# OPTIMIZATION OF MANUFACTURING OF LOGICAL ELEMENTS "AND" MANUFACTURED BY USING FIELD-EFFECT HETEROTRANSISTOR TO DECREASE THEIR DIMENSIONS

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***ABSTRACT***

*In this paper we introduce an approach to decrease dimensions of logical elements "AND" based on field-effect heterotransistors. Framework the approach one shall consider a heterostructure with specific structure. Several specific areas of the heterostructure should be doped by diffusion or ion implantation. Both types of doping should be optimized.*

***KEYWORDS***

*Field-effect heterotransystors, logical elements "And", decreasing of dimensions, optimization of manufacturing.*

## 1.INTRODUCTION

Currently density of elements of integrated circuits and their performance intensively increasing. Simultaneously with increasing of the density of the elements of integrated circuit their dimensions decreases. One way to decrease dimensions of these elements of these integrated circuit is manufacturing of these elements in thin-film heterostructures [1-4]. An alternative approach to decrease dimensions of the elements of integrated circuits is using laser and microwave types annealing [5-7]. Both types of annealing (laser and microwave) gives a possibility to obtain inhomogeneous distribution of temperature. Inhomogeneity of temperature leads to inhomogeneity of all temperature-dependent parameters (diffusion coefficient and other) due to the Arrhenius law. The inhomogeneity of properties of materials during doping gives a possibility to decrease dimensions of elements of integrated circuits. Changing of properties of electronic materials could be obtain by using radiation processing of these materials [8,9].

In this paper we consider logical element "AND" based on field-effect transistors described in Ref. [10] (see Fig.1). We assume, that the considered element has been manufactured in heterostructure from Fig. 1. The heterostructure consist of a substrate and an epitaxial layer. The epitaxial layer includes into itself several sections manufactured by using another materials. The sections should be doped for generation into these sections required type of conductivity (*n* or *p*). We consider two types of doping: diffusion of dopant and implantation of ions of dopants. Framework this paper we analyzed redistribution of dopant during annealing of dopant and/ or radiation defects to formulate conditions for decreasing of dimensions of the considered element "AND".

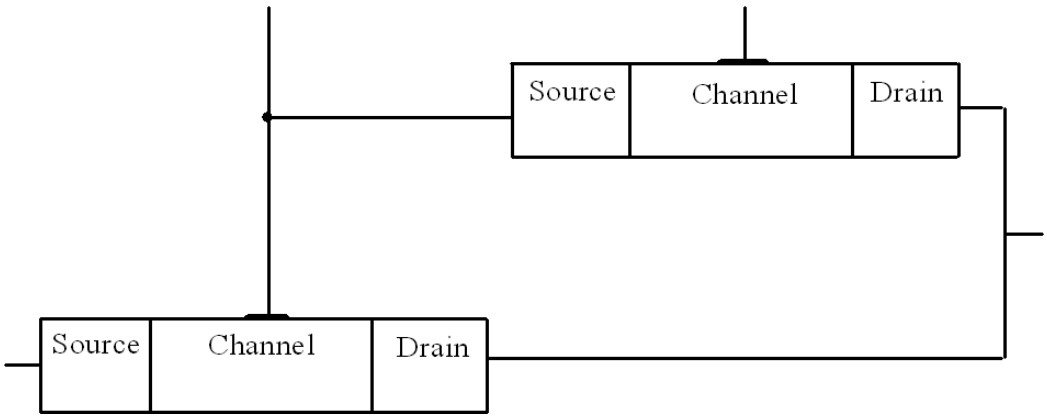


Fig. 1*a*. Structure of element "OR". View from top

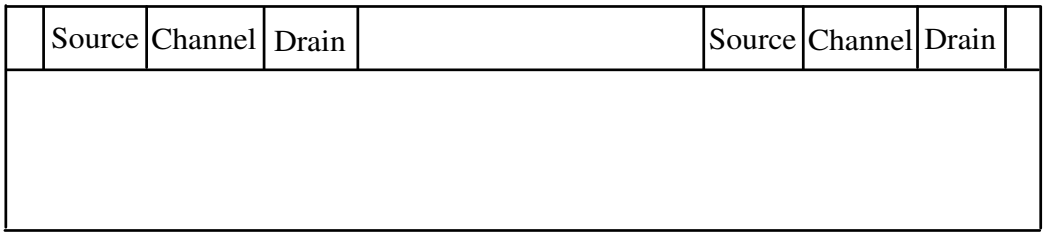


Fig. 1*b*. Heterostructure with two layers and sections in the epitaxial layer

## 2.METHOD OF SOLUTION

To solve our aim we shall analyze spatio-temporal distribution of concentration of dopant. The distribution has been determined by solving the following boundary problem

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_C\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_C\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_C\frac{\partial C(x,y,z,t)}{\partial z}\right]. \quad (1)$$

Boundary and initial conditions for the equations are

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,\ C(x,y,z,0)=f(x,y,z). \quad (2)$$

Function $C(x,y,z,t)$ describes the spatio-temporal distribution of concentration of dopant; $T$ is the temperature of annealing; $D_C$ is the dopant diffusion coefficient. Value of dopant diffusion coefficient will be different in different materials and will be changed with changing of temperature of annealing (with account Arrhenius law). The value also depends on concentrations of dopant and radiation defects. All above dependences could be accounted by the following relation [9, 11,12]

$$D_C=D_L(x,y,z,T)\left[1+\xi\frac{C^{\gamma}(x,y,z,t)}{P^{\gamma}(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]. \quad (3)$$

The function $D_L(x,y,z,T)$ gives a possibility to take into account the spatial and temperature dependences of dopant diffusion coefficient (due to presents several layers in heterostructure and Arrhenius law). The function $P(x,y,z,T)$ describes the limit of solubility of dopant. The parameter $\gamma \in [1,3]$ describes quantity of charged defects, which were interacted (in average) with atoms of dopant [11]. The function $V(x,y,z,t)$ describes the spatio-temporal distribution of concentration of radiation vacancies. The parameter $V^*$ describes the equilibrium distribution of concentration of vacancies. It should be noted, that using diffusion type of doping did not generation radiation defects. In this situation $\zeta_1 = \zeta_2 = 0$. We determine spatio-temporal distributions of concentrations of radiation defects by solving the following system of equations [9,12]

$$\frac{\partial I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[ D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right] - k_{I,I}(x,y,z,T)\times$$

$$\times I^2(x,y,z,t) + \frac{\partial}{\partial z}\left[ D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right] - k_{I,V}(x,y,z,T)\, I(x,y,z,t)\, V(x,y,z,t) \qquad (4)$$

$$\frac{\partial V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[ D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right] - k_{V,V}(x,y,z,T)\times$$

$$\times\ V^2(x,y,z,t) + \frac{\partial}{\partial z}\left[ D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right] - k_{I,V}(x,y,z,T) I(x,y,z,t)\ V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial \rho(x,y,z,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial \rho(x,y,z,t)}{\partial x}\right|_{x=L_x} = 0,\ \left.\frac{\partial \rho(x,y,z,t)}{\partial y}\right|_{y=0} = 0,\ \left.\frac{\partial \rho(x,y,z,t)}{\partial y}\right|_{y=L_y} = 0,$$

$$\left.\frac{\partial \rho(x,y,z,t)}{\partial z}\right|_{z=0} = 0,\ \left.\frac{\partial \rho(x,y,z,t)}{\partial z}\right|_{z=L_z} = 0,\ \rho(x,y,z,0) = f_\rho(x,y,z). \qquad (5)$$

Here $\rho = I, V$. The function $I(x,y,z,t)$ describes variation of distribution of concentration of radiation interstitials in space and time. The function $D_\rho(x,y,z,T)$ describes dependences of the diffusion coefficients of point radiation defects on spatial coordinates and temperature. Terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation divacancies and diinterstitials; $k_{I,V}(x,y,z,T)$ is the parameter of recombination of point radiation defects; $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of generation of simplest complexes of point radiation defects.

We determine concentrations of divacancies $\Phi_V(x,y,z,t)$ and dinterstitials $\Phi_I(x,y,z,t)$ as functions of space and time by solving the following system of equations [9,12]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[ D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right] +$$

$$+ \frac{\partial}{\partial z}\left[ D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right] + k_{I,I}(x,y,z,T) I^2(x,y,z,t) - k_I(x,y,z,T) I(x,y,z,t) \qquad (6)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[ D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}\left[ D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right] +$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t).$$

Boundary and initial conditions for these equations are

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_\rho(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\ \Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z). \quad (7)$$

Here $D_{\Phi\rho}(x,y,z,T)$ are the diffusion coefficients of the above complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of these complexes.

We used method of averaging of function corrections [13] with decreased quantity of iteration steps [14] to determine distributions of concentrations of dopant and radiation defects in space and time. Framework the initial step of the approach we consider solutions of linear Eqs. (1), (4) and (6) with averaged values of diffusion coefficients $D_{0L}$, $D_{0I}$, $D_{0V}$, $D_{0\Phi I}$, $D_{0\Phi V}$:

$$C_1(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nC}(t),$$

$$I_1(x,y,z,t)=\frac{F_{0I}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nI}c_n(x)c_n(y)c_n(z)e_{nI}(t),$$

$$V_1(x,y,z,t)=\frac{F_{0C}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{nC}c_n(x)c_n(y)c_n(z)e_{nV}(t),$$

$$\Phi_{I1}(x,y,z,t)=\frac{F_{0\Phi_I}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{n\Phi_I}c_n(x)c_n(y)c_n(z)e_{n\Phi_I}(t),$$

$$\Phi_{V1}(x,y,z,t)=\frac{F_{0\Phi_V}}{L_xL_yL_z}+\frac{2}{L_xL_yL_z}\sum_{n=1}^{\infty}F_{n\Phi_V}c_n(x)c_n(y)c_n(z)e_{n\Phi_V}(t),$$

where $e_{n\rho}(t)=\exp\left[-\pi^2n^2D_{0\rho}t\left(\frac{1}{L_x^2}+\frac{1}{L_y^2}+\frac{1}{L_z^2}\right)\right]$, $F_{n\rho}=\int_0^{L_x}c_n(u)\int_0^{L_y}c_n(v)\int_0^{L_z}c_n(v)f_\rho(u,v,w)dwdvdu$, $c_n(\chi)=\cos(\pi n\chi/L_\chi)$. We consider the above solutions as initial-order approximations of concentrations of dopant and radiation defects.

Approximations of concentrations of dopant and radiation defects with the second and higher orders could be determine framework standard iterative procedure [13,14]. The procedure based on replacement of the functions $C(x,y,z,t)$, $I(x,y,z,t)$, $V(x,y,z,t)$, $\Phi_I(x,y,z,t)$, $\Phi_V(x,y,z,t)$ in the right sides of the Eqs. (1), (4) and (6) on the following sums $\alpha_{n\rho}+\rho_{n-}$

$_1(x,y,z,t)$. Framework the standard iterative procedure we obtain equations for the second-order approximations of concentrations of dopant and radiation defects

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\right.$$

$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$

$$\left.\times\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,t)}{\partial y}\right)+\frac{\partial}{\partial z}\left(D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\times\right.$$

$$\left.\times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right) \tag{8}$$

$$\begin{cases}\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+\\ +\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-[\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)]\times\\ \times k_{I,V}(x,y,z,T)-k_{I,I}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,t)]^2\\ \frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+\\ +\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-[\alpha_{2I}+I_1(x,y,z,t)][\alpha_{2V}+V_1(x,y,z,t)]\times\\ \times k_{I,V}(x,y,z,T)-k_{V,V}(x,y,z,T)[\alpha_{2V}+V_1(x,y,z,t)]^2\end{cases} \tag{9}$$

$$\begin{cases}\frac{\partial\Phi_{I2}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\times\right.\\ \left.\times\frac{\partial\Phi_{I1}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)\times\\ \times I^2(x,y,z,t)-k_I(x,y,z,T)I(x,y,z,t)\\ \frac{\partial\Phi_{V2}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\times\right.\\ \left.\times\frac{\partial\Phi_{I1}(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)\times\\ \times V^2(x,y,z,t)-k_V(x,y,z,T)V(x,y,z,t)\end{cases} \tag{10}$$

Integration of the left and right sides of Eqs.(8)-(10) gives us possibility to obtain relations for the second-order approximations of concentrations of dopant and radiation defects in final form

$$C_2(x,y,z,t)=\frac{\partial}{\partial x}\left(\int_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\right.$$

$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial x}\right)d\tau+\frac{\partial}{\partial y}\left(\int_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times\right.$$

$$\left.\times D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial y}d\tau\right)+f_C(x,y,z)+\frac{\partial}{\partial z}\left(\int_0^t D_L(x,y,z,T)\times\right.$$

$$\left.\times\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,\tau)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\frac{\partial C_1(x,y,z,\tau)}{\partial z}d\tau\right) \quad (8a)$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$+\frac{\partial}{\partial z}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{I,I}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)]^2d\tau+$$

$$+f_I(x,y,z)-\int_0^t k_{I,V}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]d\tau \quad (9a)$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$\times\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\Bigg]+\frac{\partial}{\partial z}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{V,V}(x,y,z,T)\,[\alpha_{2I}+V_1(x,y,z,\tau)]^2d\tau+$$

$$+f_V(x,y,z)-\int_0^t k_{I,V}(x,y,z,T)[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]d\tau$$

$$\Phi_{I2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{I1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_{I,I}(x,y,z,T)\,I^2(x,y,z,\tau)d\tau+$$

$$+f_{\Phi_I}(x,y,z)-\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau \quad (10a)$$

$$\Phi_{V2}(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{V1}(x,y,z,\tau)}{\partial y}d\tau\right]+$$

$$+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial z}d\tau\right]+\int_0^t k_{V,V}(x,y,z,T)\,V^2(x,y,z,\tau)d\tau+$$

$$+f_{\Phi_V}(x,y,z)-\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)d\tau$$

Average values of the considered approximations have been determined by the following relations [13,14]

$$\alpha_{2\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)]dz\,dy\,dx\,dt\,. \tag{11}$$

Substitution of approximations (8*a*)-(10*a*) into the previous relation gives the possibility to obtain relations for the average values $\alpha_{2\rho}$ in the following final form

$$\alpha_{2C}=\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)dz\,dy\,dx\,, \tag{12}$$

$$\alpha_{2I}=\frac{1}{2A_{II00}}\left\{(1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00})^2-4A_{II00}\left[\alpha_{2V}A_{IV10}-A_{II20}+A_{IV11}-\right.\right.$$

$$\left.\left.-\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)dz\,dy\,dx\right]\right\}^{\frac{1}{2}}-\frac{1+A_{IV01}+A_{II10}+\alpha_{2V}A_{IV00}}{2A_{II00}} \tag{13a}$$

$$\alpha_{2V}=\frac{1}{2B_4}\sqrt{\frac{(B_3+A)^2}{4}-4B_4\left(y+\frac{B_3 y-B_1}{A}\right)}-\frac{B_3+A}{4B_4}\,, \tag{13b}$$

Here $A_{abij}=\frac{1}{\Theta L_x L_y L_z}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{a,b}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)dz\,dy\,dx\,dt$, $B_4=A_{IV00}^2A_{IV00}^2-$

$-2\left(A_{IV00}^2-A_{II00}A_{VV00}\right)^2$, $B_3=A_{IV00}A_{IV00}^2+A_{IV01}A_{IV00}^3+A_{IV00}A_{II10}A_{IV00}^2-4\left(A_{IV00}^2-A_{II00}A_{VV00}\right)\left[A_{IV01}\times\right.$

$\left.\times 2A_{IV00}+2A_{IV00}(1+A_{IV01}+A_{II10})-2A_{II00}(A_{IV10}+A_{VV10}+1)\right]-4A_{IV10}A_{II00}A_{IV00}^2+2A_{IV00}A_{IV01}A_{IV00}^2$,

$B_2=A_{IV00}^2\left\{(1+A_{IV01}+A_{II10})^2+A_{IV00}^2A_{IV01}^2+2A_{IV00}A_{IV00}(A_{IV00}+A_{IV00}A_{IV01}+A_{IV00}A_{II10}-4A_{IV10}A_{II00})-\right.$

$\left.-4A_{II00}\left[A_{IV11}-A_{II20}-\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)dz\,dy\,dx\right]\right\}\left\{\left[2A_{IV01}A_{IV00}+2A_{IV00}(1+A_{IV01}+A_{II10})-\right.\right.$

$\left.\left.-2A_{II00}(A_{IV10}+1+A_{VV10})\right]^2+2\left[A_{IV01}(1+A_{IV01}+A_{II10})+\frac{2}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_V(x,y,z)dz\,dy\,dx-2A_{II00}\times\right.$

$\left.\times(A_{VV20}-A_{IV11})+A_{IV01}(1+A_{IV01}+A_{II10})\right]\left[2A_{IV00}(1+A_{IV01}+A_{II10})+2A_{IV01}A_{IV00}-2(A_{IV10}+1+A_{VV10})\times\right.$

$\left.\times A_{II00}\right]\right\}$, $B_1=2A_{IV00}A_{IV01}(1+A_{IV01}+A_{II10})^2-8\left[A_{IV11}-A_{II20}-\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)dz\,dy\,dx\right]+$

$+A_{IV00}A_{IV01}A_{II00}+A_{IV01}^2(A_{IV00}+A_{IV00}A_{IV01}+A_{IV00}A_{II10}-4A_{IV10}A_{II00})-2[A_{IV01}(1+A_{IV01}+A_{II10})+$

$$+\frac{2A_{II00}}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx - 2A_{II00}\left(A_{VV20}-A_{IV11}\right)+A_{IV01}\left(1+A_{IV01}+A_{II10}\right)\Big]\Big[2A_{IV00}\left(1+A_{IV01}+\right.$$

$$\left.+A_{II10}\right)-2\left(A_{IV10}+A_{VV10}+1\right)A_{II00}+2A_{IV01}A_{IV00}\Big],\ B_0=4\Bigg[\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx - A_{IV11}+$$

$$+A_{II20}\Big]A_{II00}A_{IV01}^2+A_{IV01}^2\left(A_{IV01}+A_{II10}+1\right)^2-\left[A_{IV01}\left(1+A_{IV01}+A_{II10}\right)-2A_{II00}\left(A_{VV20}-A_{IV11}\right)+A_{IV01}\times\right.$$

$$\left.\times\left(1+A_{IV01}+A_{II10}\right)+\frac{2A_{II00}}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_V(x,y,z)\,dz\,dy\,dx\right]^2,\ y=\sqrt[3]{\sqrt{q^2+p^3}-q}-\sqrt[3]{\sqrt{q^2+p^3}+q}+\frac{B_2}{6},$$

$$q=\left(2B_1B_3-8B_0\right)\frac{B_2}{48}+\frac{B_2^3}{216}+\frac{B_0\left(4B_2-B_3^2\right)-B_1^2}{8},\ p=\frac{1}{72}\left[3\left(2B_1B_3-8B_0\right)-2B_2^2\right],\ A=\sqrt{8y+B_3^2-4B_2},$$

$$\alpha_{2\Phi_I}=A_{II20}-\frac{1}{\Theta L_xL_yL_z}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_I(x,y,z,T)I(x,y,z,t)\,dz\,dy\,dx\,dt+$$

$$+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi I}(x,y,z)\,dz\,dy\,dx \quad (14)$$

$$\alpha_{2\Phi_V}=A_{VV20}-\frac{1}{\Theta L_xL_yL_z}\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} k_V(x,y,z,T)V(x,y,z,t)\,dz\,dy\,dx\,dt+$$

$$+\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_{\Phi V}(x,y,z)\,dz\,dy\,dx.$$

After the substitution we obtain the equation for parameter $\alpha_{2C}$ for any value of parameter $\gamma$. We analyzed distributions of concentrations of dopant and radiation defects in space and time by using the second-order approximations framework the method of averaged of function corrections. The obtained analytical results have been checked by comparison with results of numerical simulation.

## 3.DISCUSSION

In this section we analyzed the spatio-temporal distribution of concentration of dopant in the considered heterostructure during annealing. Figs. 2 shows spatial distributions of concentrations of dopants infused (Fig. 2*a*) or implanted (Fig. 2*b*) in epitaxial layer. Value of annealing time is equal for all distributions framework every figure 2*a* and 2*b*. Numbers of curves increased with increasing of difference between values of dopant diffusion coefficients in layers of heterostructure. The figures show that presents of interface between layers of heterostructure gives us possibility to increase absolute value of gradient of concentration of dopant in direction, which is perpendicular to the interface. We obtain increasing of absolute value of the gradient in neighborhood of the interface. Due to the increasing one can obtain decreasing dimensions of transistors, which have been used in the element "AND". At the same time it will be increased homogeneity of concentration of dopant in enriched area.

To choose annealing time it should be accounted decreasing of absolute value of gradient of concentration of dopant in neighborhood of interface between substrate and epitaxial layer with increasing of annealing time. Decreasing of value of annealing time leads to decreasing of homogeneity of concentration of dopant in enriched area (see Fig. 3*a* for diffusion doping of materials and Fig. 3*b* for ion doping of materials). Let us determine compromise value of annealing time framework recently introduced criteria [15-20]. Framework the criteria we approximate real distributions of concentration of dopant by ideal rectangle distribution $\psi(x,y,z)$. Farther we determine compromise value of annealing time by minimization of the mean-squared error

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[ C(x,y,z,\Theta) - \psi(x,y,z) \right] dz\, dy\, dx . \tag{8}$$

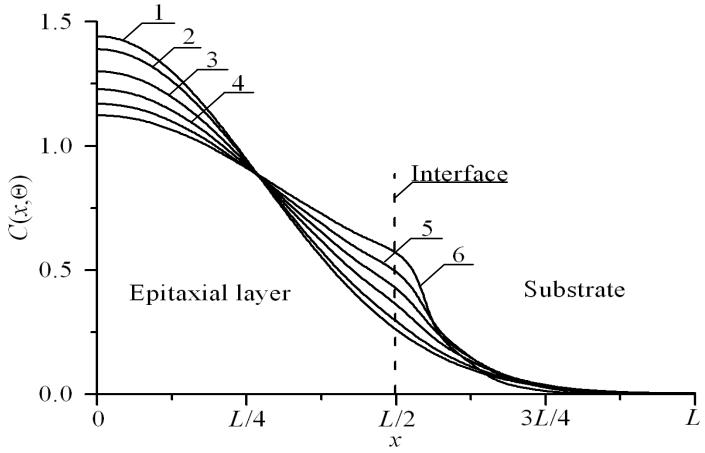


Fig.2*a*. Distributions of concentration of infused dopant near interface between layers of heterostructure for the case, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate, and for the same annealing time. Numbers of curves increased with increasing of difference between values of dopant diffusion coefficient in layers of heterostructure

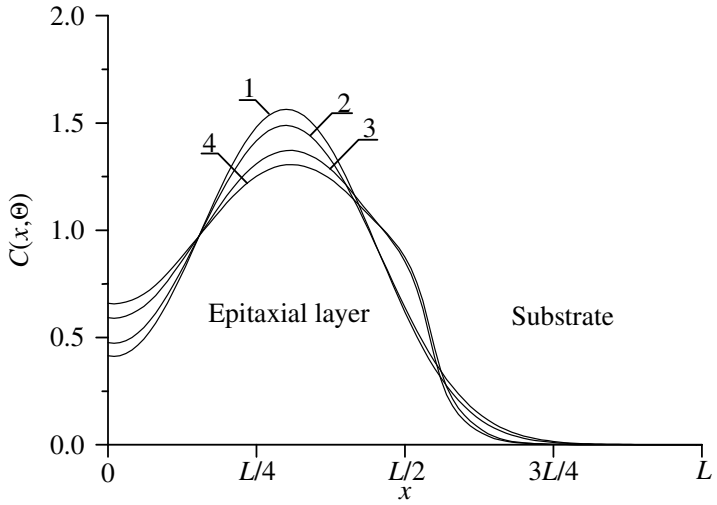

Fig.2*b*. Distributions of concentration of implanted dopant near interface between layers of heterostructure for the case, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate and for two annealing times: $\Theta = 0.0048$ $(L_x^2+L_y^2+L_z^2)/D_0$ (for curves 1 and 3) and $\Theta = 0.0057(L_x^2+L_y^2+L_z^2)/D_0$ (for curves 2 and 4). Numbers of curves increased with increasing of difference between values of dopant diffusion coefficient in layers of heterostructure

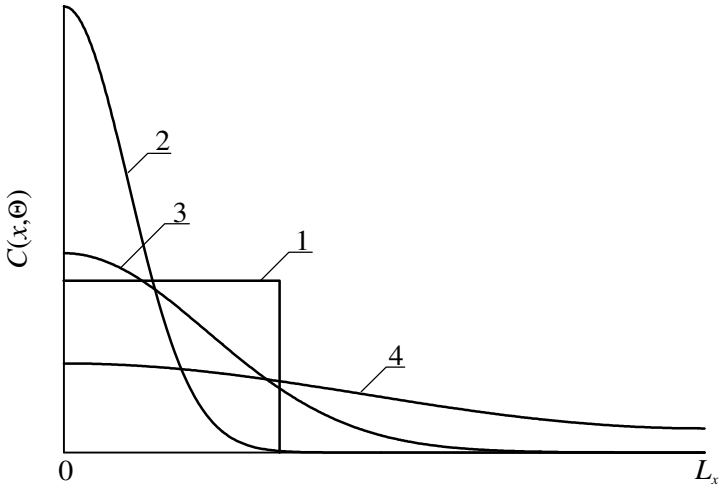


Fig. 3*a*. Spatial distributions of infused dopant in heterostructure after for different values of annealing time. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of concentration of dopant. Number of curves increases with increasing of value of annealing time

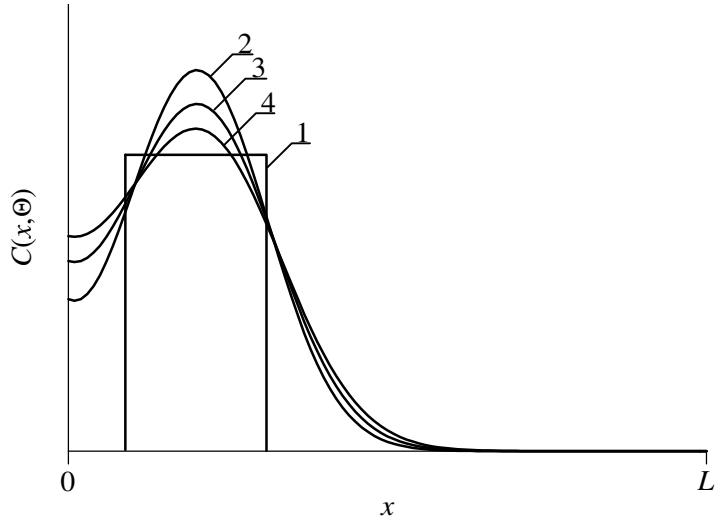


Fig. 3*b*. Spatial distributions of implanted dopant in heterostructure after for different values of annealing time. Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of concentration of dopant. Number of curves increases with increasing of value of annealing time.

We show dependences of optimal annealing time on parameters on Figs. 4. The Fig. 4*a* the show dependences for diffusion type of doping. The Fig. 4*b* the show dependences for ion type of doping. It should be noted, that one shall anneal radiation defects after ion implantation. One could

find spreading of concentration of distribution of dopant during this annealing. It will be better to optimize parameters of technological process so, that the dopant achieves appropriate interfaces between materials of heterostructure during the annealing of radiation defects. In the case, when one has no possibility to make above optimization and dopant did not achieves any interfaces during annealing of radiation defects, one shall to make additional annealing of the dopant. Optimal value of the additional annealing time is smaller, than analogous annealing time of infused dopant. At the same time ion type of doping gives us possibility to decrease mismatch-induced stress in heterostructure [21].

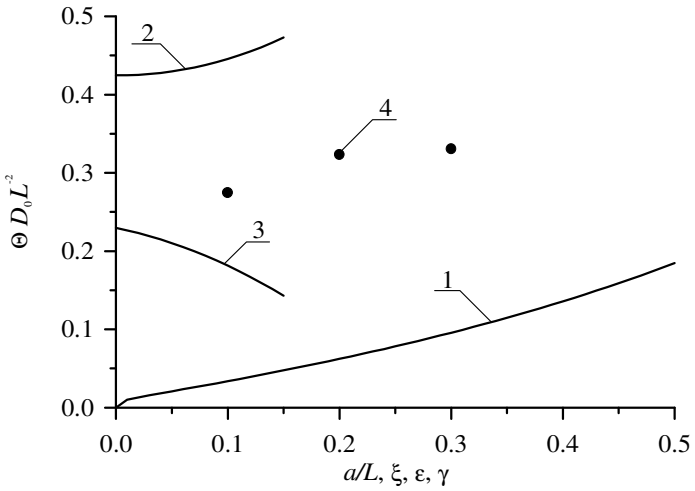


Fig.4*a*. Dependences of dimensionless optimal annealing time of infused dopant. Curve 1 describes dimensionless optimal annealing time as the function of the relation $a/L$ for $\xi=\gamma=0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes dimensionless optimal annealing time as the function of the parameter $\varepsilon$ for $a/L=1/2$ and $\xi=\gamma=0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 3 describes dimensionless optimal annealing time as the function of the parameter $\xi$ for $a/L=1/2$ and $\varepsilon=\gamma=0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 4 describes dimensionless optimal annealing time as the function of the parameter $\gamma$ for $a/L=1/2$ and $\varepsilon=\xi=0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure

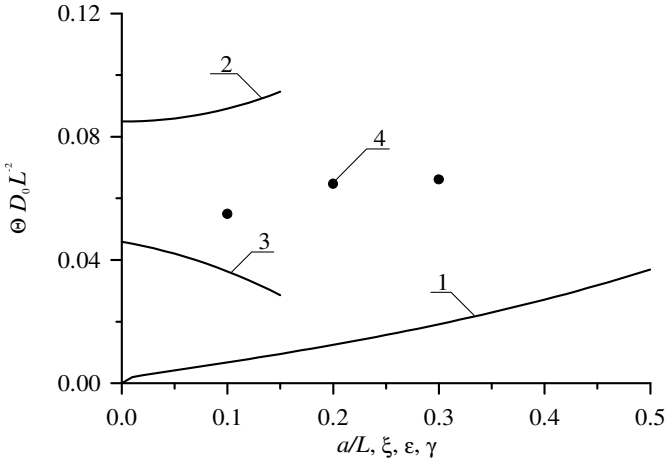

Fig.4*b*. Dependences of dimensionless optimal annealing time of implanted dopant. Curve 1 describes dimensionless optimal annealing time as the function of the relation *a/L* for $\xi = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 describes dimensionless optimal annealing time as the function of the parameter $\varepsilon$ for *a/L*=1/2 and $\xi = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 3 describes dimensionless optimal annealing time as the function of the parameter $\xi$ for *a/L*=1/2 and $\varepsilon = \gamma = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 4 describes dimensionless optimal annealing time as the function of the parameter $\gamma$ for *a/L*=1/2 and $\varepsilon = \xi = 0$ and for equal to each other values of dopant diffusion coefficient in all parts of heterostructure

# 4. CONCLUSIONS

In this paper we model redistribution of infused and implanted dopants during manufacture logical elements “OR” based on field-effect heterotransistors. Several recommendations to optimize manufacture the heterotransistors have been formulated. Analytical approach to model diffusion and ion types of doping with account concurrent changing of parameters in space and time has been introduced. At the same time the approach gives us possibility to take into account nonlinearity of doping processes.

# ACKNOWLEDGEMENTS

This work is supported by the agreement of August 27, 2013 № 02.B.49.21.0003 between The Ministry of education and science of the Russian Federation and Lobachevsky State University of Nizhni Novgorod and educational fellowship for scientific research of Government of Russian and of Nizhny Novgorod State University of Architecture and Civil Engineering.

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