NONLINEAR PROFIT MAXIMIZATION WITH ACCOUNT CHANGING OF PRICES

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ABSTRACT

In this paper, we consider the maximization of the profit of an enterprise that produces several types of products (as example we consider the output of three types of products). Maximization of profit is carried out taking into account the possibility of price changes on the example of prices, linearly depending on the number of products on the market. When considering the maximization of profit, several restrictions are taken into account.

KEYWORDS

nonlinear maximization of profit; accounting of price changing

1. INTRODUCTION

Necessity of maximization of profits leads to the need to compile an optimal output plan with available resources [1-4]. Change in the market situation leads to a change in prices depending on the volume of goods on the market [5-11]. In this situation, when solving the problem of profit maximization, the market reaction to the release of a new batch of products should be taken into account in the form of price changes for a given product (that is, feedback must be taken into account). This feedback can be taken into account as the dependence of prices on the amount of output produced as a function of profit.

In this paper, we consider the profit maximization of an enterprise that produces several types of products. This maximization is carried out on the example of output of three types of products. To maximize the profit under consideration, a generalization of the previously developed [12] methodology for solving similar problems is made, taking into account a number of limitations.

2. METHOD OF SOLUTION

We will analyze the profit of the enterprise on the basis of studying the profit function

$$L=p_1(x_1)\,x_1+p_2(x_2)\,x_2+p_3(x_3)\,x_3. \tag{1}$$

Here x_i is the quantity of the i-th product released, $p_i(x_i)$ is the price of the i-th product as a function of its quantity on the market. In the framework of this paper, let us consider as an example the simplest dependence of the price of a product on its quantity $p_i(x_i)=a_i-b_ix_i$. This dependence allows you to take into account the dependence of the price of the product on its quantity and at the same time reduce the amount of payments with increasing of quantity of product. In the framework of this paper, we consider a number of restrictions (this ratio shows that the maximum amount of products at the time of the start of the sale is fixed: for example, the volume of the warehouse is limited d_1)

$$c_1x_1+c_2x_2+c_3x_3=d_1 \tag{2a}$$

and (this ratio shows that the minimum volume d_2 of products has been reached, from which starting own production or supply of products from outside)

$$c_4x_1+c_5x_2+c_6x_3=d_2. \tag{2b}$$

Here c_i are the volumes of i-th products. Next, consider the maximization of profit within a mathematically standard procedure. In the first stage, we write the Lagrange function [12]

$$l=p_1(x_1)x_1+p_2(x_2)\,x_2+p_3(x_3)\,x_3+\lambda_1(c_1x_1+c_2\,x_2+c_3\,x_3\text{-}d_1)+\lambda_2(c_4x_1+c_5x_2+c_6x_3\text{-}d_2). \tag{3}$$

Here λ_i are the Lagrange multipliers, which are an auxiliary parameter. Further, the maximal value of profit is determined in the framework of the standard procedure for calculation of a conditional extremum [12], i.e. the extremum (in this case, the maximum) of the profit function (1) under the conditions (2). To determine of quantities of products x_i, which corresponds to maximal value of profit, one should calculate partial derivatives of Lagrange function l (3) on all quantities of products x_i and to put them to zero. These partial derivatives could be written as

$$\begin{cases} \dfrac{\partial l}{\partial x_1} = a_1 - 2b_1x_1 + \lambda_1c_1 + \lambda_2c_4 = 0 \\ \dfrac{\partial l}{\partial x_2} = a_2 - 2b_2x_2 + \lambda_1c_2 + \lambda_2c_5 = 0 \\ \dfrac{\partial l}{\partial x_3} = a_3 - 2b_3x_3 + \lambda_1c_3 + \lambda_2c_6 = 0 \\ \dfrac{\partial l}{\partial \lambda_1} = c_1x_1 + c_2x_2 + c_3x_3 - d_1 = 0 \\ \dfrac{\partial l}{\partial \lambda_1} = c_4x_1 + c_5x_2 + c_6x_3 - d_2 = 0 \end{cases} \tag{4}$$

Now let us transform Eqs. (4*a*), (4*b*), (4*c*), (4*d*), (4*e*) to the following equivalent form

$$\begin{cases} 2b_1x_1 - \lambda_1c_1 - \lambda_2c_4 = a_1 \\ 2b_2x_2 - \lambda_1c_2 - \lambda_2c_5 = a_2 \\ 2b_3x_3 - \lambda_1c_3 - \lambda_2c_6 = a_3 \\ c_1x_1 + c_2x_2 + c_3x_3 = d_1 \\ c_4x_1 + c_5x_2 + c_6x_3 = d_2 \end{cases} \tag{4a}$$

Now we solve the system of equations (4*a*) by the Cramer approach [12]. Framework the approach we calculate the following determinants

$$\Delta = \begin{vmatrix} 2b_1 & 0 & 0 & -c_1 & -c_4 \\ 0 & 2b_2 & 0 & -c_2 & -c_5 \\ 0 & 0 & 2b_3 & -c_3 & -c_6 \\ c_1 & c_2 & c_3 & 0 & 0 \\ c_4 & c_5 & c_6 & 0 & 0 \end{vmatrix} \tag{5a}$$

$$\Delta_1 = \begin{vmatrix} a_1 & 0 & 0 & -c_1 & -c_4 \\ a_2 & 2b_2 & 0 & -c_2 & -c_5 \\ a_3 & 0 & 2b_3 & -c_3 & -c_6 \\ d_1 & c_2 & c_3 & 0 & 0 \\ d_2 & c_5 & c_6 & 0 & 0 \end{vmatrix} \tag{5b}$$

$$\Delta_2 = \begin{vmatrix} 2b_1 & a_1 & 0 & -c_1 & -c_4 \\ 0 & a_2 & 0 & -c_2 & -c_5 \\ 0 & a_3 & 2b_3 & -c_3 & -c_6 \\ c_1 & d_1 & c_3 & 0 & 0 \\ c_4 & d_2 & c_6 & 0 & 0 \end{vmatrix} \tag{5c}$$

$$\Delta_3 = \begin{vmatrix} 2b_1 & 0 & a_1 & -c_1 & -c_4 \\ 0 & 2b_2 & a_2 & -c_2 & -c_5 \\ 0 & 0 & a_3 & -c_3 & -c_6 \\ c_1 & c_2 & d_1 & 0 & 0 \\ c_4 & c_5 & d_2 & 0 & 0 \end{vmatrix} \tag{5d}$$

The results of calculation of these determinants could be written as [12]

$$\Delta = 2b_1(c_2c_6 - c_3c_5)^2 + 2b_2(c_1c_6 - c_3c_4)^2 + 2b_3c_1c_5(c_1c_5 - c_2c_4)^2 \tag{6a}$$

$$\begin{aligned}\Delta_1 = a_1(c_2c_6 - c_3c_5)^2 + a_2(c_1c_6 - c_3c_4)(c_2c_6 - c_3c_5) + a_3(c_1c_5 - c_2c_4)(c_2c_6 - c_3c_5) + \\ + 2b_2(c_1c_6 - c_3c_4)(d_1c_6 - d_2c_3) + 2b_3(c_1c_5 - c_2c_4)(d_1c_5 - d_2c_2)\end{aligned} \tag{6b}$$

$$\begin{aligned}\Delta_2 = (c_3c_4 - c_1c_6)[a_1(c_2c_6 - c_3c_5) - a_2(c_1c_6 - c_3c_4) + a_3(c_1c_5 - c_2c_4)] + 2b_1d_1(c_6 - c_3)(c_2c_6 - c_3c_5) + \\ + 2b_3(c_1d_2 - c_4d_1)(c_1c_5 - c_2c_4)\end{aligned} \tag{6c}$$

$$\begin{aligned}\Delta_3 = (c_1c_5 - c_2c_4)[a_1(c_2c_6 - c_3c_5) - a_2(c_1c_6 - c_3c_4) + a_3(c_1c_5 - c_2c_4)] + 2b_1d_1(c_2 - c_5)(c_2c_6 - c_3c_5) + \\ + 2b_2(c_1d_2 - c_4d_1)(c_1c_6 - c_3c_4).\end{aligned} \tag{6d}$$

Now we have a possibility to obtain the optimal quantities of products x_i, which corresponds to maximal values of the profit function (1) under conditions (2), of the required maxima

$$x_1 = \frac{\Delta_1}{\Delta} = \left[a_1(c_2c_6 - c_3c_5)^2 + a_2(c_1c_6 - c_3c_4)(c_2c_6 - c_3c_5) + a_3(c_1c_5 - c_2c_4)(c_2c_6 - c_3c_5) + 2b_2 \times\right.$$

$$\left.\times (c_1c_6 - c_3c_4)(d_1c_6 - d_2c_3) + 2b_3(c_1c_5 - c_2c_4)(d_1c_5 - d_2c_2)\right]\left[2b_1(c_2c_6 - c_3c_5)^2 + (c_1c_6 - c_3c_4)^2 \times\right.$$

$$\left.\times 2b_2 + 2b_3c_1c_5(c_1c_5 - c_2c_4)^2\right]^{-1},$$

$$x_2 = \{(c_3c_4 - c_1c_6)[a_1(c_2c_6 - c_3c_5) - a_2(c_1c_6 - c_3c_4) + a_3(c_1c_5 - c_2c_4)] + (c_6 - c_3)(c_2c_6 - c_3c_5) \times$$

$$\times 2b_1d_1 + 2b_3(c_1d_2 - c_4d_1)(c_1c_5 - c_2c_4)\}\left[2b_1(c_2c_6 - c_3c_5)^2 + 2b_2(c_1c_6 - c_3c_4)^2 + 2b_3c_1c_5 \times\right.$$

$$\times\left(c_1c_5-c_2c_4\right)^2\Big]^{-1},$$

$$x_3=\left\{\left(c_1c_5-c_2c_4\right)\left[a_1\left(c_2c_6-c_3c_5\right)-a_2\left(c_1c_6-c_3c_4\right)+a_3\left(c_1c_5-c_2c_4\right)\right]+2b_1d_1\left(c_2c_6-c_3c_5\right)\times\right.$$

$$\left.\times\left(c_2-c_5\right)+2b_2\left(c_1d_2-c_4d_1\right)\left(c_1c_6-c_3c_4\right)\right\}\Big[2b_1\left(c_2c_6-c_3c_5\right)^2+2b_2\left(c_1c_6-c_3c_4\right)^2+$$

$$+2b_3\left(c_1c_5-c_2c_4\right)^2\Big]^{-1}.$$

These coordinates are the volumes of output corresponding to the maximum profit of the enterprise. A few typical dependencies of the profit function (1) on the output of production volume x_i are shown in Figures 1-3 for different values of parameter a_i, b_i, c_i, d_i.

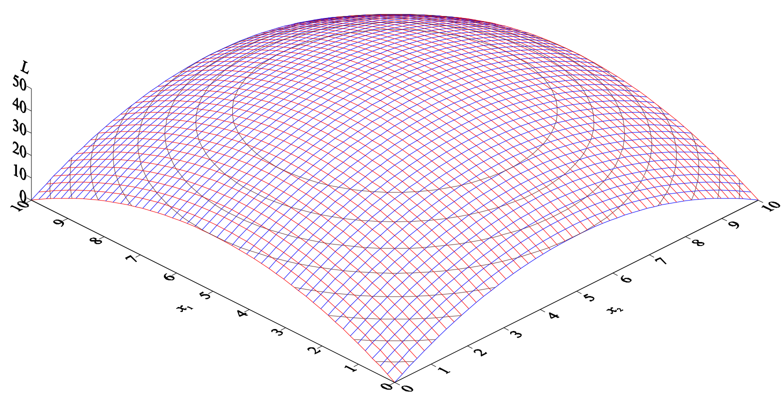

Fig. 1. Example of the dependency of the profit function on the output production volumes x_1 and x_2

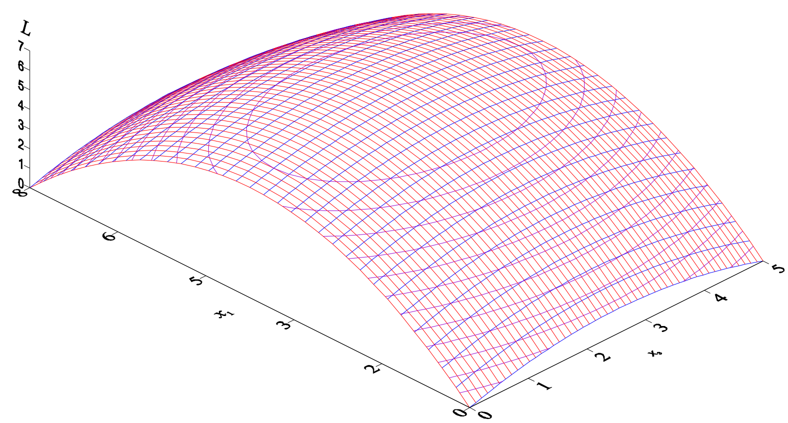

Fig. 2. Example of the dependency of the profit function on the output production volumes x_1 and x_3

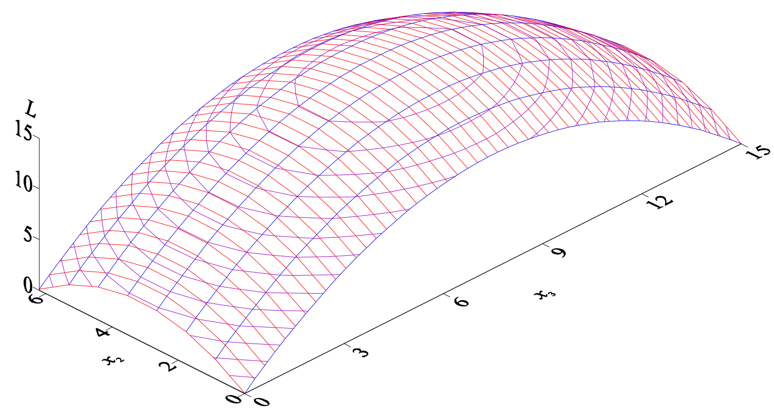

Fig. 3. Example of the dependency of the profit function on the output production volumes x_2 and x_3

Now we analyzed dependences of optimal values of products x_{iopt} on different values of parameter a_i, b_i, c_i, d_i. Fig. 4 shows dependences of product x_{1opt} on partial volume of the warehouse c_1. Dependences of optimal values of products x_{1opt} on other parameters c_i have the similar structures. However dependences of optimal value of products x_{1opt} on parameters c_2, c_3, c_5, c_6 more weak in comparison with the same dependences on c_1 and c_4. The figure shows that dependences of optimal value of products have two optimal values: for maximization of profit (at smaller value of the partial volume of the warehouse) and for maximization of damage (at larger value of the partial volume of the warehouse). Reason of obtaining of the damage could be following: total volume of the warehouse is limited and using only one product could leads to error in prognosis of demand on this concrete product.

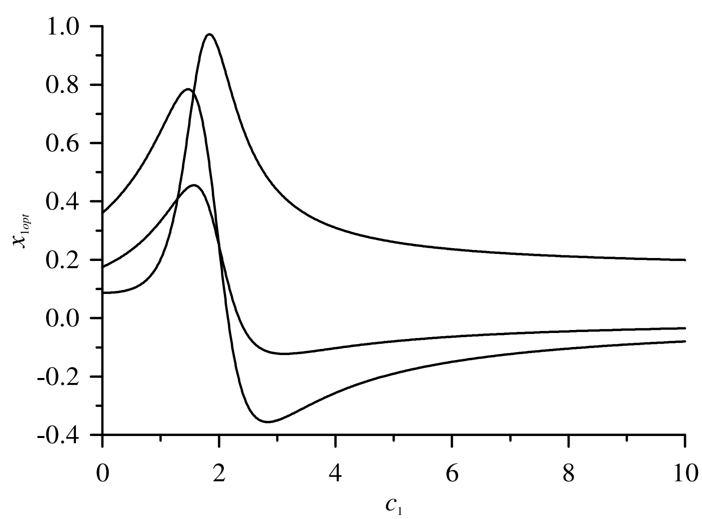

Fig. 4*a*. Dependences of optimal values of products x_{1opt} on partial volume of the warehouse c_1

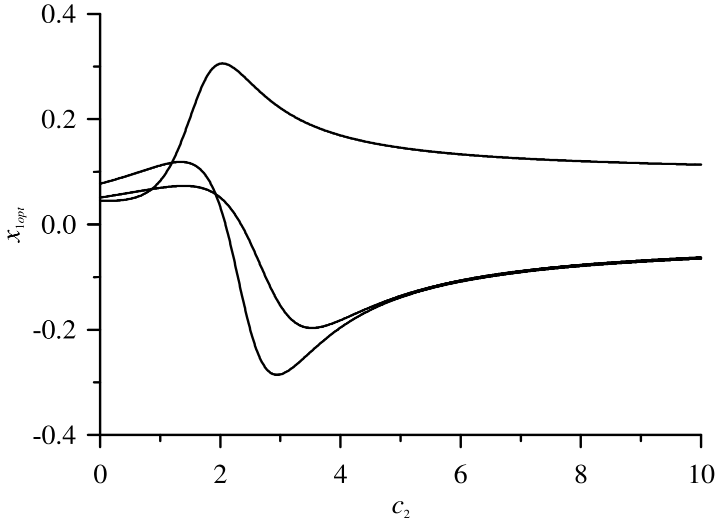

Fig. 4*b*. Dependences of optimal values of products x_{1opt} on partial volume of the warehouse c_2

The Fig. 5 shows dependences of optimal value of products x_{1opt} on base value of prices a_1. The figure shows apparent conclusion: increasing of price of product leads to increase of value of profit. Comparison of dependences of x_{1opt} on a_1 and on a_2 shows that the second dependence is weaker in comparison with the first one (see Figs. 5*a* and 5*b*).

Now we will consider dependences of optimal value of products x_{1opt} on partial value of product in price b_1. Several dependences of optimal value of products x_{1opt} are presented on Fig. 6. The figure shows decreasing of optimal value of products to increase value of profit. The result is enough clear: increasing of quantity of product leads to decreasing of price. Comparison of dependences of x_{1opt} on b_1 and on b_2 shows that the second dependence is weaker in comparison with the first one (see Figs. 6*a* and 6*b*.

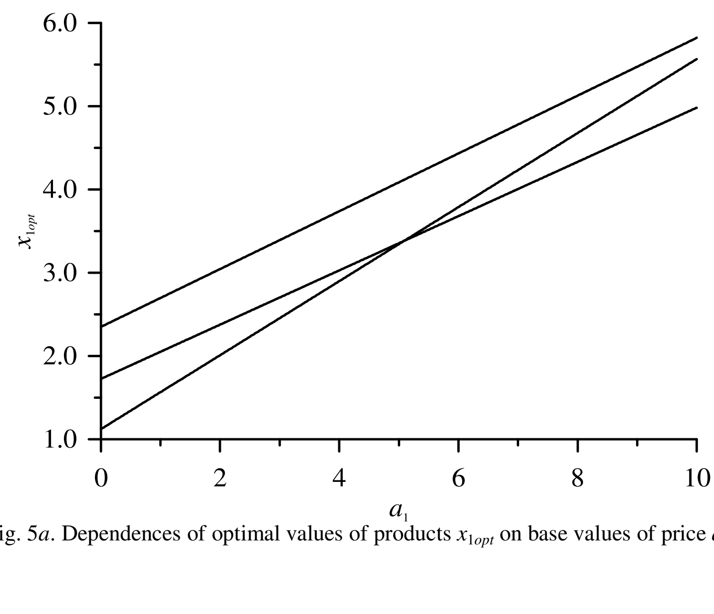

Fig. 5*a*. Dependences of optimal values of products x_{1opt} on base values of price a_1

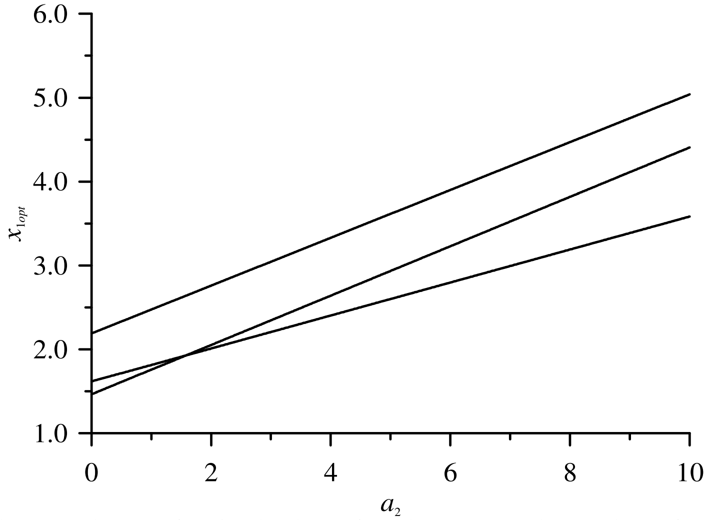

Fig. 5*b*. Dependences of optimal values of products x_{1opt} on base values of price a_2

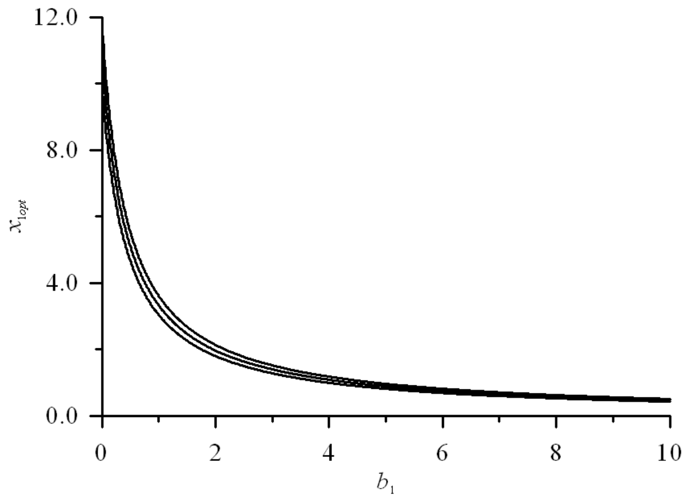

Fig. 6*a*. Dependences of optimal values of products x_{1opt} on partial values of product b_1

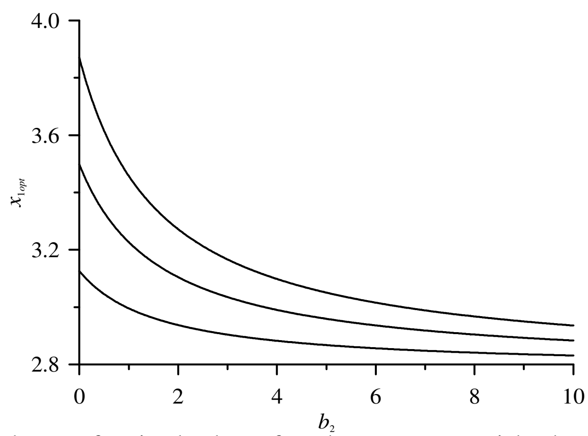

Fig. 6*b*. Dependences of optimal values of products x_{1opt} on partial values of product b_2

Further we we will consider dependences of optimal value of products x_{1opt} on volume of the warehouse d_1 and minimum volume of products d_2. Appropriate dependences are presented on Figs. 7 and 8. The Fig. 7 shows qualitatively different dependences. Type of these curves depends on values of parameters. If quantities of products too small than owner or leaseholder of the warehouse has larger damage, then profit. If quantities of products are enough large for obtaining profit, value of profit depends on quantities and prices of products. Optimization of process to decrease minimum volume of products d_2 gives a possibility to obtain profit with decreasing of expenses (see Fig. 8).

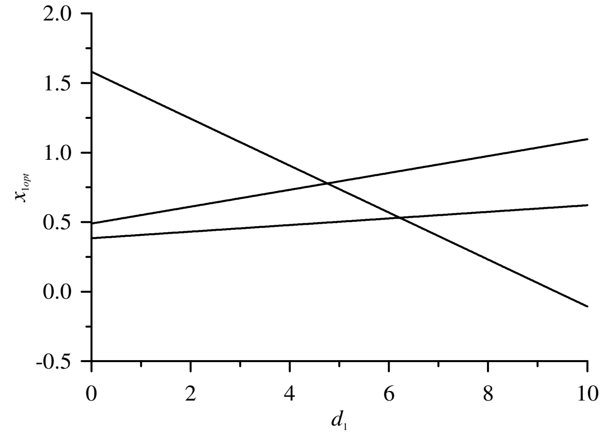

Fig. 7. Dependences of optimal values of products x_{1opt} on volume of the warehouse d_1

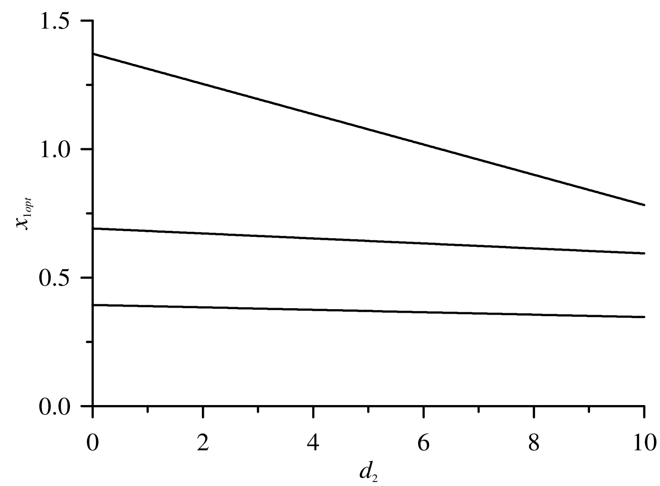

Fig. 8. Dependences of optimal values of products x_{1opt} on minimum volume of products d_2

4. CONCLUSIONS

In this paper we introduce an approach to maximize the profit of an enterprise in the conditions of existing constraints. This technique is considered on the example of three products, but it is possible to consider another quantity of products. Based on this approach we analyzed optimal values of products on several parameters.

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