

# CHAOS CONTROL VIA ADAPTIVE INTERVAL TYPE-2 FUZZY NONSINGULAR TERMINAL SLIDING MODE CONTROL

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## ABSTRACT

*In this paper, a novel robust adaptive type-2 fuzzy nonsingular sliding mode controller is proposed to stabilize the unstable periodic orbits of uncertain perturbed chaotic system with internal parameter uncertainties and external disturbances. This letter is assumed to have an affine form with unknown mathematical model, the type-2 fuzzy system is used to overcome this constraint. A global nonsingular terminal sliding mode manifold is proposed to eliminate the singularity problem associated with normal terminal sliding mode control. The proposed control law can drive system tracking error to converge to zero in finite time. The adaptive type-2 fuzzy system used to model the unknown dynamic of system is adjusted on-line by adaptation law deduced from the stability analysis in Lyapunov sense. Simulation results show the good tracking performances, and the efficiency of the proposed approach.*

## KEYWORDS

*Chaotic System, Type-2 Fuzzy Logic System, Nonsingular Terminal Sliding Mode Control, Lyapunov Stability.*

## 1. INTRODUCTION

Chaos is a particular case of nonlinear dynamics that has some specific characteristics such as extraordinary sensitivity to initial conditions and system parameter variations. The study of chaos can be introduced in several applications as: medical field, fractal theory, electrical circuits and secure communication [1]. Nowadays, the scientific community has identified two problems in chaos control: suppression and synchronization. The chaos suppression problem can be defined as the stabilization of unstable periodic orbits (UPO's) of a chaotic attractor in equilibrium points or periodic orbits with period  $n$  embedded into the chaotic attractor [2]. Many nonlinear control techniques have been applied for chaos elimination and chaos synchronization such as linear and nonlinear control techniques based on feedback [3-6], variable structure control [7-8], nonlinear control [9-11], active control [12-14], backstepping design [15-17], fuzzy logic control [18-19], and adaptive control [20-21].

Unfortunately, most of the above approaches mentioned have not considered the uncertainties and unknown parameters of the chaotic system, internal and external disturbances. Then, a useful and effective control scheme to deal with uncertainties, time varying properties, nonlinearities and bounded external disturbances is the sliding mode control (SMC). Since then, different controllers based on sliding mode control schemes have been proposed to control chaotic systems [22-23]

However, its major drawback in practical applications is the chattering problem. A lot of works have proceeded to solve this problem by using adaptive control [24-26], intelligent approach [27-29], and higher order sliding mode control [30]. In general, the sliding surface is designed as a

linear dynamic equation  $s = \dot{e} + ce$ . However, the linear sliding surface can only guarantee the asymptotic error convergence in the sliding mode, i.e., the output error cannot converge to zero in finite time. The terminal sliding mode TSM has a nonlinear surface  $s = \dot{e} + \beta e^{q/p}$ , while reaching the terminal sliding mode, the system tracking error can be converged to zero in finite time. Furthermore, TSM controller design methods have a singularity problem. Moreover, the known bounds of uncertainties is required. Based on TSM, some nonsingular terminal sliding mode (NTSM) control systems have been proposed to avoid the singularity in TSM [31-33].

The objective of this paper is to force the  $n$ -dimensional chaotic system to a desired state even if it has uncertainties system, external and internal disturbances, by incorporation the fuzzy type-2 approach and nonsingular terminal sliding mode (NTSM) control. We introduced an adaptive type-2 fuzzy system for model the unknown dynamic of system, and we use boundary layer method to avoid a chattering phenomenon.

The organization of this paper is as follows. After a description of system and problem formulation in section II, the adaptive type-2 fuzzy nonsingular terminal sliding mode control scheme is presented in section III. Simulation example demonstrate the efficiently of the proposed approach in section IV. Finally, section V gives the conclusions of the advocated design methodology.

## 2. DESCRIPTION OF SYSTEM AND PROBLEM FORMULATION

Consider  $n$ -order uncertain chaotic system which has an affine form:

$$\begin{cases} \dot{x}_i = x_{i+1}, & 1 \leq i \leq n-1, \\ \dot{x}_n = f(\underline{x}, t) + \Delta f(\underline{x}, t) + d(t) + u(t) \end{cases} \quad (1)$$

where  $\underline{x} = [x_1(t) \ x_2(t) \ \dots \ x_n(t)] \in \mathfrak{R}^n$  is the measurable state vector,  $f(\underline{x}, t)$  is unknown nonlinear continuous and bounded function,  $u(t) \in \mathfrak{R}$  is control input of the system,  $d(t)$  is the external bounded disturbance, and  $Df(\underline{x}, t)$  represents the uncertainties,

$$|f(\underline{x}, t)| < F \quad , \quad |\Delta f(\underline{x}, t)| \leq \Delta_f \quad , \quad |d(t)| \leq \Delta_d \quad (2)$$

where  $F$ ,  $\Delta_f$  and  $\Delta_d$  are positive constants.

The control problem is to get the system to track an  $n$ - dimensional desired vector  $\underline{y}_d(t)$  which belong to a class of continuous functions on  $[t_0, \infty]$ . Let's the tracking error as;

$$\begin{aligned} \underline{e}(t) &= \underline{x}(t) - \underline{y}_d(t) \\ &= [x(t) - y_d(t) \quad \dot{x}(t) - \dot{y}_d(t) \quad \dots \quad x^{(n-1)}(t) - y_d^{(n-1)}(t)] \\ &= [e(t) \quad \dot{e}(t) \quad \dots \quad e^{(n-1)}(t)] \end{aligned} \quad (3)$$

Therefore, the dynamic errors of system can be obtained as;

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3, \\ \vdots \\ \dot{e}_n = f(\underline{x}, t) - y_d^{(n)}(t) + \Delta f(\underline{x}, t) + d(t) + u(t) \end{cases} \quad (4)$$

The control goal considered is that;

$$\lim_{t \rightarrow \infty} \|\underline{e}(t)\| = \lim_{t \rightarrow \infty} \|\underline{x}(t) - \underline{y}_d(t)\| \rightarrow 0, \quad (5)$$

## 2.1. Terminal Sliding Mode Control

We consider a second order nonlinear system (1), the conventional TSM is described by the following first order terminal sliding variable;

$$s = \dot{e} + \beta e^{q/p} \quad (6)$$

where  $\beta > 0$  is a design constant, and  $p, q, (p > q)$  are positive odd integers. The sufficient condition to ensure the transition trajectory of the tracking error from approaching phase to the sliding one is:

$$\frac{1}{2} \frac{d}{dt} s^2(\underline{e}, t) = s(\underline{e}, t) \dot{s}(\underline{e}, t) \leq -\eta |s(\underline{e}, t)| \quad (7)$$

where  $\eta > 0$  is a constant.

If  $f(\underline{x}, t)$  is known and free of uncertainties and external disturbances, and when the system (1) is restricted to the  $s(\underline{e}, t) = 0$ , it will be governed by an equivalent control  $u_{eq}$  obtained by:

$$u_{eq} = - \left[ f(\underline{x}, t) - \ddot{y}_d + \beta \frac{q}{p} e^{q/p-1} \dot{e} \right] \quad (8)$$

The global control is composed of the equivalent control and discontinuous term, such that;

$$u_{dis} = -k_s \operatorname{sgn}(s) \quad (9)$$

where  $k_s$  ( $k_s > 0$ ) is switching gain, by adding this term to (8), we obtain the global control:

$$u_{TSM} = - \left[ f(\underline{x}, t) - \ddot{y}_d + \beta \frac{q}{p} e^{q/p-1} \dot{e} + k_s \operatorname{sign}(s) \right] \quad (10)$$

which ensures that TSM occurs. Then, we can choose switching gain as follows:

$$k_s = \eta + D \quad (11)$$

Where  $D = \Delta_d + \Delta_f$ . If  $s(0) \neq 0$ , it's clear that the tracking errors will reach the sliding mode ( $s = 0$ ) within the finite time  $t_r$ , which satisfies;

$$t_r \leq \frac{|s(0)|}{\eta} \quad (12)$$

Suppose the attaining time is  $t_s$  from  $e(t_r) \neq 0$  to  $e(t_s + t_r) = 0$ . In this phase, the sliding mode ( $s=0$ ) is reached, i.e., the system dynamics is determined by the following nonlinear differential equation:

$$\dot{e} + \beta e^{q/p} = 0 \quad (13)$$

By integrating the differential equation  $\dot{e} = -\beta e^{q/p}$ , we have:

$$t_s = \frac{p}{\beta(p-q)} |e(t_r)|^{1-\frac{q}{p}} \quad (14)$$

From TSM control (10), the term containing  $e^{q/p-1}\dot{e}$  may cause a singular problem.

## 2.2. Non Singular Terminal Sliding Mode Control

In order to overcome the singularity problem in the conventional TSM systems, the proposed NTSM model is described as follows:

$$s = e + \frac{1}{\beta} \dot{e}^{p/q} \quad (15)$$

where  $\beta, q$  and  $p$  ( $1 < p/q < 2$ ) have been defined in (6). For system (1) with the nonsingular sliding mode manifold (15), the control is designed as;

$$u_{NTSM} = - \left[ f(\underline{x}, t) - \ddot{y}_d(t) + \beta \frac{q}{p} \dot{e}^{2-p/q} + k_s \text{sign}(s) \right] \quad (16)$$

Thus to satisfy the transition condition (7), the time derivative of  $s$  is:

$$\dot{s} = \dot{e} + \frac{1}{\beta} \frac{p}{q} \dot{e}^{\frac{p}{q}-1} (f(\underline{x}, t) - \ddot{y}_d(t) + u(t) + \Delta f(\underline{x}, t) + d(t)) \quad (17)$$

Using control law (16),

$$\dot{s} = \frac{1}{\beta} \frac{p}{q} \dot{e}^{\frac{p}{q}-1} (-k_s \text{sgn}(s) + \Delta f(\underline{x}, t) + d(t))$$

After some manipulations, we obtain:

$$\begin{aligned} s\dot{s} &\leq \frac{1}{\beta} \frac{p}{q} \dot{e}^{\frac{p}{q}-1} [-k_s \text{sgn}(s) + (\Delta_f + \Delta_d)|s|] \\ &\leq -\frac{1}{\beta} \frac{p}{q} \dot{e}^{\frac{p}{q}-1} \eta |s| \end{aligned} \quad (18)$$

Since  $\beta > 0$ ,  $p$  and  $q$  are positive odd integers ( $1 < p/q < 2$ ), we have  $\dot{e}^{p/q-1} > 0$  (when  $\dot{e} \neq 0$ ), then;

$$\begin{cases} s\dot{s} \leq -\rho(\dot{e})|s| \\ \rho(\dot{e}) = \left( \frac{1}{\beta} \frac{p}{q} \dot{e}^{\frac{p}{q}-1} \eta \right) > 0, \quad \text{for } \dot{e} \neq 0 \end{cases} \quad (19)$$

Therefore, the condition for Lyapunov stability is satisfied when  $\dot{e} \neq 0$ , and the tracking errors can reach the sliding mode  $s=0$  within finite time. Substituting the control (16) into system (4) yields;

$$\ddot{e} = -\beta \frac{q}{p} \dot{e}^{2-\frac{p}{q}} + \Delta f(\underline{x}, t) + d(t) - k_s \operatorname{sgn}(s)$$

When  $\dot{e} = 0$ , we obtain ,

$$\begin{aligned} \ddot{e} &= \Delta f(\underline{x}, t) + d(t) - D + \eta \operatorname{sgn}(s) , \\ \begin{cases} \ddot{e} \leq -\eta & \text{for } s > 0 \\ \ddot{e} \geq \eta & \text{for } s < 0 \end{cases} \end{aligned}$$

Then, for a small  $\varepsilon (\varepsilon > 0)$ , there exists a vicinity of  $\dot{e} = 0$ , such that  $|\dot{e}| < \varepsilon$ , therefore, it is concluded that the NTSM manifold (15)  $s=0$  can be reached in the phase plane in finite time. Note that in control law (16), the nonlinear function  $f(\underline{x}, t)$  is unknown. Then, the purpose of this paper is to approximate  $f(\underline{x}, t)$  by interval type-2 fuzzy logic system and to eliminate chattering, a saturation function can be used to replace the sign function in switching term. The adaptation law of adjustable parameter of the fuzzy system is deduced from the Lyapunov stability.

### 3. ADAPTIVE INTERVAL TYPE-2 FUZZY NON-SINGULAR TERMINAL SLIDING MODE CONTROL

In this section, the adaptive fuzzy system used to approximate the unknown function  $f(\underline{x}, t)$  has the same structure as the output fuzzy system using the center of set method [34], then we replace  $f(\underline{x}, t)$  by  $\hat{f}_{-f}(\underline{x}, \theta_{-f})$ , such as:

$$\hat{f}_{-f}(\underline{x}, \theta_{-f}) = \theta_{-f}^T \xi_{-f}(\underline{x}) \quad (20)$$

where  $\theta_{-f}$  is adjustable vector parameters.

In order to guarantee the global stability of closed loop system (1) with the convergence of tracking error to zero, we propose the following control law:

$$u_{NTSM} = - \left[ \hat{f}_{-f}(\underline{x}, \theta_{-f}) - \ddot{y}_d(t) + \beta \frac{q}{p} \dot{e}^{2-p/q} + k_s \operatorname{sign}(s) \right] \quad (21)$$

To derive the adaptive laws of  $q_f$ , we define the optimal parameter vector  $\theta_{-f}^*$  as;

$$\theta_{-f}^* = \operatorname{argmin}_{\theta_{-f} \in \Omega_f} \left[ \sup_{x \in \Omega_x} \left| \hat{f}_{-f}(\underline{x}, \theta_{-f}) - f(\underline{x}, t) \right| \right],$$

where  $\Omega_f$  and  $\Omega_x$  are constraint sets of suitable bounds on  $\theta_{-f}$  and  $x$ , respectively, they are defined as;

$$\Omega_f = \left\{ \theta_{-f} : \left| \theta_{-f} \right| \leq M_f \right\}, \quad \Omega_x = \{x : |x| \leq M_x\},$$

where  $M_f$  and  $M_x$  are positive constants.

We define the minimum approximation error as;

$$w = f(\underline{x}, t) - \hat{f}(\underline{x}, \underline{\theta}_{-f}^*)$$

We can write,

$$\begin{aligned} |w| &\leq \left| f(\underline{x}, t) - \left\| \underline{\theta}_{-f}^{*T} \right\| \left\| \underline{\xi}_{-f}(\underline{x}) \right\| \right| \\ &\leq F - M_f \end{aligned}$$

By using  $F - M_f = \alpha$ , it can be easily concluded that  $w$  is bounded  $w \leq \alpha$ , (i.e.  $w \in L_\infty$ ).

To study the closed loop stability and to find the adaptation law of adjustable parameter, we consider the following Lyapunov function:

$$V(s, \tilde{\theta}_{-f}) = \frac{1}{2} s^2 + \frac{1}{2\gamma_f} \tilde{\theta}_{-f}^T \tilde{\theta}_{-f} \quad (22)$$

where  $\tilde{\theta}_{-f} = \theta_{-f} - \theta_{-f}^*$ , and  $\gamma_f$  is arbitrary positive constant, so the time derivative of (22) is:

$$\dot{V} = s\dot{s} + \frac{1}{\gamma_f} \tilde{\theta}_{-f}^T \dot{\tilde{\theta}}_{-f} \quad (23)$$

Using the control law (21), and (20), the time derivative of the NTSM manifold (15) becomes:

$$\begin{aligned} \dot{s} &= \rho'(\dot{e}) \left( f(\underline{x}, t) - \hat{f}(\underline{x}, \underline{\theta}_{-f}) - k_s \operatorname{sgn}(s) + \Delta f(\underline{x}, t) + d(t) \right) \\ &= \rho'(\dot{e}) \left( f(\underline{x}, t) - \hat{f}(\underline{x}, \underline{\theta}_{-f}) + \hat{f}(\underline{x}, \underline{\theta}_{-f}^*) - \hat{f}(\underline{x}, \underline{\theta}_{-f}^*) - k_s \operatorname{sgn}(s) + \Delta f(\underline{x}, t) + d(t) \right) \\ \dot{s} &= \rho'(\dot{e}) \left( w - (\theta_{-f} - \theta_{-f}^*) \underline{\xi}_{-f}(\underline{x}) - k_s \operatorname{sgn}(s) + \Delta f(\underline{x}, t) + d(t) \right) \end{aligned} \quad (24)$$

$$\text{such that } \rho'(\dot{e}) = \frac{1}{\beta} \frac{p}{q} \dot{e}^{\frac{p}{q}-1}.$$

The substitution of (24) in (23) will be:

$$\dot{V} = s\rho'(\dot{e}) \left( w - k_s \operatorname{sgn}(s) + \Delta f(\underline{x}, t) + d(t) \right) + \frac{1}{\gamma_f} \tilde{\theta}_{-f}^T \left( \dot{\tilde{\theta}}_{-f} - \gamma_f s\rho'(\dot{e}) \underline{\xi}_{-f}(\underline{x}) \right) \quad (25)$$

By choosing the following adaptation law:

$$\dot{\tilde{\theta}}_{-f} = \gamma_f s\rho'(\dot{e}) \underline{\xi}_{-f}(\underline{x}) \quad (26)$$

where  $\dot{\tilde{\theta}}_{-f} = \dot{\theta}_{-f}$ , therefore, we obtain:

$$\begin{aligned} \dot{V} &= s\rho'(\dot{e}) \left( w - k_s \operatorname{sgn}(s) + \Delta f(\underline{x}, t) + d(t) \right) \\ &= \rho'(\dot{e}) \left( sw - k_s |s| + s\Delta f(\underline{x}, t) + sd(t) \right) \\ &= \rho'(\dot{e}) \left( |w| - k_s + D \right) |s| \end{aligned}$$

Then,

$$\begin{aligned} \dot{V} &\leq \rho'(\dot{e}) (|w| - \eta) |s| \\ &\leq \rho'(\dot{e}) (\alpha - \eta) |s| \end{aligned} \tag{27}$$

From the universal approximation theorem, it is expected that  $\alpha$  will be very small (if not equal to zero) in the adaptive fuzzy system, and  $\rho'(\dot{e}) > 0$ . So, we have  $\dot{V} < 0$ .

The overall scheme of the adaptive type-2 fuzzy nonsingular terminal sliding mode control in presence of uncertainties, external disturbance and the training data is corrupted with internal noise is shown in Figure 1.

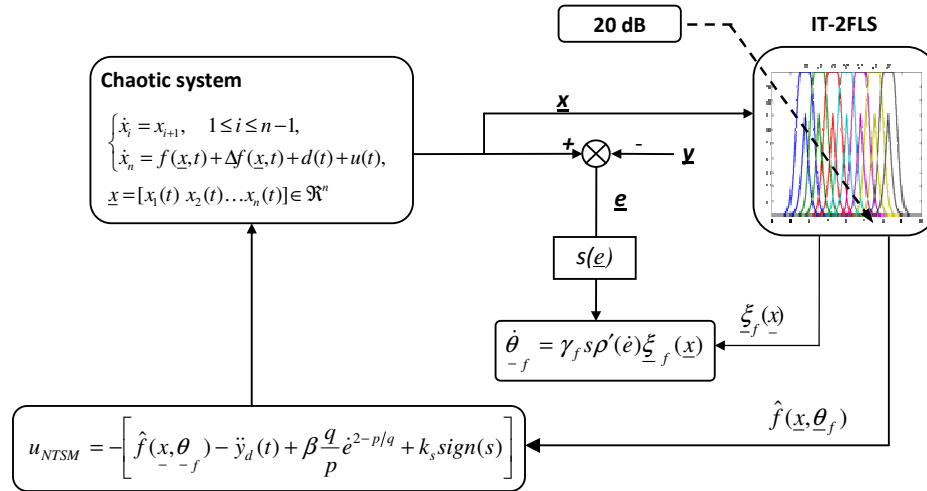


Figure1. Overall scheme of the adaptive type-2 fuzzy nonsingular terminal sliding mode control system.

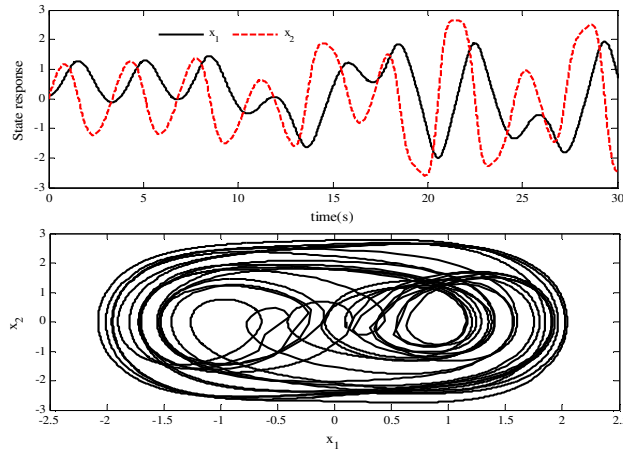
#### 4. SIMULATION EXAMPLE

The above described control scheme is now used to control the states of chaotic system which is defined as follows;

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -0.4x_2 - 1.1x_1 - x_1^3 - 2.1\cos(1.8t) \end{cases} \tag{28}$$

With initial states:  $x(0) = [0.1 \ 0]^T$ .

For free input, the simulation results of system are shown in Figure 2.


 Figure 2. Time response  $(x_1, x_2)$  and typical chaotic behavior of duffing oscillator

In order to force the states system  $x_i(t), i=1,2$  to track the reference trajectories  $y_d(t)$  and  $\dot{y}_d(t)$  in finite time, such as  $y_d(t) = (\pi/30)(\sin(t) + 0.3\sin(3t))$ , the adaptive interval type-2 fuzzy nonsingular terminal sliding mode control  $u(t)$  is added into the system as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -0.4x_2 - 1.1x_1 - x_1^3 - 2.1\cos(1.8t) + \Delta f(\underline{x}, t) + d(t) + u(t) \end{cases} \quad (29)$$

We choose  $\gamma_f = 15, q = 3, p = 5$  and  $\beta = 1$ , the TSM and NTSM manifolds are selected as,  $s_{TSM} = \dot{e} + \beta e^{q/p}$  and  $s_{NTSM} = e + (1/\beta)e^{p/q}$ , respectively. To design the fuzzy system  $\hat{f}(\underline{x}, \underline{\theta}_f)$ , we define seven type-2 Gaussian membership functions depending  $x_i(t), i=1,2$  selected as  $F_i^l, l=1, \dots, 7$  are shown in table. 1, with variance  $\sigma = 0.5$  and initial values  $\theta_f(0) = \mathbf{0}_{2 \times 7}$ .

 Table 1. Interval Type-2 Fuzzy Membership Functions For  $x_i (i = 1, 2)$ .

	Mean			Mean	
	$m_1$	$m_2$		$m_1$	$m_2$
$\mu_{F_1^1}(x_i)$	-3.5	-2.5	$\mu_{F_5^5}(x_i)$	0.5	1.5
$\mu_{F_2^2}(x_i)$	-2.5	-1.5	$\mu_{F_6^6}(x_i)$	1.5	2.5
$\mu_{F_3^3}(x_i)$	-1.5	-0.5	$\mu_{F_7^7}(x_i)$	2.5	3.5
$\mu_{F_4^4}(x_i)$	-0.5	0.5			

In this section, two control laws are adopted, adaptive type-2 fuzzy nonsingular terminal sliding mode control (AT- 2FNTSM) described in (21), and adaptive type-2 fuzzy terminal sliding mode control (AT-2FTSM), which is designed as follow;

$$u_{TSM} = - \left[ \hat{f}(\underline{x}, \underline{\theta}_f) - \ddot{y}_d + \beta \frac{q}{p} e^{q/p-1} \dot{e} + k_s \text{sign}(s) \right] \quad (1)$$



The simulation results are presented in the presence of uncertainties  $\Delta f(\underline{x},t) = \sin(2\pi x_1(t))\sin(3\pi x_2(t))$ , external disturbance  $d(t) = \sin(t)$ , and white Gaussian noise is applied to the measured signal  $x_i(t), i = 1, 2$  with Signal to Noise Ratios (SNR=20dB). A boundary layer method is used to eliminate chattering.

#### 4.1. Adaptive Interval Type-2 Fuzzy Terminal Sliding Mode Control (AT-2FTSM)

The tracking performance of states  $\underline{x}(t)$  is shown in Figure 3. The control input  $u(t)$  and the phase-plane trajectories of system are represented in Figures 4-5.

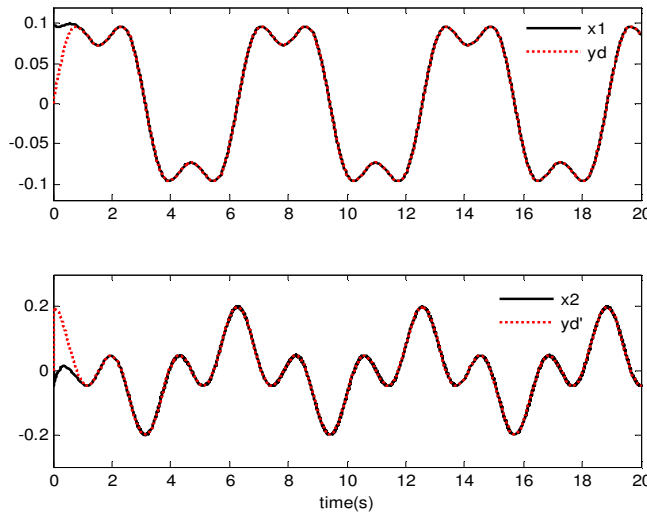


Figure 3. The output trajectories of  $(x_1, x_2)$ .

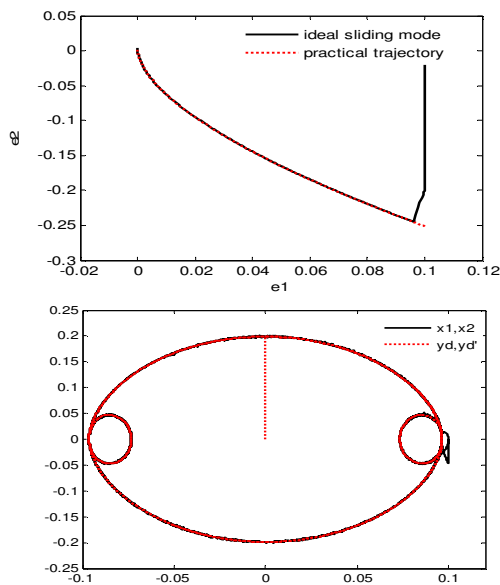


Figure 4. Phase-plane of tracking error and typical chaotic behavior of duffing oscillator

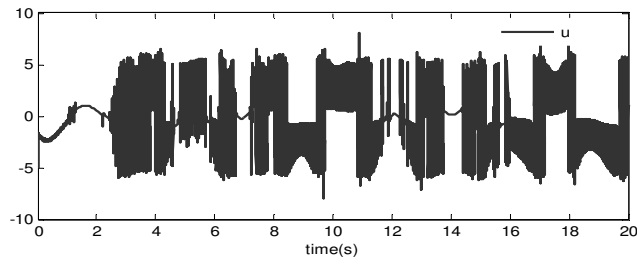


Figure5. Control input  $u(t)$

#### 4.2. Adaptive Interval Type-2 Fuzzy Non-singular Terminal Sliding Mode Control (AT-2FNTSM)

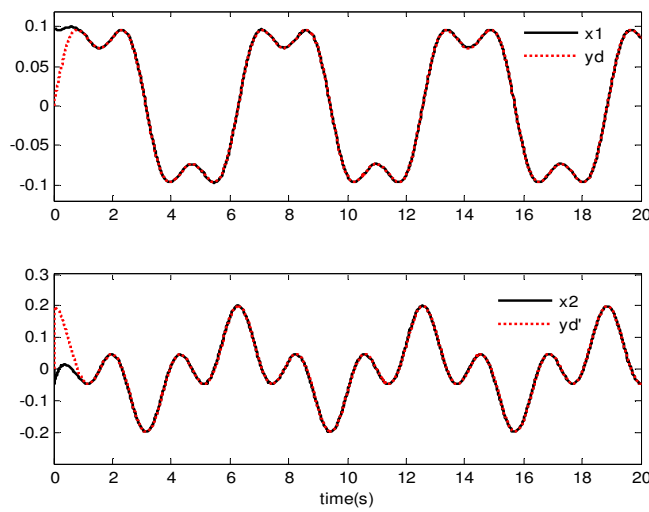


Figure6. The output trajectories of  $(x_1, x_2)$

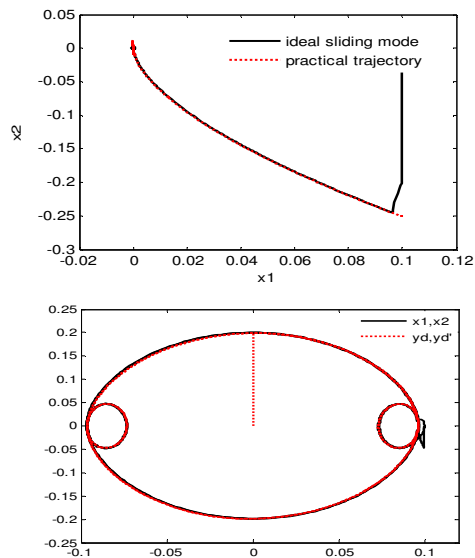
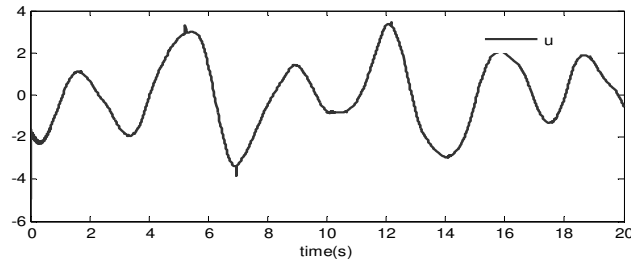


Figure7. Phase-plane of tracking error and typical chaotic behavior of duffing oscillator

Figure8. Control input  $u(t)$ 

According to the above simulation results, we can see that both controller provide a good tracking of outputs system  $(x_1, x_2)$  to their trajectories in finite time. Furthermore, a singularity problem occurs in the case of AT-2FTSM control as shown in Figure 5. The proposed approach allows obtaining a smooth control signal (Figure 8), then, the NTSM manifold (15) can eliminate the singularity problem associated with conventional TSM manifold.

## 5. CONCLUSION

In this paper, the problem of stabilization orbit of nonlinear uncertain chaotic system in the presence of external, internal disturbances and disturbances is solved by incorporation of interval type-2 fuzzy approach and non-singular terminal sliding mode control. In order to eliminate the chattering phenomenon efficiently, a boundary layer method is used, and an adaptive interval type-2 fuzzy system is introduced to approximate the unknown part of system. Based on the Laypunov stability criterion, the adaptation law of adjustable parameters of the type-2 fuzzy system and the stability of closed loop system are ensured. A simulation example has been presented to illustrate the effectiveness and the robustness of the proposed approach.

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