λ is the coefficient of thermal conductivity; $p(r,z,t)$ is the density of the power generated in the considered system; $\vec{V}$ is the velocity of heat carrier described by the Navier-Stokes equation

$$\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}=-\nabla\left(\frac{P}{C}\right)+\nu\Delta\vec{v},\quad (2)$$

where P is the pressure in pipe; ν is the kinematics viscosity. Boundary and initial conditions

$$-\lambda\left.\frac{\partial T(r,\varphi,z<a,t)}{\partial r}\right|_{r=R}=0;\ \left.\frac{\partial T(r,\varphi,z,t)}{\partial\varphi}\right|_{\varphi=0}=\left.\frac{\partial T(r,\varphi,z,t)}{\partial\varphi}\right|_{\varphi=2\pi};\ T(r,0,z,t)=T(r,2\pi,z,t);$$

$$T(0,\varphi,z,t)\neq\infty;\ T(r,\varphi,z,0)=T_0;\ T(r,\varphi,0,t)=T(r,\varphi,L,t)=T_{b1};\ T(r,\varphi,z\in[a,b],t)=T_{b2};\quad (3)$$

$$\left.\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial\varphi}\right|_{\varphi=0}=\left.\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial\varphi}\right|_{\varphi=2\pi},\ -\lambda\left.\frac{\partial T(r,\varphi,z>b,t)}{\partial r}\right|_{r=R}=0;\ \left.\frac{\partial v_r(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,$$

$$v_\varphi(r,0,z,t)=v_\varphi(r,2\pi,z,t),\ v_r(0,\varphi,z,t)\neq\infty,\ v_z(r,\varphi,0,t)=v_z(r,\varphi,L,t)=V_0,\ v_r(r,\varphi,z,t)=v_\varphi(r,\varphi,z,t)=v_z(r,\varphi,z,t)=V_0,$$

where $\sigma=5{,}67\cdot10^{-8}\ W\cdot m^{-2}\cdot K^{-4}$; the boundary conditions on the metal section could be transferred to a separate term in the form of power density to the equation (1) $p(r,z,t)/c=T_{b2}sign([z-b+a]/L)\delta((r-R)/L)/\vartheta$, where ϑ is the time scale for achievement of stationary temperature distribution. This time scale has been calculated by using previously introduced approach [2] and takes the following value $\vartheta=\lambda/c\pi R^2L$. In a cylindrical coordinate system, the equations for the velocity projections takes the form

$$\frac{\partial v_r}{\partial t}=\nu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial v_r(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 v_r(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 v_r(r,\varphi,z,t)}{\partial z^2}\right\}-$$
$$-v_r\frac{\partial v_r}{\partial r}-\frac{\upsilon_\varphi}{r}\frac{\partial v_\varphi}{\partial\varphi}-\upsilon_z\frac{\partial v_z}{\partial z}-\frac{\partial}{\partial r}\left(\frac{P}{C}\right)\quad (4a)$$

$$\frac{\partial v_\varphi}{\partial t}=\nu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 v_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 v_\varphi(r,\varphi,z,t)}{\partial z^2}\right\}-$$
$$-v_r\frac{\partial v_r}{\partial r}-\frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial\varphi}-v_z\frac{\partial v_z}{\partial z}-\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{P}{C}\right)\quad (4b)$$

$$\frac{\partial v_z}{\partial t} = \nu \left\{ \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\partial v_z(r,\varphi,z,t)}{\partial r} \right] + \frac{1}{r^2}\frac{\partial^2 v_z(r,\varphi,z,t)}{\partial \varphi^2} + \frac{\partial^2 v_z(r,\varphi,z,t)}{\partial z^2} \right\} - \\ - v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} - v_z \frac{\partial v_z}{\partial z} - \frac{\partial}{\partial z}\left(\frac{P}{C}\right). \quad (4c)$$

Due to the symmetry of the system, to describe the velocity field, it suffices to consider only the axial velocity component, i.e. v_z. To solve the equation (4*c*) we transform it to the following integral form

$$v_z(z,t) = v_z(z,t) + \frac{1}{L^2}\left\{ V_0 z^2 - \int_0^t \int_0^z v_z^2(u,\tau)\, d\,u\, d\,\tau - \int_0^t \int_0^z \frac{P}{C}\, d\,u\, d\,\tau - \int_0^z (z-u)\, v_z(u,t)\, d\,u - \nu V_0 t + \right. \\ \left. + \nu \int_0^t v_z(z,\tau)\, d\,\tau + z\int_0^L \left(1-\frac{u}{L}\right) v_z(u,t)\, d\,u + \frac{z}{L}\int_0^t \int_0^L v_z^2(u,\tau)\, d\,u\, d\,\tau + \frac{z}{L}\int_0^t \int_0^L \frac{P}{LC}\, d\,u\, d\,\tau - z V_0 L \right\}. (4d)$$

To solve the Eq. (4*d*) we use the method of averaging functional corrections [3,4]. In the framework of the method to determine the first-order approximation of the projection of the velocity v_z on shall to replace it in the right-hand side of equation (4*d*) on not yet known average value $v_z \to \alpha_{1z}$. After this replacement one can obtain relation for the first-order approximation of the considered component in the following form

$$v_{1z}(z,t) = \alpha_{1z} + \frac{\alpha_{1z}}{2L^2}\left(z L + 2\nu t - z^2\right) + \frac{1}{L^2}\left[V_0 z^2 - \nu V_0 t - z V_0 L^2 - \left(1 - \frac{z}{L}\right) \int_0^t \int_0^z \frac{P}{C}\, d\,u\, d\,\tau \right]. \quad (5)$$

The not yet known average value α_{1z} was determined by using the following standard relation

$$\alpha_{1z} = \frac{1}{\Theta L}\int_0^\Theta \int_0^L v_{1z}\, d\,z\, d\,t\,, \quad (6)$$

where Θ is the continues of monitoring of transport of heat carrier. Substitution of relation (5) into relation (6) gives a possibility to obtain relation for the required average value α_{1z} in the following form

$$\alpha_{1z} = \frac{12}{\Theta L}\int_0^\Theta (\Theta - t) \int_0^L \frac{L-z}{L^2 + 6\nu\,\Theta}\frac{P}{C}\, d\,z\, d\,t - \frac{6}{\Theta L^2}\int_0^\Theta (\Theta - t)\int_0^L \frac{L^2 - u^2}{L^2 + 6\nu\,\Theta}\frac{P}{C}\, d\,u\, d\,t + 2V_0 \frac{L^2 + 3\nu\,\Theta}{L^2 + 6\nu\,\Theta}. (7)$$

The second-order approximation of the velocity projection could be obtained by replacing the required projection on the right-hand side of equation (4*d*) on the following sum $v_z \to \alpha_{2z} + v_{1z}$. The considered approximation could be written as

$$v_{2z}(z,t) = \alpha_{2z} + v_{1z}(z,t) + \frac{1}{L^2}\left\{ \nu \int_0^t v_{1z}(z,\tau)\, d\,\tau - \int_0^t \int_0^z \left[\alpha_{2z}^2 + 2\alpha_{2z} v_{1z}(u,\tau) + v_{1z}^2(u,\tau)\right] d\,u\, d\,\tau - \alpha_{2z}\frac{z^2}{2} - \right. \\ - \int_0^z (z-u) v_{1z}(u,t)\, d\,u + \alpha_{2z} z \frac{L}{2} + \frac{z}{L}\int_0^t \int_0^L \left[\alpha_{2z}^2 + 2\alpha_{2z} v_{1z}(u,\tau) + v_{1z}^2(u,\tau)\right] d\,u\, d\,\tau + \frac{z}{L}\int_0^t \int_0^L \frac{P}{LC} d\,u\, d\,\tau + \\ \left. - \nu V_0 t + V_0 z^2 - z V_0 L - \int_0^t \int_0^z \frac{P}{C} d\,u\, d\,\tau + \alpha_{2z}\nu\ t + z\int_0^L \left(1 - \frac{u}{L}\right) v_{1z}(u,t)\, d\,u \right\}. \quad (8)$$

We determine average value α_{2z} by the following standard relation

$$\alpha_{2z} = \frac{1}{\Theta L}\int_0^{\Theta}\int_0^{L}\left(v_{2z} - v_{1z}\right) d\,z\,d\,t\,. \tag{9}$$

Substitution of the first- and the second-order approximations of the considered velocity projection into relation (9) gives a possibility to obtain the required average value α_{2z} in the following form

$$\alpha_{2z} = \frac{\sqrt{a_1^2 - 4a_0 a_2} - a_1}{a_2}\,, \tag{10}$$

where $a_2 = \int_0^{\Theta}(\Theta - t)\int_0^{L}\left(z - \frac{L}{2}\right) d\,z\,d\,t$, $a_1 = 2\int_0^{\Theta}(\Theta - t)\int_0^{L}\left(z - \frac{L}{2}\right) v_{1z}(z,t)\,d\,z\,d\,t + L\frac{\Theta}{2}\left(\frac{L^2}{6} + \Theta\nu\right)$, $a_0 = -\nu \times$ $\times V_0 L\frac{\Theta^2}{2} + \int_0^{\Theta}(\Theta - t)\int_0^{L}\left(z - \frac{L}{2}\right) v_{1z}^2(z,t)\,d\,z\,d\,t + \nu\int_0^{\Theta}(\Theta - t)\int_0^{L} v_{1z}(z,t)\,d\,z\,d\,t - \int_0^{\Theta}(\Theta - t)\int_0^{L}(L - z)\frac{P}{C}\,d\,z\,d\,t -$ $-\frac{1}{2}\int_0^{\Theta}\int_0^{L}(L - z)^2\,v_{1z}(z,t)\,d\,z\,d\,t - V_0\Theta\frac{L^3}{6} + \Theta\frac{L^2}{2}\int_0^{L}\left(1 - \frac{z}{L}\right) v_{1z}(z,t)\,d\,z + \frac{L}{2}\int_0^{\Theta}(\Theta - t)\int_0^{L}\frac{P}{LC}\,d\,z\,d\,t - \frac{L^3}{6} \times$ $\times V_0\Theta$.

In this section the projection of the heat carrier velocity v_z was calculated as the second-order approximation in the framework of the method of averaging functional corrections. Usually the second-order approximation is sufficient to make a qualitative analysis of the solution obtained and to obtained some quantitative estimates. Next, we write equation (1) in a cylindrical coordinate system

$$c\frac{\partial T(r,z,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,\lambda\frac{\partial T(r,z,t)}{\partial r}\right] + \frac{\partial}{\partial z}\left[\lambda\frac{\partial T(r,z,t)}{\partial z}\right] - c\cdot C\frac{\partial\, v_z(z,t)\cdot T(r,z,t)}{\partial r} -$$
$$-c\cdot C\frac{\partial v_z(z,t)\cdot T(r,z,t)}{\partial z} + p(r,z,t)\,. \tag{11}$$

After that we transform the above equation to the following integro-differential form

$$T(r,z,t) = \frac{1}{c\cdot r}\frac{\partial}{\partial r}\left[r\int_0^{t}\lambda\frac{\partial T(r,z,\tau)}{\partial r}\,d\,\tau\right] + \frac{1}{c}\frac{\partial}{\partial z}\left[\int_0^{t}\lambda\frac{\partial T(r,z,\tau)}{\partial z}\,d\,\tau\right] + \frac{1}{c}\int_0^{t} p(r,z,\tau)\,d\,\tau + T_0 -$$
$$-C\frac{\partial}{\partial\,r}\int_0^{t} v_z(z,\tau)\cdot T(r,z,\tau)\,d\,\tau - C\frac{\partial}{\partial\,z}\int_0^{t} v_z(z,\tau)\cdot T(r,z,\tau)\,d\,\tau\,. \tag{12}$$

To calculate distribution of temperature in space and time we again used the same method of averaging functional corrections. To calculate the first-order approximation of the required function we replace it with an as yet unknown mean value α_{1T} in the right side of Eq. (12). Using the above algorithm we obtain the relation for the first-order approximation of temperature in the following form

$$T_1(r,z,t)=\frac{1}{c}\int_0^t p(r,z,\tau)\,d\,\tau-\alpha_{1T}C\frac{\partial}{\partial\,z}\int_0^t v_z(z,\tau)\,d\,\tau+T_0\,. \tag{13}$$

Now let us calculate the not yet known average value of the first-order approximation of the required function by using the following relation

$$\alpha_{1T}=\frac{2}{\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^L T_1(r,z,t)d\,z\,d\,r\,d\,t\,. \tag{14}$$

Substitution of the first-order approximation of temperature (13) into the relation (14) leads to the following result

$$\alpha_{1T}=\frac{2}{c\Theta R^2 L}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^L p(r,z,t)\,d\,z\,d\,r\,d\,t+T_0\,. \tag{15}$$

We calculate the second-order approximation by using the standard procedure of method of averaging of function corrections [3,4], i.e. by replacement of the required function in the right side of Eq. (12) on the following sum: $T\rightarrow\alpha_{2T}+T_1$. In this case the second-order approximation of the required function could be written as

$$T_2(r,z,t)=\frac{1}{c\cdot r}\frac{\partial}{\partial\,r}\left[r\int_0^t\lambda\frac{\partial T_1(r,z,\tau)}{\partial\,r}d\,\tau\right]+\frac{1}{c}\frac{\partial}{\partial\,z}\left[\int_0^t\lambda\frac{\partial T_1(r,z,\tau)}{\partial\,z}d\,\tau\right]+\frac{1}{c}\int_0^t p(r,z,\tau)\,d\,\tau+T_0-$$
$$-C\frac{\partial}{\partial\,r}\int_0^t v_z(z,\tau)[\alpha_{2T}+T_1(r,z,\tau)]d\,\tau-C\frac{\partial}{\partial\,z}\int_0^t v_z(z,\tau)[\alpha_{2T}+T_1(r,z,\tau)]d\,\tau\,. \tag{16}$$

We calculate the average value of the second-order approximation of temperature of heat carrier α_{2T} by using the following standard relation [3,4]

$$\alpha_{2T}=\frac{2}{\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^L (T_2-T_1)d\,z\,d\,r\,d\,t\,. \tag{17}$$

Substitution of the first- and the second-order approximations of temperature into the relation (17) gives a possibility to obtain relation for the required average value in the following form

$$\alpha_{2T}=\frac{2}{\Theta R^2 L c}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^L\lambda\frac{\partial T_1(r,z,\tau)}{\partial z}d\,z\,d\,r\,d\,t\,. \tag{18}$$

Analysis of the distribution of temperature in space and time has been done analytically by using the second-order approximation using the method of averaging functional corrections. The approximation is usually sufficient to obtain a qualitative analysis and to obtain some quantitative results. The results of analytical calculations were verified by comparing them with the results of numerical simulation.

3. DISCUSSION

In this section we analyzed temperature field in the system heat carrier - transportation system. Figures 2-4 show dependence of the temperature of the heat carrier on axial coordinate at different values of width of section in the transportation system, velocity of inlet flow of heat carrier, monitoring time on flow of heat carrier.

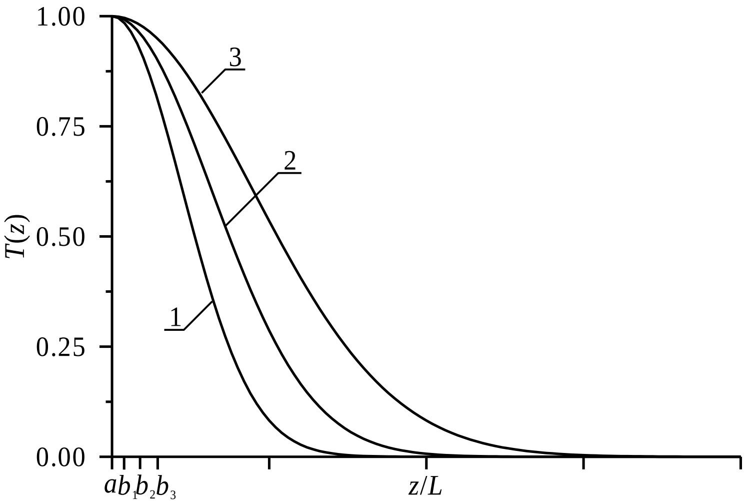

Fig. 2*a*. Dependences of temperature of heat carrier on the axial coordinate at several values of width of section. Increasing of number of the curves corresponds to increasing of the considered width under condition: temperature of wall of transportation system is larger, than the heat carrier temperature

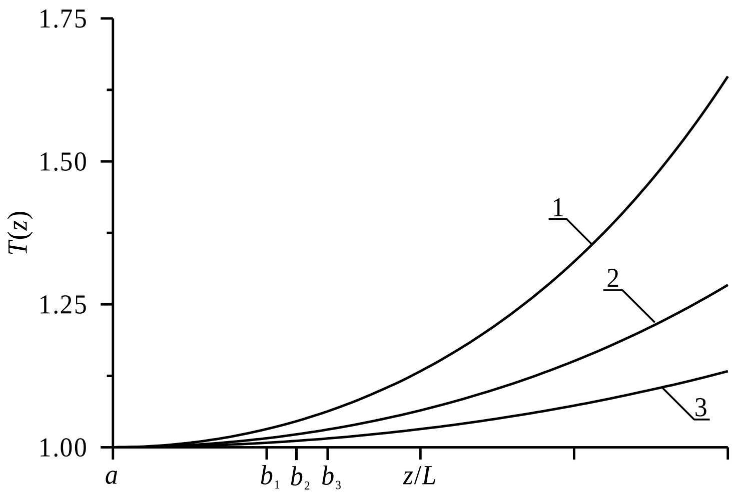

Fig. 2*b*. Dependences of temperature of heat carrier on the axial coordinate at several values of width of section. Increasing of number of the curves corresponds to increasing of the considered width under condition: temperature of wall of transportation system is smaller, than the heat carrier temperature

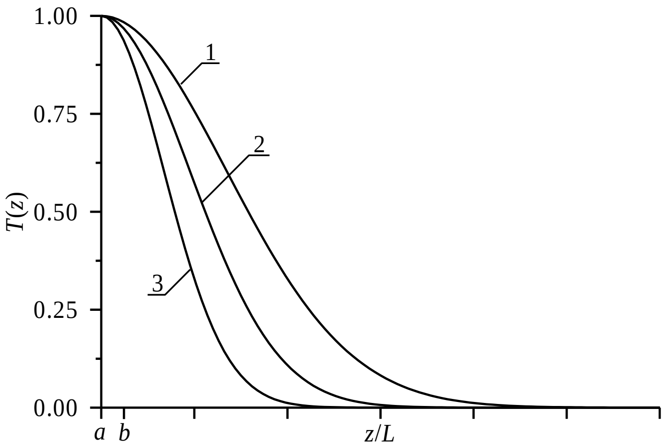

Fig. 3*a*. Dependences of temperature of heat carrier on the axial coordinate at several values of velocity of inlet flow of heat carrier. Increasing of number of the curves corresponds to increasing of the considered velocity under condition: temperature of wall of transportation system is larger, than the heat carrier temperature

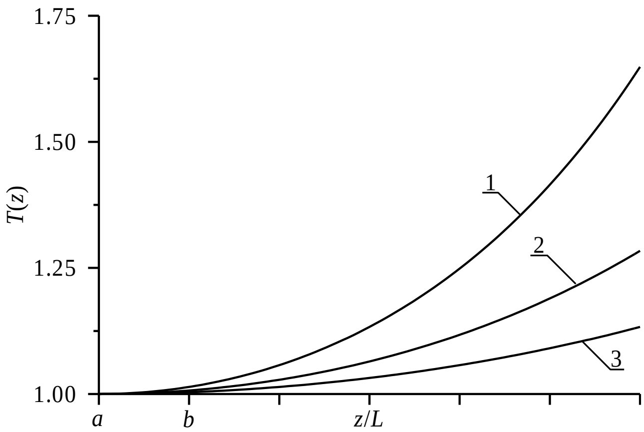

Fig. 3*b*. Dependences of temperature of heat carrier on the axial coordinate at several values of velocity of inlet flow of heat carrier. Increasing of number of the curves corresponds to increasing of the considered velocity under condition: temperature of wall of transportation system is smaller, than the heat carrier temperature

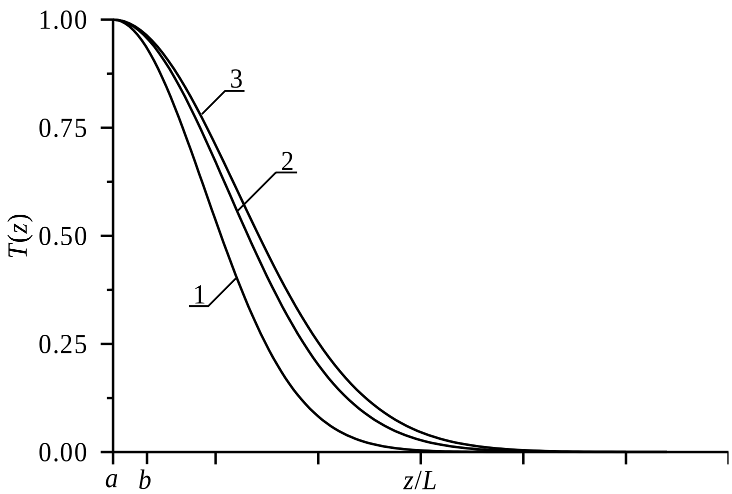

Fig. 4*a*. Dependences of temperature of heat carrier on the axial coordinate at several values of monitoring time on flow of heat carrier (at $t < \vartheta$). Increasing of number of the curves corresponds to increasing of the considered velocity under condition: temperature of wall of transportation system is larger, than the heat carrier temperature

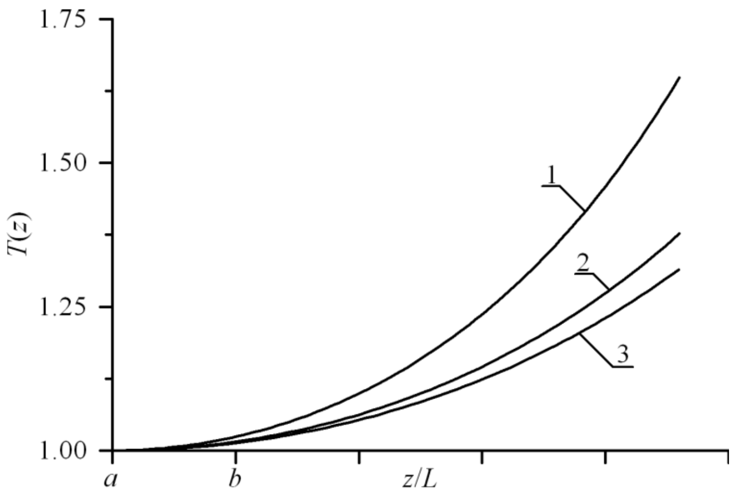

Fig. 4*b*. Dependences of temperature of heat carrier on the axial coordinate at several values of monitoring time on flow of heat carrier (at $t < \vartheta$). Increasing of number of the curves corresponds to increasing of the considered velocity under condition: temperature of wall of transportation system is smaller, than the heat carrier temperature

4. CONCLUSIONS

In this paper we introduce an approach to control of temperature in the system heat carrier - transportation system due to local heating or cooling. We introduced a model with account forced convection and possible variation of several parameters of the considered system. Also we consider an analytical approach for analysis of mass and heat transport with account possible variation of several parameters of the above system.

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