

ON OPTIMIZATION OF TECHNOLOGICAL PROCESS TO INCREASE DENSITY OF ELEMENTS IN CIRCUITS NAND AND AND, BASED ON FIELD-EFFECT HETEROTRANSISTORS

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ABSTRACT

In this paper we analyzed possibility to increase density of elements in circuits NAND and AND based on field-effect heterotransistors. We introduce an approach to increase density of the considered elements. Framework the approach it is necessary to manufacture a heterostructure with specific configuration. After the manufacturing it is necessary to dope required areas of the heterostructure by diffusion or ion implantation. The doping finished by optimized annealing of dopant and/or radiation defects. We compare manufacturing of these transistors, manufactured by diffusion and ion implantation. We analyzed possibility to decrease mismatch-induced stress in the considered heterostructure. Some comparison of calculated results with experimental one has been done.

KEYWORDS

Circuits NAND and AND, increasing of density of elements

1. INTRODUCTION

In the present time one of actual questions of solid state electronic devices is increasing of density of elements of integrated circuits. The increasing leads to necessity of decreasing of dimensions of these elements. To date, several methods to decrease dimensions of elements of integrated circuits have been developed. One of them is growth of thin films structures [1-5]. The second approach based on doping by diffusion or ion implantation of required areas of samples or heterostructures. After that the dopants and/or radiation defects annealed by laser or microwave irradiation [6-8]. Using of the above approaches of annealing leads to generation of inhomogenous distribution of temperature and consequently to decreasing of dimensions of elements of integrated circuits. Another approach to change properties of doped materials is radiation processing [9,10].

In this paper we introduce a method of increasing of density of elements of circuits NAND and AND. Manufacturing of these circuits based on field-effect heterotransistors. Manufacture of these transistors based on a heterostructure, which includes into itself a substrate and an epitaxial layer. Several sections have been manufactured by using another materials framework the epitaxial layer. Now we consider doping of the sections by diffusion or ion implantation to generation required types of conductivity (*p* or *n*). It is necessary to manufacture field-effect transistors so as it is shown on Fig. 1. After finishing of the doping we consider annealing of dopant and/or radiation defects. Our main aim framework the present paper is optimization of annealing process of the dopant to increase density of transistors framework the considered circuits. An accompany aim of the present paper is analysis of possibility to decrease mismatch-induced stress in the considered heterostructure.

2. METHOD OF SOLUTION

First of all we calculate distribution of concentration of dopant in space and time by solving the following boundary problem [9-13]

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[D \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[D \frac{\partial C(x, y, z, t)}{\partial z} \right] + \quad (1)$$

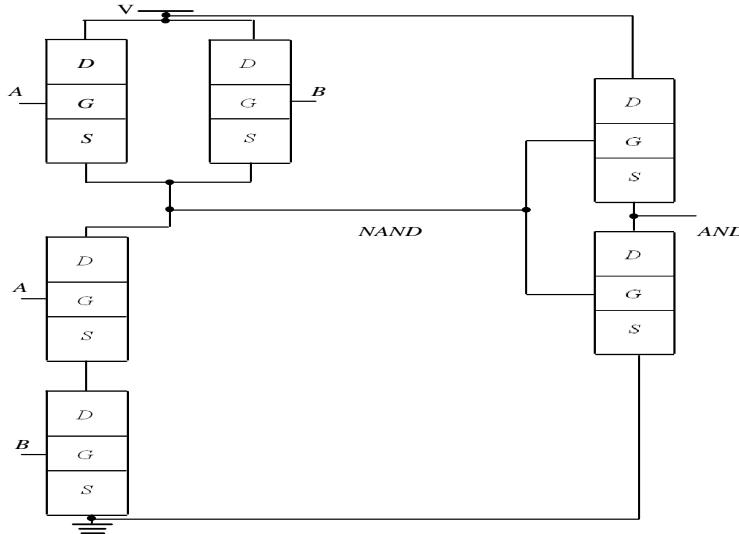


Fig. 1. Structure of circuits NAND and AND. View from top

$$+ \Omega \frac{\partial}{\partial x} \left[\frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} C(x, y, W, t) dW \right] + \\ + \Omega \frac{\partial}{\partial y} \left[\frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} C(x, y, W, t) dW \right]$$

Boundary and initial conditions are

$$\frac{\partial C(x, y, z, t)}{\partial x} \Big|_{x=0} = 0, \frac{\partial C(x, y, z, t)}{\partial x} \Big|_{x=L_x} = 0, \frac{\partial C(x, y, z, t)}{\partial y} \Big|_{y=0} = 0, \frac{\partial C(x, y, z, t)}{\partial y} \Big|_{x=L_y} = 0, \\ \frac{\partial C(x, y, z, t)}{\partial z} \Big|_{z=0} = 0, \frac{\partial C(x, y, z, t)}{\partial z} \Big|_{x=L_z} = 0, C(x, y, z, 0) = f_C(x, y, z).$$

Function $C(x, y, z, t)$ describe distribution of concentration of dopant in space and time; Ω is the atomic volume of the dopant; symbol ∇_s is the surficial gradient; function $\int_0^{L_z} C(x, y, z, t) dz$ describe distribution of surficial concentration of dopant in space and time in the neighborhood of the interface between materials of heterostructure (in this case we assume, that Z-axis is perpendicular to the interface between materials of heterostructure); $\mu(x, y, z, t)$ is the chemical potential; D are D_s are the coefficients of volumetric and surficial diffusions (the surficial diffusion is the consequences of mismatch-induced stress). Values of diffusion coefficients depend on properties of materials of heterostructure, speed of heating and cooling of heterostructure, concentration of

dopant. Dependences of dopant diffusion coefficients on parameters could be approximated by the following relations [11]

$$D_c = D_L(x, y, z, T) \left[1 + \xi \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right],$$

$$D_s = D_{sL}(x, y, z, T) \left[1 + \xi_s \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \right] \left[1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right]. \quad (2)$$

Functions $D_L(x, y, z, T)$ and $D_{sL}(x, y, z, T)$ describe dependences of dopant diffusion coefficients on coordinate (if we consider heterostructure with several layers) and temperature; T describes annealing temperature; function $P(x, y, z, T)$ describe dependence of limit of solubility of dopant on coordinate and temperature; parameter γ depends on properties of materials and could be integer in the following interval $\gamma \in [1, 3]$ [11]; function $V(x, y, z, t)$ describe dependence of radiation vacancies in space and time with equilibrium distribution of vacancies V^* . Concentrational dependence of dopant diffusion coefficient has been described in details in [11]. Distributions of concentration of point radiation defects in space and time have been determined by solving the following system of equations [10, 12, 13]

$$\begin{aligned} \frac{\partial I(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_{I,V}(x, y, z, T) I(x, y, z, t) \times \\ &\times V(x, y, z, t) + \Omega \frac{\partial}{\partial x} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right] + \\ &+ \Omega \frac{\partial}{\partial y} \left[\frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} I(x, y, W, t) dW \right], \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial V(x, y, z, t)}{\partial t} &= \frac{\partial}{\partial x} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + \\ &+ \frac{\partial}{\partial z} \left[D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{V,V}(x, y, z, T) V^2(x, y, z, t) - k_{I,V}(x, y, z, T) I(x, y, z, t) \times \\ &\times V(x, y, z, t) + \Omega \frac{\partial}{\partial x} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} V(x, y, W, t) dW \right] + \\ &+ \Omega \frac{\partial}{\partial y} \left[\frac{D_{VS}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} V(x, y, W, t) dW \right] \end{aligned}$$

Boundary and initial conditions are

$$\left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0,$$

$$\left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0,$$

$$\left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \\ I(x, y, z, 0) = f_I(x, y, z), V(x, y, z, 0) = f_V(x, y, z). \quad (4)$$

Function $I(x, y, z, t)$ describes distribution of concentration of radiation interstitials in space and time with equilibrium distribution I^* ; functions $D_I(x, y, z, T)$, $D_V(x, y, z, T)$, $D_{IS}(x, y, z, T)$, $D_{VS}(x, y, z, T)$ describe dependences of coefficients of volumetric and surficial diffusions of interstitials and vacancies on coordinate and temperature; terms $V^2(x, y, z, t)$ and $I^2(x, y, z, t)$ correspond to generation of divacancies and diinterstitials, respectively (see, for example, [10] and appropriate references in this book); functions $k_{I,V}(x, y, z, T)$, $k_{I,I}(x, y, z, T)$ and $k_{V,V}(x, y, z, T)$ describe dependences of parameters of recombination of point radiation defects and generation of their complexes on coordinate and temperature.

We determine distributions of concentrations of divacancies $\Phi_V(x, y, z, t)$ and diinterstitials $\Phi_I(x, y, z, t)$ on coordinate and time by solving the following system of equations [10,12,13]

$$\begin{aligned} \frac{\partial \Phi_I(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right] + \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{IS}}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} \Phi_I(x, y, W, t) dW \right] + \\ & + \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{IS}}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} \Phi_I(x, y, W, t) dW \right] + k_{I,I}(x, y, z, T) I^2(x, y, z, t) + \\ & + k_I(x, y, z, T) I(x, y, z, t), \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial \Phi_V(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right] + \Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{VS}}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} \Phi_V(x, y, W, t) dW \right] + \\ & + \Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{VS}}}{kT} \nabla_s \mu(x, y, z, t) \int_0^{L_z} \Phi_V(x, y, W, t) dW \right] + k_{V,V}(x, y, z, T) V^2(x, y, z, t) + \\ & + k_V(x, y, z, T) V(x, y, z, t) \end{aligned}$$

Boundary and initial conditions are

$$\begin{aligned} \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \left. \frac{\partial I(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \\ \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \left. \frac{\partial V(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \left. \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \end{aligned}$$

$$\Phi_I(x, y, z, 0) = f_{\Phi_I}(x, y, z), \Phi_V(x, y, z, 0) = f_{\Phi_V}(x, y, z). \quad (6)$$

Functions $D_{\phi l}(x,y,z,T)$, $D_{\phi V}(x,y,z,T)$, $D_{\phi lS}(x,y,z,T)$ and $D_{\phi VS}(x,y,z,T)$ describe dependences of coefficients of volumetric and surficial diffusions of complexes of radiation defects on coordinate and temperature; functions $k_l(x,y,z,T)$ and $k_V(x,y,z,T)$ describe dependences of parameters of decay of these complexes on coordinate and temperature. Chemical potential in Eq.(1) could be determined by the following relation [12]

$$\mu = E(z)\Omega \sigma_{ij} [u_{ij}(x,y,z,t) + u_{ji}(x,y,z,t)]/2. \quad (7)$$

Here $E(z,C) = E_1(z) + E_2(z)[C(x,y,z,t)/P(x,y,z,T)]$ [15] is the Young modulus, σ_{ij} is the stress tensor; $u_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ is the deformation tensor; u_i , u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; variables x_i , x_j equal to x , y , z depends on number of equation for components of displacement vector. The Eq. (7) could be transformed to the following form

$$\begin{aligned} \mu(x,y,z,t) = & \left[\frac{\partial u_i(x,y,z,t)}{\partial x_j} + \frac{\partial u_j(x,y,z,t)}{\partial x_i} \right] \left\{ \frac{1}{2} \left[\frac{\partial u_i(x,y,z,t)}{\partial x_j} + \frac{\partial u_j(x,y,z,t)}{\partial x_i} \right] - \right. \\ & \left. - \varepsilon_0 \delta_{ij} + \frac{\sigma(z) \delta_{ij}}{1 - 2\sigma(z)} \left[\frac{\partial u_k(x,y,z,t)}{\partial x_k} - 3\varepsilon_0 \right] - K(z) \beta(z) [T(x,y,z,t) - T_0] \delta_{ij} \right\} \frac{\Omega}{2} E(z,C), \end{aligned}$$

where σ is the Poisson coefficient; $\varepsilon_0 = (a_s - a_{EL})/a_{EL}$ is the mismatch parameter; a_s , a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_0 is the equilibrium temperature which coincides (for our case) with the room temperature. Components of displacement vector could be obtained by solution of the following equations [14]

$$\begin{cases} \rho(z) \frac{\partial^2 u_x(x,y,z,t)}{\partial t^2} = \frac{\partial \sigma_{xx}(x,y,z,t)}{\partial x} + \frac{\partial \sigma_{xy}(x,y,z,t)}{\partial y} + \frac{\partial \sigma_{xz}(x,y,z,t)}{\partial z} \\ \rho(z) \frac{\partial^2 u_y(x,y,z,t)}{\partial t^2} = \frac{\partial \sigma_{yx}(x,y,z,t)}{\partial x} + \frac{\partial \sigma_{yy}(x,y,z,t)}{\partial y} + \frac{\partial \sigma_{yz}(x,y,z,t)}{\partial z} \\ \rho(z) \frac{\partial^2 u_z(x,y,z,t)}{\partial t^2} = \frac{\partial \sigma_{zx}(x,y,z,t)}{\partial x} + \frac{\partial \sigma_{zy}(x,y,z,t)}{\partial y} + \frac{\partial \sigma_{zz}(x,y,z,t)}{\partial z} \end{cases}$$

Here $\sigma_{ij} = \frac{E(z)}{2[1 + \sigma(z)]} \left[\frac{\partial u_i(x,y,z,t)}{\partial x_j} + \frac{\partial u_j(x,y,z,t)}{\partial x_i} - \frac{\delta_{ij}}{3} \frac{\partial u_k(x,y,z,t)}{\partial x_k} \right] + \delta_{ij} \frac{\partial u_k(x,y,z,t)}{\partial x_k} \times K(z) - \beta(z) K(z) [T(x,y,z,t) - T_r]$, $\rho(z)$ is the density of materials of heterostructure, δ_{ij} is the Kronecker symbol. Accounting relation for σ_{ij} in the previous system of equations last system of equation could be written as

$$\begin{aligned} \rho(z) \frac{\partial^2 u_x(x,y,z,t)}{\partial t^2} = & \left\{ K(z) + \frac{5E(z,C)}{6[1 + \sigma(z)]} \right\} \frac{\partial^2 u_x(x,y,z,t)}{\partial x^2} + \left\{ K(z) - \frac{E(z,C)}{3[1 + \sigma(z)]} \right\} \times \\ & \times \frac{\partial^2 u_y(x,y,z,t)}{\partial x \partial y} + \frac{E(z,C)}{2[1 + \sigma(z)]} \left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u_z(x,y,z,t)}{\partial z^2} \right] + \left[K(z) + \frac{E(z,C)}{3[1 + \sigma(z)]} \right] \times \end{aligned}$$

$$\begin{aligned}
 & \times \frac{\partial^2 u_z(x, y, z, t)}{\partial x \partial z} - K(z)\beta(z) \frac{\partial T(x, y, z, t)}{\partial x} \\
 \rho(z) \frac{\partial^2 u_y(x, y, z, t)}{\partial t^2} &= \frac{E(z, C)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_y(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_x(x, y, z, t)}{\partial x \partial y} \right] - \frac{\partial T(x, y, z, t)}{\partial y} \times \\
 & \times K(z)\beta(z) + \frac{\partial}{\partial z} \left\{ \frac{E(z, C)}{2[1+\sigma(z)]} \left[\frac{\partial u_y(x, y, z, t)}{\partial z} + \frac{\partial u_z(x, y, z, t)}{\partial y} \right] \right\} + \frac{\partial^2 u_y(x, y, z, t)}{\partial y^2} \times (8) \\
 & \times \left\{ \frac{5E(z, C)}{12[1+\sigma(z)]} + K(z) \right\} + \left\{ K(z) - \frac{E(z, C)}{6[1+\sigma(z)]} \right\} \frac{\partial^2 u_y(x, y, z, t)}{\partial y \partial z} + K(z) \frac{\partial^2 u_y(x, y, z, t)}{\partial x \partial y} \\
 \rho(z) \frac{\partial^2 u_z(x, y, z, t)}{\partial t^2} &= \frac{E(z, C)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_z(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_z(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_x(x, y, z, t)}{\partial x \partial z} + \right. \\
 & \left. + \frac{\partial^2 u_y(x, y, z, t)}{\partial y \partial z} \right] + \frac{\partial}{\partial z} \left\{ K(z) \left[\frac{\partial u_x(x, y, z, t)}{\partial x} + \frac{\partial u_y(x, y, z, t)}{\partial y} + \frac{\partial u_z(x, y, z, t)}{\partial z} \right] \right\} + \\
 & + \frac{1}{6} \frac{\partial}{\partial z} \left\{ \frac{E(z, C)}{1+\sigma(z)} \left[6 \frac{\partial u_z(x, y, z, t)}{\partial z} - \frac{\partial u_x(x, y, z, t)}{\partial x} - \frac{\partial u_y(x, y, z, t)}{\partial y} - \frac{\partial u_z(x, y, z, t)}{\partial z} \right] \right\} - \\
 & - K(z)\beta(z) \frac{\partial T(x, y, z, t)}{\partial z}.
 \end{aligned}$$

Conditions for the system of these equations are

$$\begin{aligned}
 \frac{\partial \bar{u}(x, y, z, t)}{\partial x} \Big|_{x=0} &= 0; \quad \frac{\partial \bar{u}(x, y, z, t)}{\partial x} \Big|_{x=L_x} = 0; \quad \frac{\partial \bar{u}(x, y, z, t)}{\partial y} \Big|_{y=0} = 0; \quad \frac{\partial \bar{u}(x, y, z, t)}{\partial y} \Big|_{y=L_y} = 0; \\
 \frac{\partial \bar{u}(x, y, z, t)}{\partial z} \Big|_{z=0} &= 0; \quad \frac{\partial \bar{u}(x, y, z, t)}{\partial z} \Big|_{z=L_z} = 0; \quad \bar{u}(x, y, z, 0) = \bar{u}_0; \quad \bar{u}(x, y, z, \infty) = \bar{u}_0.
 \end{aligned}$$

We determine spatio-temporal distributions of concentrations of dopant and radiation defects by solving the Eqs.(1), (3) and (5) simultaneously by the standard method of averaging of function corrections [16,17]. On the first step of solving of our aim we take into account initial distributions of the considered concentrations in the Eqs. (1), (3) and (5). In this situation initial conditions after appropriate equations will be zero. After that we replace the required functions in right sides of Eqs. (1a), (3a) and (5a) on their not yet known average values α_{lp} . In this situation we obtain equations for the first-order approximations of the considered concentrations in the following form

$$\begin{aligned}
 \frac{\partial C_1(x, y, z, t)}{\partial t} &= \alpha_{lc}\Omega \frac{\partial}{\partial x} \left[z \frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \right] + \alpha_{lc}\Omega \frac{\partial}{\partial y} \left[z \frac{D_s}{kT} \nabla_s \mu(x, y, z, t) \right] + \\
 & + f_c(x, y, z) \delta(t),
 \end{aligned} \tag{1b}$$

$$\begin{aligned}
 \frac{\partial I_1(x, y, z, t)}{\partial t} &= f_I(x, y, z) \delta(t) + \alpha_{I_l} z \Omega \frac{\partial}{\partial x} \left[\frac{D_{ls}}{kT} \nabla_s \mu(x, y, z, t) \right] + \\
 & + \alpha_{I_l} \Omega \frac{\partial}{\partial y} \left[z \frac{D_{ls}}{kT} \nabla_s \mu(x, y, z, t) \right] - \alpha_{I_l}^2 k_{I,l}(x, y, z, T) - \alpha_{I_l} \alpha_{IV} k_{I,V}(x, y, z, T),
 \end{aligned} \tag{3b}$$

$$\begin{aligned}
 \frac{\partial V_1(x, y, z, t)}{\partial t} &= f_v(x, y, z)\delta(t) + \alpha_{IV}z\Omega \frac{\partial}{\partial x} \left[\frac{D_{vs}}{kT} \nabla_s \mu(x, y, z, t) \right] + \\
 &\quad + \alpha_{IV}\Omega \frac{\partial}{\partial y} \left[z \frac{D_{vs}}{kT} \nabla_s \mu(x, y, z, t) \right] - \alpha_{IV}^2 k_{V,V}(x, y, z, T) - \alpha_{II}\alpha_{IV} k_{I,V}(x, y, z, T), \\
 \frac{\partial \Phi_{II}(x, y, z, t)}{\partial t} &= f_{\Phi_I}(x, y, z)\delta(t) + \alpha_{I\Phi_I}z\Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{I,S}}}{kT} \nabla_s \mu(x, y, z, t) \right] + \\
 &\quad + \alpha_{I\Phi_I}z\Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{I,S}}}{kT} \nabla_s \mu(x, y, z, t) \right] + k_{I,I}(x, y, z, T)I^2(x, y, z, t) + k_I(x, y, z, T)I(x, y, z, t), \\
 \frac{\partial \Phi_{IV}(x, y, z, t)}{\partial t} &= f_{\Phi_V}(x, y, z)\delta(t) + \alpha_{I\Phi_V}z\Omega \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{V,S}}}{kT} \nabla_s \mu(x, y, z, t) \right] + \\
 &\quad + \alpha_{I\Phi_V}z\Omega \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{V,S}}}{kT} \nabla_s \mu(x, y, z, t) \right] + k_{V,V}(x, y, z, T)V^2(x, y, z, t) + k_V(x, y, z, T)V(x, y, z, t). \tag{5b}
 \end{aligned}$$

Integration of the left and right sides of Eqs. (1b), (3b) and (5b) gives us the possibility to obtain relations for above first-order approximations in the following form

$$\begin{aligned}
 C_1(x, y, z, t) &= \alpha_{IC}\Omega \frac{\partial}{\partial x} \int_0^t D_{SL}(x, y, z, \tau) \nabla_s \mu(x, y, z, \tau) \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\
 &\quad \times \frac{z}{kT} \left[1 + \frac{\xi_s \alpha_{IC}^r}{P^r(x, y, z, T)} \right] d\tau + \alpha_{IC} \frac{\partial}{\partial y} \int_0^t D_{SL}(x, y, z, \tau) \left[1 + \frac{\xi_s \alpha_{IC}^r}{P^r(x, y, z, T)} \right] \left[1 + \frac{\xi_s \alpha_{IC}^r}{P^r(x, y, z, T)} \right] \times \\
 &\quad \times z \frac{\Omega}{kT} \left[1 + \varsigma_1 \frac{V(x, y, z, \tau)}{V^*} + \varsigma_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] d\tau + f_C(x, y, z), \tag{1c}
 \end{aligned}$$

$$\begin{aligned}
 I_1(x, y, z, t) &= f_I(x, y, z) - \alpha_{II}^2 \int_0^t k_{I,I}(x, y, z, \tau) d\tau - \alpha_{II}\alpha_{IV} \int_0^t k_{I,V}(x, y, z, \tau) d\tau \\
 &\quad + \alpha_{II}z\Omega \frac{\partial}{\partial x} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \alpha_{II}z\Omega \frac{\partial}{\partial y} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau, \tag{3c}
 \end{aligned}$$

$$\begin{aligned}
 V_1(x, y, z, t) &= f_V(x, y, z) - \alpha_{IV}^2 \int_0^t k_{V,V}(x, y, z, \tau) d\tau - \alpha_{II}\alpha_{IV} \int_0^t k_{I,V}(x, y, z, \tau) d\tau + \\
 &\quad + \alpha_{IV}z\Omega \frac{\partial}{\partial x} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \alpha_{IV}z\Omega \frac{\partial}{\partial y} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau,
 \end{aligned}$$

$$\begin{aligned}
 \Phi_{II}(x, y, z, t) &= f_{\Phi_I}(x, y, z) + \alpha_{I\Phi_I}z\Omega \frac{\partial}{\partial x} \int_0^t \frac{D_{\Phi_{I,S}}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \int_0^t k_I(x, y, z, \tau) I(x, y, z, \tau) d\tau + \\
 &\quad + \alpha_{I\Phi_I}z\Omega \frac{\partial}{\partial y} \int_0^t \frac{D_{\Phi_{I,S}}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \int_0^t k_{I,I}(x, y, z, \tau) I^2(x, y, z, \tau) d\tau, \tag{5c}
 \end{aligned}$$

$$\Phi_{IV}(x, y, z, t) = f_{\Phi_V}(x, y, z) + \alpha_{I\Phi_V}z\Omega \frac{\partial}{\partial x} \int_0^t \frac{D_{\Phi_{V,S}}}{kT} \nabla_s \mu(x, y, z, \tau) d\tau + \int_0^t k_V(x, y, z, \tau) V(x, y, z, \tau) d\tau +$$

$$+ \alpha_{\Phi_V} z \Omega \frac{\partial}{\partial y} \int_0^t \frac{D_{\Phi_V S}}{k T} \nabla_s \mu(x, y, z, \tau) d\tau + \int_0^t k_{V,V}(x, y, z, T) V^2(x, y, z, \tau) d\tau.$$

We determine average values of the first-order approximations of required functions by using the following relations [16,17]

$$\alpha_{1\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \int_0^{\Theta L_x L_y L_z} \rho_1(x, y, z, t) dz dy dx dt. \quad (9)$$

Substitution of these relations (1c), (3c) and (5c) into relation (9) gives us a possibility to obtain the following relations

$$\begin{aligned} \alpha_{1C} &= \frac{1}{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_C(x, y, z) dz dy dx, \quad \alpha_{1I} = \sqrt{\frac{(a_3 + A)^2}{4 a_4^2} - 4 \left(B + \frac{\Theta a_3 B + \Theta^2 L_x L_y L_z a_1}{a_4} \right)} - \\ &- \frac{a_3 + A}{4 a_4}, \quad \alpha_{1V} = \frac{1}{S_{IV00}} \left[\frac{\Theta}{\alpha_{1I}} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_I(x, y, z) dz dy dx - \alpha_{1I} S_{II00} - \Theta L_x L_y L_z \right], \end{aligned}$$

where $S_{\rho\rho ij} = \int_0^{\Theta} (\Theta - t) \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} k_{\rho, \rho}(x, y, z, T) I_i^j(x, y, z, t) V_1^j(x, y, z, t) dz dy dx dt$, $a_4 = S_{II00} (S_{IV00}^2 - S_{II00} S_{VV00})$,

$$\begin{aligned} a_3 &= S_{IV00} S_{II00} + S_{IV00}^2 - S_{II00} S_{VV00}, \quad a_2 = S_{IV00} S_{IV00}^2 \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_V(x, y, z) dz dy dx + 2 S_{VV00} S_{II00} \times \\ &\times \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_I(x, y, z) dz dy dx - S_{IV00}^2 \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_I(x, y, z) dz dy dx - \Theta L_x^2 L_y^2 L_z^2 S_{VV00} + S_{IV00} \Theta L_x^2 L_y^2 L_z^2, \\ a_1 &= S_{IV00} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_I(x, y, z) dz dy dx, \quad a_0 = S_{VV00} \left[\int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_I(x, y, z) dz dy dx \right]^2, \quad A = \sqrt{8y + \Theta^2 \frac{a_3^2 - 4\Theta a_2}{a_4^2}}, \quad B = \frac{\Theta a_2}{6a_4} + \\ &+ \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q}, \quad q = \frac{\Theta^3 a_2}{24 a_4^2} \left(4a_0 - \Theta L_x L_y L_z \frac{a_1 a_3}{a_4} \right) - L_x^2 L_y^2 L_z^2 \frac{\Theta^4 a_1^2}{8 a_4^2} - \\ &- \Theta^2 \frac{a_0}{8 a_4^2} \left(4\Theta a_2 - \Theta^2 \frac{a_3^2}{a_4} \right) - \frac{\Theta^3 a_2^3}{54 a_4^3}, \quad p = \frac{a_0 \Theta^2}{3a_4} - L_x L_y L_z \Theta^3 \frac{a_1 a_3}{12 a_4^2} - \frac{\Theta a_2}{18 a_4}, \end{aligned}$$

$$\alpha_{1\Phi_I} = \frac{R_{I1}}{\Theta L_x L_y L_z} + \frac{S_{II20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_{\Phi_I}(x, y, z) dz dy dx,$$

$$\alpha_{1\Phi_V} = \frac{R_{V1}}{\Theta L_x L_y L_z} + \frac{S_{VV20}}{\Theta L_x L_y L_z} + \frac{1}{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} f_{\Phi_V}(x, y, z) dz dy dx,$$

where $R_{\rho i} = \int_0^{\Theta} (\Theta - t) \int_0^{L_x L_y L_z} \int_0^{L_x L_y L_z} k_i(x, y, z, T) I_i^j(x, y, z, t) dz dy dx dt$.

Approximations of the above concentrations of the second and higher orders we obtain framework standard iterative procedure of method of averaging of function corrections [16,17]. Framework the procedure to determine approximations with the n -th-order approximations of concentrations of dopant and radiation defects we replace the required concentrations $C(x, y, z, t)$, $I(x, y, z, t)$, $V(x, y, z, t)$, $\Phi_I(x, y, z, t)$ and $\Phi_V(x, y, z, t)$ in right sides of Eqs.(1), (4) and (6) on sums of the following sums $\alpha_{n\rho} + \rho_{n-1}(x, y, z, t)$, where $\alpha_{n\rho}$ are the average values of n -th-order approximations of concentrations of dopant and defects $\rho_{n-1}(x, y, z, t)$. The replacement leads to the following results

$$\begin{aligned}
 \frac{\partial C_2(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left(\frac{\partial C_1(x, y, z, t)}{\partial x} \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, t)]^r}{P^r(x, y, z, T)} \right\} \right] \left[1 + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} + \right. \\
 & \left. + \zeta_1 \frac{V(x, y, z, t)}{V^*} \right] D_L(x, y, z, T) \right) + \frac{\partial}{\partial y} \left(D_L(x, y, z, T) \left[1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \times \right. \\
 & \times \left. \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, t)]^r}{P^r(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, t)}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_L(x, y, z, T) \left[1 + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} + \right. \right. \\
 & \left. \left. + \zeta_1 \frac{V(x, y, z, t)}{V^*} \right] \frac{\partial C_1(x, y, z, t)}{\partial z} \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, t)]^r}{P^r(x, y, z, T)} \right\} \right) + \Omega \frac{\partial}{\partial x} \{ \nabla_s \mu_2(x, y, z, t) \times \right. \\
 & \times \left. \frac{D_{cs}}{kT} \int_0^{L_z} [\alpha_{2c} + C_1(x, y, W, t)] dW \right\} + \frac{\partial}{\partial y} \left\{ \Omega \nabla_s \mu_2(x, y, z, t) \frac{D_{cs}}{kT} \int_0^{L_z} [\alpha_{2c} + C_1(x, y, W, t)] dW \right\} + \\
 & + \frac{\partial}{\partial x} \left[\frac{D_{cs}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{cs}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{cs}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial z} \right] \quad (1d) \\
 \frac{\partial I_2(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial y} \right] - \\
 & - k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, t)] [\alpha_{2V} + V_1(x, y, z, t)] + \frac{\partial}{\partial z} \left[D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial z} \right] + \\
 & + \frac{\partial}{\partial x} \left[\frac{D_{IS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{IS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{IS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial z} \right] - \\
 & + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{IS}}{kT} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2I} + I_1(x, y, W, t)] dW \right\} - k_{I,I}(x, y, z, T) \times \\
 & \times [\alpha_{2I} + I_1(x, y, z, t)]^2 + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{IS}}{kT} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2I} + I_1(x, y, W, t)] dW \right\} \quad (3d) \\
 \frac{\partial V_2(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial y} \right] - \\
 & - k_{V,V}(x, y, z, T) [\alpha_{2V} + V_1(x, y, z, t)] [\alpha_{2V} + V_1(x, y, z, t)] + \frac{\partial}{\partial z} \left[D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial z} \right] + \\
 & + \frac{\partial}{\partial x} \left[\frac{D_{VS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{VS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{VS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, t)}{\partial z} \right] - \\
 & + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{VS}}{kT} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2V} + V_1(x, y, W, t)] dW \right\} - k_{V,V}(x, y, z, T) [\alpha_{2V} + V_1(x, y, z, t)]^2 + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{VS}}{kT} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2V} + V_1(x, y, W, t)] dW \right\}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \Phi_{I_2}(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, t)}{\partial y} \right] + \\
 & + \frac{\partial}{\partial z} \left[D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, t)}{\partial z} \right] + k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_I(x, y, z, T) I(x, y, z, t) + \\
 & + \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{IS}}}{\bar{V} k T} \frac{\partial \mu_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{IS}}}{\bar{V} k T} \frac{\partial \mu_I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{\Phi_{IS}}}{\bar{V} k T} \frac{\partial \mu_I(x, y, z, t)}{\partial z} \right] + \\
 & + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{\Phi_{IS}}}{k T} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2\Phi_I} + \Phi_{II}(x, y, W, t)] dW \right\} + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{\Phi_{IS}}}{k T} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2\Phi_I} + \Phi_{II}(x, y, W, t)] dW \right\} \quad (5d) \\
 \frac{\partial \Phi_{V_2}(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{VI}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{VI}(x, y, z, t)}{\partial y} \right] + \\
 & + \frac{\partial}{\partial z} \left[D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{VI}(x, y, z, t)}{\partial z} \right] + k_{V,V}(x, y, z, T) V^2(x, y, z, t) - k_V(x, y, z, T) V(x, y, z, t) + \\
 & + \frac{\partial}{\partial x} \left[\frac{D_{\Phi_{VS}}}{\bar{V} k T} \frac{\partial \mu_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{D_{\Phi_{VS}}}{\bar{V} k T} \frac{\partial \mu_I(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[\frac{D_{\Phi_{VS}}}{\bar{V} k T} \frac{\partial \mu_I(x, y, z, t)}{\partial z} \right] + \\
 & + \Omega \frac{\partial}{\partial x} \left\{ \frac{D_{\Phi_{VS}}}{k T} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2\Phi_V} + \Phi_{VI}(x, y, W, t)] dW \right\} + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \frac{D_{\Phi_{VS}}}{k T} \nabla_s \mu_2(x, y, z, t) \int_0^{L_z} [\alpha_{2\Phi_V} + \Phi_{VI}(x, y, W, t)] dW \right\}.
 \end{aligned}$$

Integration of the left and right sides of Eqs. (1d), (3d), (5d) gives us possibility to obtain relations for the second-order approximations of dopant and radiation defects in the following form

$$\begin{aligned}
 C_2(x, y, z, t) = & \frac{\partial}{\partial x} \left(\int_0^t D_L(x, y, z, \tau) \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \frac{\partial C_1(x, y, z, \tau)}{\partial x} \times \right. \\
 & \times \left. \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right\} d\tau + \frac{\partial}{\partial y} \left(\int_0^t \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \right. \right. \\
 & \times D_L(x, y, z, T) \left. \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, \tau)}{\partial y} d\tau \right) + \frac{\partial}{\partial z} \left(\int_0^t D_L(x, y, z, T) \times \right. \\
 & \times \left. \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, \tau)}{\partial z} d\tau \right) + \\
 & + \frac{\partial}{\partial x} \left(\int_0^t D_{LS}(x, y, z, T) \left[1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right\} \times \right.
 \end{aligned}$$

$$\begin{aligned}
 & \times \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} d\tau \Bigg) + \frac{\partial}{\partial y} \left[\int_0^t \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} \right] \left[1 + \xi_1 \frac{V(x, y, z, \tau)}{V^*} + \xi_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \times \\
 & \times D_{LS}(x, y, z, T) \left\{ 1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right\} d\tau \Bigg) + \frac{\partial}{\partial z} \left(\int_0^t D_{LS}(x, y, z, T) \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} \right. \\
 & \times \left. \left[1 + \xi_1 \frac{V(x, y, z, \tau)}{V^*} + \xi_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \right\} \left[1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right] \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} d\tau \Bigg) + \\
 & + \Omega \frac{\partial}{\partial x} \left(\int_0^t \left[1 + \xi_1 \frac{V(x, y, z, \tau)}{V^*} + \xi_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \right\} \left[1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right] \times \\
 & \times \frac{D_{LS}(x, y, z, T)}{kT} \nabla_s \mu_2(x, y, z, \tau) \int_0^{L_z} [\alpha_{2c} + C_1(x, y, W, \tau)] dW \Bigg) d\tau + \frac{\partial}{\partial y} \left(\int_0^t D_{LS}(x, y, z, T) \right. \\
 & \times \left. \nabla_s \mu_2(x, y, z, \tau) \left[1 + \xi_1 \frac{V(x, y, z, \tau)}{V^*} + \xi_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \right\} \left[1 + \xi \frac{[\alpha_{2c} + C_1(x, y, z, \tau)]^r}{P^r(x, y, z, T)} \right] \times \\
 & \times \Omega \int_0^{L_z} [\alpha_{2c} + C_1(x, y, W, \tau)] dW \Bigg) d\tau + f_c(x, y, z) \quad (1e)
 \end{aligned}$$

$$\begin{aligned}
 I_2(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial y} d\tau \right] + \\
 & + \frac{\partial}{\partial z} \left[\int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial z} d\tau \right] - \int_0^t k_{I,I}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)]^2 d\tau - \\
 & + \frac{\partial}{\partial x} \int_0^t \frac{D_{IS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_0^t \frac{D_{IS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial y} d\tau + \frac{\partial}{\partial z} \int_0^t \frac{\partial \mu_1(x, y, z, \tau)}{\partial z} \frac{D_{IS} d\tau}{\bar{V} kT} - \\
 & - \int_0^t k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)] [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau + \Omega \frac{\partial}{\partial x} \int_0^t \frac{D_{IS}}{kT} \nabla_s \mu_2(x, y, z, \tau) \times \\
 & \times \int_0^{L_z} [\alpha_{2I} + I_1(x, y, W, \tau)] dW d\tau + \Omega \frac{\partial}{\partial y} \int_0^t \frac{D_{IS}}{kT} \int_0^{L_z} [\alpha_{2I} + I_1(x, y, W, \tau)] dW \nabla_s \mu_2(x, y, z, \tau) d\tau \\
 & \times \int_0^{L_z} [\alpha_{2I} + I_1(x, y, W, \tau)] dW d\tau + \Omega \frac{\partial}{\partial z} \int_0^t \frac{D_{IS}}{kT} \int_0^{L_z} [\alpha_{2I} + I_1(x, y, W, \tau)] dW \nabla_s \mu_2(x, y, z, \tau) d\tau \quad (3e)
 \end{aligned}$$

$$\begin{aligned}
 V_2(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial y} d\tau \right] + \\
 & + \frac{\partial}{\partial z} \left[\int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial z} d\tau \right] - \int_0^t k_{V,V}(x, y, z, T) [\alpha_{2V} + V_1(x, y, z, \tau)]^2 d\tau - \\
 & + \frac{\partial}{\partial x} \int_0^t \frac{D_{VS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_0^t \frac{D_{VS}}{\bar{V} kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial y} d\tau + \frac{\partial}{\partial z} \int_0^t \frac{\partial \mu_1(x, y, z, \tau)}{\partial z} \frac{D_{VS} d\tau}{\bar{V} kT} - \\
 & - \int_0^t k_{V,V}(x, y, z, T) [\alpha_{2V} + V_1(x, y, z, \tau)] [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau + \Omega \frac{\partial}{\partial x} \int_0^t \frac{D_{VS}}{kT} \nabla_s \mu_2(x, y, z, \tau) \times
 \end{aligned}$$

$$\begin{aligned}
 & \times \int_0^{L_z} [\alpha_{2V} + V_1(x, y, W, \tau)] dW d\tau + \Omega \frac{\partial}{\partial y} \int_0^t \frac{D_{VS}}{kT} \int_0^{L_z} [\alpha_{2V} + V_1(x, y, W, \tau)] dW \nabla_s \mu_2(x, y, z, \tau) d\tau \\
 \Phi_{I2}(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, \tau)}{\partial y} d\tau \right] + \\
 & + \frac{\partial}{\partial z} \left[\int_0^t D_{\Phi I}(x, y, z, T) \frac{\partial \Phi_{II}(x, y, z, \tau)}{\partial z} d\tau \right] + \frac{\partial}{\partial x} \int_0^t \frac{D_{\Phi IS}}{V kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_0^t \frac{\partial \mu_1(x, y, z, \tau)}{\partial y} d\tau \times \\
 & \times \frac{D_{\Phi IS}}{V kT} d\tau + \Omega \frac{\partial}{\partial x} \left\{ \int_0^t \frac{D_{\Phi IS}}{kT} \nabla_s \mu_2(x, y, z, \tau) \int_0^{L_z} [\alpha_{2\Phi I} + \Phi_{II}(x, y, W, \tau)] dW d\tau \right\} + \frac{\partial}{\partial z} \int_0^t \frac{D_{\Phi IS}}{V kT} \times \\
 & \times \frac{\partial \mu_1(x, y, z, \tau)}{\partial z} d\tau + \Omega \frac{\partial}{\partial y} \left\{ \int_0^t \frac{D_{\Phi IS}}{kT} \nabla_s \mu_2(x, y, z, \tau) \int_0^{L_z} [\alpha_{2\Phi I} + \Phi_{II}(x, y, W, \tau)] dW d\tau \right\} - \\
 & - \int_0^t k_I(x, y, z, T) I(x, y, z, \tau) d\tau + \int_0^t k_{I,I}(x, y, z, T) I^2(x, y, z, \tau) d\tau + f_{\Phi I}(x, y, z) \quad (5e) \\
 \Phi_{V2}(x, y, z, t) = & \frac{\partial}{\partial x} \left[\int_0^t D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[\int_0^t D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial y} d\tau \right] + \\
 & + \frac{\partial}{\partial z} \left[\int_0^t D_{\Phi V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial z} d\tau \right] + \frac{\partial}{\partial x} \int_0^t \frac{D_{\Phi VS}}{V kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \int_0^t \frac{D_{\Phi VS}}{V kT} \times \\
 & \times \frac{\partial \mu_1(x, y, z, \tau)}{\partial y} d\tau + \Omega \frac{\partial}{\partial x} \left\{ \int_0^t \frac{D_{\Phi VS}}{kT} \nabla_s \mu_2(x, y, z, \tau) \int_0^{L_z} [\alpha_{2\Phi V} + \Phi_{V1}(x, y, W, \tau)] dW d\tau \right\} + \\
 & + \Omega \frac{\partial}{\partial y} \left\{ \int_0^t \frac{D_{\Phi VS}}{kT} \nabla_s \mu_2(x, y, z, \tau) \int_0^{L_z} [\alpha_{2\Phi V} + \Phi_{V1}(x, y, W, \tau)] dW d\tau \right\} + \frac{\partial}{\partial z} \int_0^t \frac{D_{\Phi VS}}{V kT} \frac{\partial \mu_1(x, y, z, \tau)}{\partial z} d\tau + \\
 & + \int_0^t k_{V,V}(x, y, z, T) V^2(x, y, z, \tau) d\tau - \int_0^t k_V(x, y, z, T) V(x, y, z, \tau) d\tau + f_{\Phi V}(x, y, z).
 \end{aligned}$$

We determine average values of the second-order approximations of required functions by using of the standard relation [16,17]

$$\alpha_{2\rho} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta L_x} \int_0^{L_y} \int_0^{L_z} [\rho_2(x, y, z, t) - \rho_1(x, y, z, t)] dz dy dx dt. \quad (10)$$

Substitution of the relations (1e), (3e), (5e) into relation (10) gives us possibility to obtain relations for the parameter $\alpha_{2\rho}$

$$\begin{aligned}
 \alpha_{2C} = 0, \alpha_{2\Phi I} = 0, \alpha_{2\Phi V} = 0, \alpha_{2V} = & \sqrt{\frac{(b_3 + E)^2}{4b_4^2} - 4 \left(F + \frac{\Theta a_3 F + \Theta^2 L_x L_y L_z b_1}{b_4} \right)} - \frac{b_3 + E}{4b_4}, \\
 \alpha_{2I} = & \frac{C_v - \alpha_{2V}^2 S_{VV00} - \alpha_{2V} (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) - S_{VV02} - S_{IV11}}{S_{IV01} + \alpha_{2V} S_{IV00}},
 \end{aligned}$$

$$\begin{aligned}
 \text{where } b_4 = & \frac{1}{\Theta L_x L_y L_z} S_{IV00}^2 S_{VV00} - \frac{1}{\Theta L_x L_y L_z} S_{VV00}^2 S_{II00}, \quad b_3 = -\frac{S_{II00} S_{VV00}}{\Theta L_x L_y L_z} (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) + \\
 & + \frac{1}{\Theta L_x L_y L_z} (S_{IV01} + 2S_{II10} + S_{IV01} + \Theta L_x L_y L_z) S_{IV00} S_{VV00} + \frac{S_{IV00}^2}{\Theta L_x L_y L_z} (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) -
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{S_{IV00}^2 S_{IV10}}{\Theta^3 L_x^3 L_y^3 L_z^3}, b_2 = \frac{S_{H00} S_{VV00}}{\Theta L_x L_y L_z} (S_{VV02} + S_{IV11} + C_V) - (\Theta L_x L_y L_z - 2S_{VV01} + S_{IV10})^2 + \frac{S_{IV01} S_{VV00}}{\Theta L_x L_y L_z} \times \\
 & + \frac{S_{IV01} S_{VV00}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{H10} + S_{IV01}) + S_{IV00} \frac{\Theta L_x L_y L_z + S_{IV01} + 2S_{H10} + 2S_{IV01}}{\Theta L_x L_y L_z} (2S_{VV01} + S_{IV10} + \\
 & + \Theta L_x L_y L_z) - S_{IV00}^2 \frac{C_V - S_{VV02} - S_{IV11}}{\Theta L_x L_y L_z} + \frac{C_I S_{IV00}^2}{\Theta^2 L_x^2 L_y^2 L_z^2} - \frac{2}{\Theta L_x L_y L_z} S_{IV10} S_{IV00} S_{IV01}, b_1 = S_{H00} \times \\
 & \times (2S_{VV01} + S_{IV10} + \Theta L_x L_y L_z) \frac{S_{IV11} + S_{VV02} + C_V}{\Theta L_x L_y L_z} + \frac{S_{IV01}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{H10} + S_{IV01}) (2S_{VV01} + \\
 & + S_{IV10} + \Theta L_x L_y L_z) - S_{IV00} \frac{3S_{IV01} + 2S_{H10} + \Theta L_x L_y L_z}{\Theta L_x L_y L_z} (C_V - S_{VV02} - S_{IV11}) + 2C_I S_{IV00} S_{IV01} - \frac{S_{IV10}}{\Theta L_x} \times \\
 & \times \frac{S_{IV01}^2}{L_y L_z}, b_0 = \frac{S_{H00}}{\Theta L_x L_y L_z} (S_{IV00} + S_{VV02})^2 - S_{IV01} \frac{C_V - S_{IV11} - S_{VV02}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{H10} + S_{IV01}) + \\
 & + 2C_I S_{IV01}^2 - \frac{S_{IV01}}{\Theta L_x L_y L_z} (\Theta L_x L_y L_z + 2S_{H10} + S_{IV01}) (C_V - S_{VV02} - S_{IV11}), C_I = \frac{\alpha_{II} \alpha_{IV} S_{IV00}}{\Theta L_x L_y L_z} + \\
 & + \frac{\alpha_{II}^2 S_{H00}}{\Theta L_x L_y L_z} - \frac{S_{H20} S_{H20}}{\Theta L_x L_y L_z} - \frac{\alpha_{IV} S_{IV11} S_{IV00}}{\Theta L_x L_y L_z} + \frac{\alpha_{II}^2 S_{H00}}{\Theta L_x L_y L_z} - \frac{S_{H20} S_{H20}}{\Theta L_x L_y L_z} - \frac{S_{IV11}}{\Theta L_x L_y L_z}, C_V = \alpha_{II} \alpha_{IV} \times \\
 & \times S_{IV00} + \alpha_{IV}^2 S_{VV00} - S_{VV02} - S_{IV11}, E = \sqrt[3]{8y + \Theta^2 \frac{a_3^2}{a_4^2} - 4\Theta \frac{a_2}{a_4}}, F = \sqrt[3]{\sqrt{r^2 + s^3} - r} - \sqrt[3]{\sqrt{r^2 + s^3} + r} + \\
 & + \frac{\Theta a_2}{6 a_4}, r = \frac{\Theta^3 b_2}{24 b_4^2} \left(4b_0 - \Theta L_x L_y L_z \frac{b_1 b_3}{b_4} \right) - b_0 \frac{\Theta^2}{8 b_4^2} \left(4\Theta b_2 - \Theta^2 \frac{b_3^2}{b_4} \right) - \frac{\Theta^3 b_2^3}{54 b_4^3} - L_x^2 L_y^2 L_z^2 \frac{\Theta^4 b_1^2}{8 b_4^2}, \\
 & s = -\Theta [2b_2 - 3\Theta (4b_0 b_4 - \Theta L_x L_y L_z b_1 b_3)]/36b_4.
 \end{aligned}$$

After that we determine solutions of the system of Eqs.(8). The solutions physically correspond to components of displacement vector. To determine components of displacement vector we used the method of averaging of function corrections in its standard form. By using this approach we replace the above components in right sides of Eqs. (8) by their not yet known average values α_i . The substitution leads to the following results

$$\begin{aligned}
 \rho(z) \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial t^2} &= -K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial x}, \quad \rho(z) \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial t^2} = -K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial y}, \\
 \rho(z) \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial t^2} &= -K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial z}.
 \end{aligned}$$

Integration of the left and right sides of the above equations on time t leads to final relations for the first-order approximations of components of displacement vector in the following form

$$\begin{aligned}
 u_{1x}(x, y, z, t) &= K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial x} \int_0^t \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0x}, \\
 u_{1y}(x, y, z, t) &= K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial y} \int_0^t \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0y}, \\
 u_{1z}(x, y, z, t) &= K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_0^t \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0z}.
 \end{aligned}$$

Approximations of the second-order and higher orders of components of displacement vector could be calculated by replacement of the required components in the Eqs. (8) by the following sums $\alpha_i + u_i(x, y, z, t)$ [16,17]. The replacement leads to the following result

$$\begin{aligned}
 \rho(z) \frac{\partial^2 u_{2x}(x, y, z, t)}{\partial t^2} &= \left\{ K(z) + \frac{5E(z, C)}{6[1+\sigma(z)]} \right\} \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial x^2} + \left\{ K(z) - \frac{E(z, C)}{3[1+\sigma(z)]} \right\} \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial x \partial y} + \\
 &+ \frac{E(z, C)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial z^2} \right] + \left\{ K(z) + \frac{E(z, C)}{3[1+\sigma(z)]} \right\} \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial x \partial z} - \\
 &- K(z) \beta(z) \frac{\partial T(x, y, z, t)}{\partial x}, \\
 \rho(z) \frac{\partial^2 u_{2y}(x, y, z, t)}{\partial t^2} &= \frac{E(z, C)}{2[1+\sigma(z)]} \left[\frac{\partial^2 u_{1y}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial x \partial y} \right] - \beta(z) \frac{\partial T(x, y, z, t)}{\partial y} \times \\
 &\times K(z) + \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y^2} \left\{ \frac{5E(z, C)}{12[1+\sigma(z)]} + K(z) \right\} + \frac{1}{2} \frac{\partial}{\partial z} \left\{ \frac{E(z, C)}{1+\sigma(z)} \left[\frac{\partial u_{1y}(x, y, z, t)}{\partial z} + \frac{\partial u_{1z}(x, y, z, t)}{\partial y} \right] \right\} + \\
 &+ \left\{ K(z) - \frac{E(z, C)}{6[1+\sigma(z)]} \right\} \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y \partial z} + K(z) \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial x \partial y}, \\
 \rho(z) \frac{\partial^2 u_{2z}(x, y, z, t)}{\partial t^2} &= \frac{1}{2} \left[\frac{\partial^2 u_{1z}(x, y, z, t)}{\partial x^2} + \frac{\partial^2 u_{1z}(x, y, z, t)}{\partial y^2} + \frac{\partial^2 u_{1x}(x, y, z, t)}{\partial x \partial z} + \frac{\partial^2 u_{1y}(x, y, z, t)}{\partial y \partial z} \right] \times \\
 &\times \frac{E(z, C)}{1+\sigma(z)} + \frac{\partial}{\partial z} \left\{ K(z) \left[\frac{\partial u_{1x}(x, y, z, t)}{\partial x} + \frac{\partial u_{1y}(x, y, z, t)}{\partial y} + \frac{\partial u_{1x}(x, y, z, t)}{\partial z} \right] \right\} + \frac{E(z, C)}{6[1+\sigma(z)]} \times \\
 &\times \frac{\partial}{\partial z} \left\{ \left[6 \frac{\partial u_{1z}(x, y, z, t)}{\partial z} - \frac{\partial u_{1x}(x, y, z, t)}{\partial x} - \frac{\partial u_{1y}(x, y, z, t)}{\partial y} - \frac{\partial u_{1z}(x, y, z, t)}{\partial z} \right] \right\} - \frac{\partial T(x, y, z, t)}{\partial z} \times \\
 &\times K(z) \beta(z).
 \end{aligned}$$

Integration of left and right sides of the above equations on time t leads to the following results

$$\begin{aligned}
 u_{2x}(x, y, z, t) &= \frac{1}{\rho(z)} \left\{ K(z) + \frac{5E(z, C)}{6[1+\sigma(z)]} \right\} \frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \left\{ K(z) - \frac{E(z, C)}{3[1+\sigma(z)]} \right\} \times \\
 &\times \frac{1}{\rho(z)} \frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \left[\frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta}{\partial y^2} + \frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta}{\partial z^2} \right]}{\partial x \partial y} \times \\
 &\times \frac{1}{2\rho(z)} \frac{E(z, C)}{1+\sigma(z)} + \frac{1}{\rho(z)} \frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta}{\partial x \partial z} \left\{ K(z) + \frac{E(z, C)}{3[1+\sigma(z)]} \right\} - K(z) \frac{\beta(z)}{\rho(z)} \times \\
 &\times \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta - \frac{1}{\rho(z)} \frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta}{\partial x^2} \left\{ K(z) + \frac{5E(z, C)}{6[1+\sigma(z)]} \right\} - \\
 &- \frac{1}{\rho(z)} \left\{ K(z) - \frac{E(z)}{3[1+\sigma(z)]} \right\} \frac{\partial^2 \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + K(z) \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta \times
 \end{aligned}$$

$$\begin{aligned}
 & \times \frac{\beta(z)}{\rho(z)} - \frac{1}{2\rho(z)} \frac{E(z, C)}{1+\sigma(z)} \left[\frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial z^2} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] + \\
 & - \frac{1}{\rho(z)} \frac{\partial^2}{\partial x \partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \left\{ K(z) + \frac{E(z, C)}{3[1+\sigma(z)]} \right\} + u_{0x}, \\
 u_{2y}(x, y, z, t) = & \frac{1}{2\rho(z)} \frac{E(z, C)}{1+\sigma(z)} \left[\frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \right] + \\
 & + \frac{K(z)}{\rho(z)} \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \left\{ \frac{5E(z, C)}{12[1+\sigma(z)]} + K(z) \right\} + \\
 & + \frac{1}{2\rho(z)} \frac{\partial}{\partial z} \left\{ \frac{E(z, C)}{1+\sigma(z)} \left[\frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \right\} - K(z) \frac{\beta(z)}{\rho(z)} \times \\
 & \times \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + \frac{1}{\rho(z)} \left\{ K(z) - \frac{E(z, C)}{6[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial y \partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta - \frac{E(z, C)}{1+\sigma(z)} \times \\
 & \times \frac{1}{2\rho(z)} \left\{ \frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \right\} - \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta \times \\
 & \times K(z) \frac{\beta(z)}{\rho(z)} - \frac{K(z)}{\rho(z)} \frac{\partial^2}{\partial x \partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta - \frac{1}{\rho(z)} \frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \left\{ K(z) + \right. \\
 & \left. + \frac{5E(z, C)}{12[1+\sigma(z)]} \right\} - \frac{1}{2\rho(z)} \frac{\partial}{\partial z} \left\{ \frac{E(z, C)}{1+\sigma(z)} \left[\frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \right\} - \\
 & - \frac{1}{\rho(z)} \left\{ K(z) - \frac{E(z, C)}{6[1+\sigma(z)]} \right\} \frac{\partial^2}{\partial y \partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta + u_{0y}, \\
 \frac{\partial^2 u_{2z}}{\partial t^2}(x, y, z, t) = & \frac{1}{2\rho(z)} \frac{E(z, C)}{1+\sigma(z)} \left[\frac{\partial^2}{\partial x^2} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial y^2} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta + \right. \\
 & + \frac{\partial^2}{\partial x \partial z} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial^2}{\partial y \partial z} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta \left. \right] + \frac{\partial}{\partial z} \left\{ \left[\frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \right. \right. \\
 & + \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta + \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta \left. \right] K(z) \left. \right\} \frac{1}{\rho(z)} + \frac{1}{6\rho(z)} \frac{\partial}{\partial z} \left\{ \frac{E(z, C)}{1+\sigma(z)} \times \right. \\
 & \left. \left[6 \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta - \frac{\partial}{\partial x} \int_0^\vartheta \int_0^\vartheta u_{1x}(x, y, z, \tau) d\tau d\vartheta - \frac{\partial}{\partial y} \int_0^\vartheta \int_0^\vartheta u_{1y}(x, y, z, \tau) d\tau d\vartheta - \right. \right. \\
 & \left. \left. - \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta u_{1z}(x, y, z, \tau) d\tau d\vartheta \right] \right\} - K(z) \frac{\beta(z)}{\rho(z)} \frac{\partial}{\partial z} \int_0^\vartheta \int_0^\vartheta T(x, y, z, \tau) d\tau d\vartheta + u_{0z}.
 \end{aligned}$$

In this paper we determine concentration of dopant, concentrations of radiation defects and components of displacement vector by using the second-order approximations, which were calculated by using method of averaging of function corrections. This approximation is usually good enough approximation to carry out a qualitative analysis and to obtain some quantitative results. All ob-

tained analytical results have been checked by comparison with results of numerical simulations. In this paper we used explicit difference scheme to solve Eqs.(1), (3) and (5).

3. DISCUSSION

In this section we analyzed distribution of concentration of dopant, infused (see Fig. 2) or implanted (see Fig. 3) into epitaxial layer. Annealing time is the same for all curves of these figures. Increasing of number of curves corresponds to increasing of difference between values of dopant diffusion coefficients in layers of hetero structure. The figures show, that presents of interface between layers of hetero structure gives a possibility to increase absolute value of gradient of concentration of dopant in direction, which is perpendicular to the considered interface. A consequence of this result is decreasing of the dimensions of transistors included in the considered schemes. At the same time with increasing of the considered gradient homogeneity of concentration of dopant in enriched area.

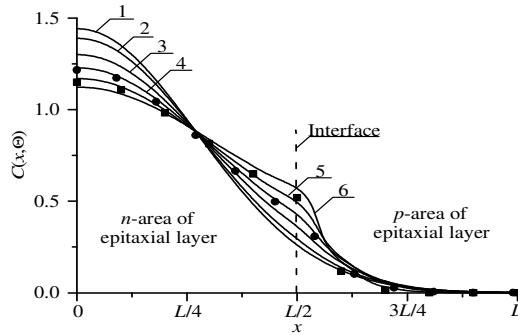


Fig.2. Distributions of concentration of infused dopant in heterostructure from Figs. 1 in direction, which is perpendicular to interface between epitaxial layer substrate. Increasing of number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate. Squares are the experimental data from [24]. Circles are the experimental data from [25]. Both experimental data have been obtained for heterostructures

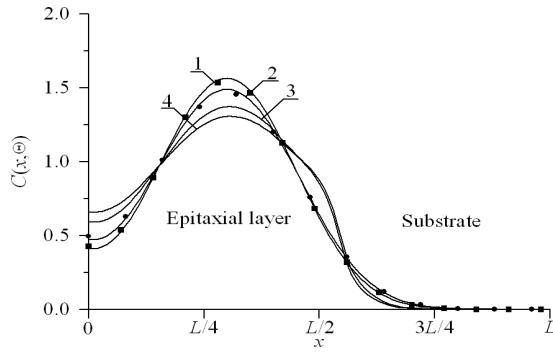


Fig.3. Distributions of concentration of implanted dopant in heterostructure from Figs. 1 in direction, which is perpendicular to the interface between epitaxial layer substrate. Curves 1 and 3 correspond to annealing time $\Theta = 0.0048(L_x^2 + L_y^2 + L_z^2)/D_0$. Curves 2 and 4 correspond to annealing time $\Theta = 0.0057(L_x^2 + L_y^2 + L_z^2)/D_0$. Curves 1 and 2 correspond to homogenous sample ($D_1/D_0 = D_0/D_2 = 1$). Curves 3 and 4 correspond to heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate ($D_1/D_0 = D_0/D_2 = 1.2$). Squares are the experimental data from [26]. Circles are the experimental data from [27]. Both experimental data have been obtained for homogenous samples

To estimate annealing time it is necessary to estimate decreasing of absolute value of gradient of concentration of dopant near interface between substrate and epitaxial layer with increasing of annealing time. Decreasing of annealing time leads to manufacturing more inhomogenous distribution of concentration of dopant (see Fig. 4 for diffusion type of doping and Fig. 5 for ion type of doping). We determine compromise value of annealing time framework recently introduced criterion [14-20]. Framework the criterion we approximate real distribution of concentration of dopant by idealized step-wise function $\psi(x,y,z)$. Further we determine the required annealing time by minimization of mean-square error

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [C(x, y, z, t) - \psi(x, y, z)] dz dy dx. \quad (15)$$

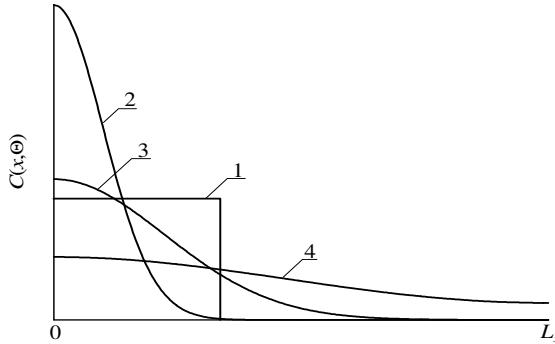


Fig. 4. Spatial distributions of dopant in heterostructure after dopant infusion. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

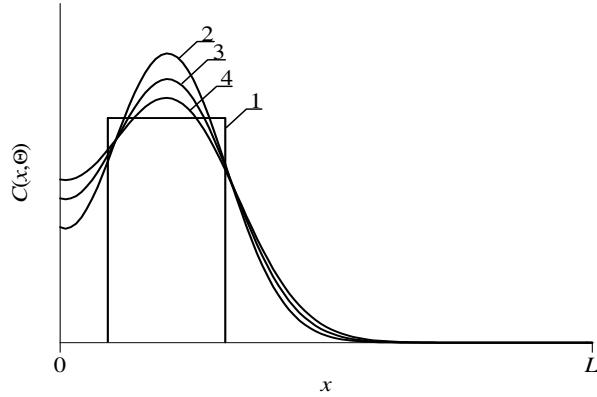


Fig. 5. Spatial distributions of dopant in heterostructure after ion implantation. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

After the minimization we obtain optimal value of annealing time Θ . Dependences of optimal values of annealing time on parameters are presented in Figs. 6 and 7 for diffusion and ion types of doping, respectively. It should be noted, that it is necessary to anneal radiation defects after ion implantation. One could find spreading of concentration of the distribution of concentration of dopant during this annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant does not achieve to the nearest interface during annealing of radiation defects, it is practical to additionally anneal the dopant. In this situation the optimal value of the additional annealing time of implanted dopant is smaller, than the annealing time of infused dopant.

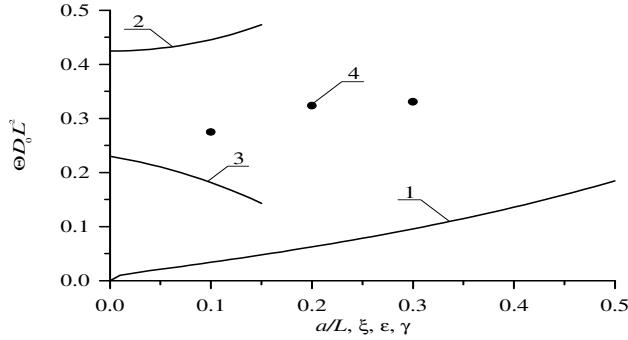


Fig.6. Optimal annealing time of infused dopant as dependences of several parameters. Curve 1 is the dependence of the considered annealing time on dimensionless thickness of epitaxial layer a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of the considered annealing time on the parameter ε for $a/L=1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of the considered annealing time on the parameter ξ for $a/L=1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of the considered annealing time on parameter γ for $a/L=1/2$ and $\varepsilon = \xi = 0$

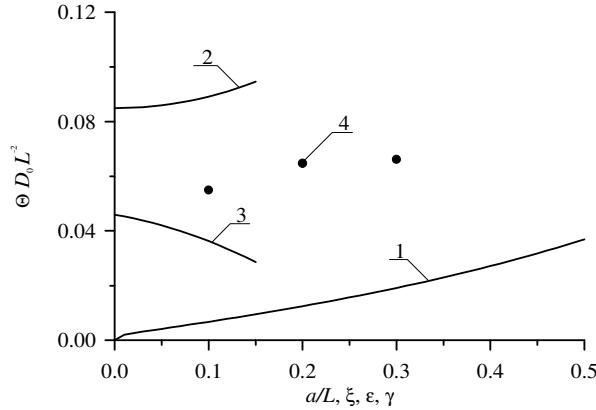


Fig.7. Optimal annealing time of implanted dopant as dependences of several parameters. Curve 1 is the dependence of the considered annealing time on dimensionless thickness of epitaxial layer a/L and $\xi = \gamma = 0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of the considered annealing time on the parameter ε for $a/L=1/2$ and $\xi = \gamma = 0$. Curve 3 is the dependence of the considered annealing time on the parameter ξ for $a/L=1/2$ and $\varepsilon = \gamma = 0$. Curve 4 is the dependence of the considered annealing time on parameter γ for $a/L=1/2$ and $\varepsilon = \xi = 0$

Next we analyzed the influence of relaxation of mechanical stress on distribution of dopant in doped areas of heterostructure. Under following condition $\varepsilon_0 < 0$ one can find compression of distribution of concentration of dopant near the interface between materials of heterostructure. Contrary (at $\varepsilon_0 > 0$) one can find spreading of distribution of concentration of dopant in this area. Accounting relaxation of mismatch-induced stress in heterostructure could leads to changing of optimal values of annealing time.

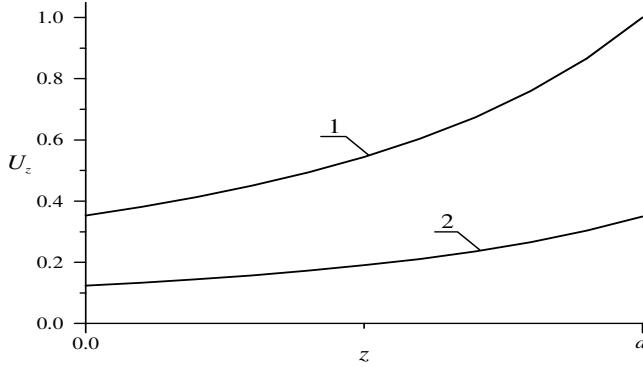


Fig.8. Normalized dependences of perpendicular to interface between materials of heterostructure components of displacement vector u_z on coordinate. Curve 1 corresponds to diffusive type of doping. Curve 2 corresponds to ion type of doping

Farther we compare relaxation of mismatch-induced stress for diffusion and ion types of doping. The Fig. 8 shows dependences of component of displacement vector, which is perpendicular to interface between materials of heterostructure, on coordinate. Curve 1 corresponds to diffusive type of doping. Curve 2 corresponds to ion type of doping. Reason of decreasing of mismatch-induced stress is radiation processing of materials during ion implantation [23].

4. CONCLUSIONS

In this paper we consider a possibility to increase density of elements in circuits NAND and AND. The circuits manufactured based on field-effect heterotransistors. As an accompanying effect we consider an approach to decrease mismatch-induced stress.

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REFERENCES

- [1] G. Volovich. Modern chips UM3Ch class D manufactured by firm MPS. Modern Electronics. Issue 2. P. 10-17 (2006).
- [2] A. Kerentsev, V. Lanin. Constructive-technological features of MOSFET-transistors. Power Electronics. Issue 1. P. 34 (2008).
- [3] A.O. Ageev, A.E. Belyaev, N.S. Boltovets, V.N. Ivanov, R.V. Konakova, Ya.Ya. Kudrik, P.M. Litvin, V.V. Milenin, A.V. Sachenko. Semiconductors. Vol. 43 (7). P. 897-903 (2009).
- [4] N.I. Volokobinskaya, I.N. Komarov, T.V. Matiouchkina, V.I. Rechetniko, A.A. Rush, I.V. Falina, A.S. Yastrebov. Au-TiBx-n-6H-SiC Schottky barrier diodes: the features of current flow in rectifying and nonrectifying contacts. Semiconductors. Vol. 35 (8). P. 1013-1017 (2001).
- [5] A. Subramaniam, K. D. Cantley, E.M. Vogel. Logic Gates and Ring Oscillators Based on Ambi-polar Nanocrystalline-Silicon TFTs. Active and Passive Electronic Components. Vol. 2013, ID 525017 (2013).
- [6] K.K. Ong, K.L. Pey, P.S. Lee, A.T.S. Wee, X.C. Wang, Y.F. Chong. Dopant distribution in the recrystallization transient at the maximum melt depth induced by laser annealing. Appl. Phys. Lett. 89 (17), 172111-172114 (2006).
- [7] H.T. Wang, L.S. Tan, E. F. Chor. Pulsed laser annealing of Be-implanted GaN. J. Appl. Phys. 98 (9), 094901-094905 (2006).
- [8] Yu.V. Bykov, A.G. Yeremeev, N.A. Zharova, I.V. Plotnikov, K.I. Rybakov, M.N. Drozdov, Yu.N. Drozdov, V.D. Skupov. Diffusion processes in semiconductor structures during microwave annealing. Radio-physics and Quantum Electronics. Vol. 43 (3). P. 836-843 (2003).

- [9] V.V. Kozlivsky. Modification of semiconductors by proton beams (Nauka, Sant-Peterburg, 2003, in Russian).
- [10] V.L. Vinetskiy, G.A. Kholodar', Radiative physics of semiconductors. ("Naukova Dumka", Kiev, 1979, in Russian).
- [11] Z.Yu. Gotra. Technology of microelectronic devices (Radio and communication, Moscow, 1991).
- [12] Y.W. Zhang, A.F. Bower. Numerical simulations of island formation in a coherent strained epi-taxial thin film system. Journal of the Mechanics and Physics of Solids. Vol. 47 (11). P. 2273-2297 (1999).
- [13] P.M. Fahey, P.B. Griffin, J.D. Plummer. Point defects and dopant diffusion in silicon. Rev. Mod. Phys. Vol. 61 (2). P. 289-388 (1989).
- [14] L.D. Landau, E.M. Lifshits. Theoretical physics. 7 (Theory of elasticity) (Physmatlit, Moscow, 2001, in Russian).
- [15] K. Zhang, Y. Li, B. Zheng. Effects of concentration-dependent elastic modulus on Li-ions diffusion and diffusion-induced stresses in spherical composition-gradient electrodes. J. Appl. Phys. Vol. 118 (10). P. 105102-1--105102-9 (2015).
- [16] Yu.D. Sokolov. About the definition of dynamic forces in the mine lifting. Applied Mechanics. Vol.1 (1). P. 23-35 (1955).
- [17] E.L. Pankratov, E.A. Bulaeva. On optimization of regimes of epitaxy from gas phase. some analytical approaches to model physical processes in reactors for epitaxy from gas phase during growth films. Reviews in Theoretical Science. Vol. 3 (2). P. 177-215 (2015).
- [18] E.L. Pankratov. Russian Microelectronics. 2007. V.36 (1). P. 33-39.
- [19] E.L. Pankratov, E.A. Bulaeva. Int. J. Nanoscience. Vol. 11 (5). P. 1250028-1--1250028-8 (2012).
- [20] E.L. Pankratov, E.A. Bulaeva. An approach to decrease dimentions of logical elements based on bipolar transistor. Int. J. Comp. Sci. Appl. Vol. 5 (4). P. 1-18 (2015).
- [21] E.L. Pankratov, E.A. Bulaeva. On optimization of manufacturing of field-effect heterotransistors with several channels. Nano Science and Nano Technology: An Indian Journal. Vol. 9 (4). P. 43-60 (2015).
- [22] E.L. Pankratov, E.A. Bulaeva. Optimization of manufacturing of emitter-coupled logic to decrease surface of chip. International Journal of Modern Physics B. Vol. 29 (5). P. 1550023-1-1550023-12 (2015).
- [23] E.L. Pankratov, E.A. Bulaeva. Decreasing of mechanical stress in a semiconductor heterostruc-ture by ra-diation processing. Journal of Computational and Theoretical Nanoscience. Vol. 11 (1). P. 91-101 (2014).
- [24] T.I. Voronina, T.S. Lagunova, S.S. Kizhaev, S.S. Molchanov, B.V. Pushnyi, Yu. P. Yakovlev. MOCVD growth and Mg-doping of InAs layers. Semiconductors. Vol. 38. P. 556 (2004).
- [25] G. Masse, K. Djessas. p-n-junctions in (In,Se)/Cu(In,Ga)(Se,S)2 photovoltaic systems. J. Appl. Phys. Vol. 94. P. 6985 (2003).
- [26] T. Ahlgren, J. Likonen, J. Slotte, J. Räisänen, M. Rajatore, J. Keinonen. Concentration dependent and in-dependent Si diffusion in ion-implanted GaAs. Phys. Rev. B. Vol. 56 (8). P. 4597 (1997).
- [27] T. Noda. Indium segregation to dislocation loops induced by ion implantation damage in silicon. J. Appl. Phys. Vol. 93. P. 1428 (2003).

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