COMPENSATION OF DATA-LOSS IN ATTITUDE CONTROL OF SPACECRAFT SYSTEMS

Naeem Khan[1], and Dawei Gu[2]

[1]Assistant Professors in Electrical Engineering Department., UET Peshawar, Pakistan
[2]Professor at Department of Engineering, University of Leicester, UK

ABSTRACT

In this paper, a comprehensive comparison of two robust estimation techniques namely, compensated closed-loop Kalman filtering and open-loop Kalman filtering is presented. A common problem of data loss in a real-time control system is investigated through these two schemes. The open-loop scheme, dealing with the data-loss, suffers from several shortcomings. These shortcomings are overcome using compensated scheme, where an accommodating observation signal is obtained through linear prediction technique -- a closed-loop setting and is adopted at a posteriori update step. The calculation and employment of accommodating observation signal causes computational complexity. For simulation purpose, a linear time invariant spacecraft model is however, obtained from the nonlinear spacecraft attitude dynamics through linearization at nonzero equilibrium points -- achieved off-line through Levenberg-Marguardt iterative scheme. Attempt has been made to analyze the selected example from most of the perspectives in order to display the performance of the two techniques.

KEYWORDS

State Estimation, Kalman filtering, Open-loop Kalman filtering, Intermittent Observations, Compensated Closed-loop KF

1. INTRODUCTION

State estimation in uncertain environments [15, 9] or noisy information [29] is a broad field of communication and control theory. This is because the problem of state or parameter estimation is of paramount importance in the analysis and design of control systems [14]. The most celebrated techniques for state estimation are Kalman filtering and its adaptive forms, particle filtering, and H_∞ filtering [8] etc. For a linear system, Kalman filter is an optimal approach where state of an LTI system is estimated based on an optimal Kalman filter gain.

For various reasons including understanding of system behaviour, designing and implementation of an optimal control scheme, state (attitude) estimation has remained an important research topic for spacecraft control. Spacecraft systems in particular, mainly depend on data achieved and processed from the ground that ultimately results in time delay [25]. However, perfect communication is a valuable and the most desired asset in the event of fault and failure. To handle such unfavourable conditions, several techniques like hardware redundancy, including duplicate, triplicate and voting schemes, has remained consistently adopted [24]. But issues like weight, complexity and cost of the supplementary elements in these hardware-based techniques have diverted the attention toward software based approaches (Model Based FDI) to overwhelm the aforementioned limitations [31].

A precise communication is a fundamental factor in achieving fruitful results in any control system. However, scenarios including finite gateways of networks, bounded spaces and overpopulated networks might cause the data packets to be lost. As a consequence, the spacecraft performance may be significantly degraded specially in terms of delay and failure due to any of these diverse conditions [26]. Hence the loss of observation or output data plays a vital role in spacecraft attitude control and remedies need to be explored in order to provide reliable state (attitude etc.) estimation.

Open-loop Kalman filtering (OLKF), also known as Open-loop estimation is, perhaps, the predominant scheme in the literature since it is frequently utilized for data-loss cases. There are numerous research articles which elaborate this technique in details such as [28, 4, 30, 8] and the references therein, to name a few. Some of these literatures have demonstrated the associated limitations of this technique too. It is an intense need to propose some novel techniques that could handle data-loss situations more efficiently and to overcome those limitations. For such reasons, a compensated closed-loop Kalman filtering algorithm is proposed in [17, 18]. In the compensated closed-loop scheme, an accommodating observation signal is reconstructed using linear prediction coding, for which one parameter is crucial to decides i.e. the order of linear prediction filter.

The present study is aimed to elaborate two objectives: (a) to provide handsome details of this recently proposed scheme and (b) the extensive study and comparison of these two state estimation schemes (Open-loop Kalman filtering and compensated closed-loop Kalman filtering) for a rigid body spacecraft system which is subjected to an induced data-loss. A minimum mean square error based algorithm is proposed to decide the computation of linear prediction filter order. In order to provide the true and complete picture, these two schemes are compared with conventional estimation scheme (normal Kalman filtering without any data-loss). In fact it provides a common base for the comparison. Both rotational dynamic and kinematic equations are used to derive the state-space equations for the spacecraft system [1, 20], contrary to the normal trend of using `Kinematic equation' as discussed by [10, 22, 26, 6, 13] and the references therein.

The remaining paper is organized as follows: Section 2 presents the nonlinear model of a rigid body spacecraft system using Modified Rodrigues Parameterizations i.e. MRP representation. Section 3 is devoted to a brief discussion of an existing control system design. A detailed discussion of the accommodating closed-loop Kalman filtering scheme is described in Section 4. The performance of open-loop estimation and the recently introduced accommodating estimation schemes is analyzed through a numerical case study in Section 5. The paper is concluded with suggestions for the future work in Section 6.

2. SPACECRAFT RIGID BODY

It is common to observe spacecraft analysis while employing kinematic equations and/or dynamic equations in Euler angles and quaternion parameterizations. These two parameterizations have certain limitations; nonlinear trigonometric functions and singularity issues are linked with Euler angles while a redundant element and unit constraint are associated with quaternion parameterizations. To overcome these shortcomings, Modified Rodrigues Parameters (MRPs) are utilized in this work which is found advancement to the parameterization's family.

2.1 Spacecraft Dynamics

As mentioned before spacecraft dynamics are usually specified by its *'Kinematic'* system only [10, 22, 6, 13] In this manuscript, however, both Euler equations of rotational dynamics and the Kinematic equations in order to examine the complete behaviour of the spacecraft system.

2.1.1 Kinematic Equations

In the compact form, Kinematic equations are

$$\dot{\sigma} = T(\sigma) \tag{1}$$

where σ is the Modified Rodrigues Parameter and $T(\sigma)$ is defined as

$$T(\sigma) = 0.5\left[\left(\frac{1-\sigma^T\sigma}{2}\right)I_{3\times3} + S(\sigma) + \sigma\sigma^T\right] \tag{2}$$

wherein $S(\sigma)$ denotes the skew symmetric matrix defined as

$$S(\sigma) = \begin{bmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{bmatrix} \tag{3}$$

The attitude vector σ and noisy angular vector $\overline{\omega}$ are of dimension 3 x 1 with

$$\overline{\omega} := \begin{bmatrix} \overline{\omega}_1 \\ \overline{\omega}_2 \\ \overline{\omega}_3 \end{bmatrix} = \begin{bmatrix} \omega_1 - n_1 \\ \omega_2 - n_2 \\ \omega_3 - n_3 \end{bmatrix} \tag{4}$$

The gyroscope output model is $y_j(t)$ is selected as

$$y_j = c_j\dot{\theta}_j(t) + n_j(t) \qquad \forall \qquad j = \{1,2,3\} \tag{5}$$

where c_j, θ_j *and* n_j represent the scale coefficient, the angular position and gyroscope noise respectively. The noise is assumed to be Gaussian white noise with zero mean, i.e.

$$n_j \sim N(0,\Pi) \tag{6}$$

where Π is the bias variance.

2.1.2 Dynamic Equations

The dynamics are defined using Euler's equations as

$$J\dot{\overline{\omega}} = -S(\overline{\omega})J\overline{\omega} + \tau \tag{7}$$

where J is the spacecraft's inertia, τ is control input and "$S(\overline{\omega}) = \overline{\omega} \times \overline{\omega}$" is the skew symmetric matrix which shows the cross-product operation as

$$S(\omega) = \begin{bmatrix} 0 & -\overline{\omega}_3 & \overline{\omega}_2 \\ \overline{\omega}_3 & 0 & -\overline{\omega}_1 \\ -\overline{\omega}_2 & \overline{\omega}_1 & 0 \end{bmatrix} \tag{8}$$

Equations (1) and (7), collectively constitute the full state vector for the spacecraft plant model, i.e. $x = [\sigma\ \omega]^T$. The linearized attitude dynamics of the plant are represented as

$$\dot{x}(t) = Ax(t) + Bu(t) + G\xi(t) \tag{9}$$

where A, B and G are Jacobian matrices of the linear spacecraft dynamics which are derived in a straightforward manner. Equations (1) and (2) can be considered six coupled equations, from which the linearized plant model can be achieved through Jacobian linearizations. Some related theory can be found in [11].

3. Control System Design

It is important to mention that in this paper special attention is paid to obtain efficient state estimation in case of loss of observation and not the control issues related to the spacecraft system. Hence, an already established and employed control scheme [1] is briefly recalled for the sake of complete view. This control technique consists of two loops. An inner loop comprises a simple transfer function while an outer loop is merely a unity feedback gain. The recalled control system design is:

$$\tau = T(\sigma)^T [S_P \tilde{\sigma}(t) - \sigma^*(t)] \tag{10}$$

where τ is the control input and

$$\begin{aligned} S_P &= diag\ (s_{p1}, s_{p2}, s_{p3}), \\ \tilde{\sigma} &= \sigma_d(t) - \hat{\sigma}(t), \\ \sigma^*(t) &= N\sigma(t), \\ N &= diag\left(s_{d1}\frac{\alpha_1 s}{s+\alpha_1}, s_{d2}\frac{\alpha_2 s}{s+\alpha_2}, s_{d3}\frac{\alpha_3 s}{s+\alpha_3} \right) \end{aligned} \tag{11}$$

The positive definite matrices (S_P, N) are the tuning elements need to be tuned. The candidate Lyapunov function is

$$V(\sigma^*, \dot{\sigma}, \tilde{\sigma}) = 0.5(\dot{\sigma}^T H^* \dot{\sigma} + \tilde{\sigma}^T S_p \tilde{\sigma} + \sigma^{*T}\{\alpha S_d\}^{-1}\sigma^*) \tag{12}$$

with H^* *and* S_d are defined as

$$H^{*} = (T(\sigma)^{-1})^{T}\, JT(\sigma)^{-1} \quad \& \qquad (13)$$
$$S_d = diag\ (s_{d1}, s_{d2}, s_{d3})$$

The derivative of Lyapunov function w.r.t time is

$$\begin{aligned}\dot{V} &= -\dot{\sigma}^{T}\sigma^{*} + \sigma^{*T}\{\alpha S_d\}^{-1}(S_d\alpha\dot{\sigma} - \alpha\sigma^{*}) \\ &= -\sigma^{*T} S_d^{-1}\sigma^{*}\end{aligned} \qquad (14)$$

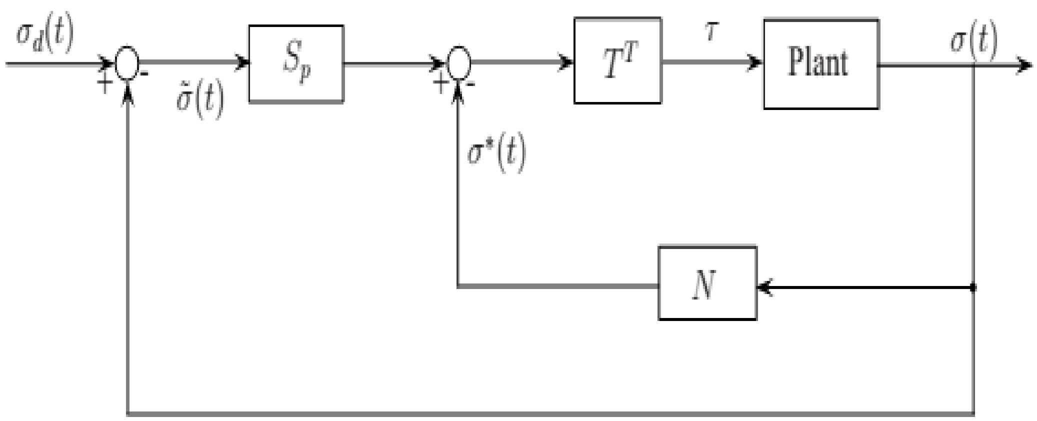

Figure 1. Schematic of Feedback Controller

The choice of gain parameters $(s_{pi}, s_{di}\ and\ \alpha_i)$ to stabilize the plant model could be any suitable positive values as long as convergence is associated. Such detailed discussion can be found in [1]. In the following section, the two robust algorithms are discussed along with the normal Kalman filtering (without any data-loss) scheme.

4. Robust Kalman Filtering Techniques

Loss of observations or data-loss might produce adverse scenarios for state estimation in Kalman filtering, as it heavily depends on measured data [7] and [3]. Such conditions are sometimes unavoidable, result in poor estimation and could lead Kalman filter to diverge very swiftly. A usual technique to avoid such shortcomings is the so-called Open-Loop Estimation (OLE) algorithm when observations are subjected to random loss -- see e.g. [27, 23, 33, 28]. In these references, the authors have presented the Kalman filter running in an open-loop method, when the plant is subjected to data-loss. Simply the predicted quantities (state and covariance) are processed without any measurement update to the next step. It is considered helpful to present a brief discussion of this OLE along with its related drawbacks.

4.1 Open-Loop Estimation

Open-Loop Estimation (OLE) or Open-Loop Kalman Filtering (OLKF) scheme is an effective, simpler and computationally efficient method in practice to accommodate data-loss [12]. In this scheme Kalman filter gain K_k is forced to zero if, a data-loss is observed at instant 'k' and hence no update step is performed. Therefore, estimation is performed with zero sensitivity matrix [12]. Open-loop estimation is briefly introduced in the following lines with necessary comments.

4.1.1 Time update step

The *a priori* step or time update quantities (state and covariance) are

$$ {}_o x_{k+1|k} = A x_{k|k} + B u_k \tag{15} $$

$$ {}_o P_{k+1|k} = A P_{k|k} A^T + Q_k \tag{16} $$

4.1.2 Accommodating measurement vector

The pseudo-observational vector in Open-loop estimation is

$$ {}_o z_{k+1} = C\, {}_o x_{k+1|k} $$

where the leading subscript 'o' shows Open-loop Kalman filtering approach. It causes zero residual vector (2-norm) and hence, the Kalman gain would be

$$ {}_o K_{k+1} = 0 \tag{17} $$

4.1.3 Measurement update step

Since no data is available for measurement update, hence the *a posteriori* state and covariance quantities will be

$$ {}_o x_{k+1|k+1} \leftarrow {}_\mathrm{o} x_{k+1|k} \tag{18} $$

$$ {}_o P_{k+1|k+1} \leftarrow {}_\mathrm{o} P_{k+1|k} \tag{19} $$

Therefore, in this approach the a *posteriori* parameters strictly follow the *a priori* quantities respectively.

4.2 Shortcomings of the OLE

Although Open-loop scheme is a fast remedy to accommodate data-loss in state estimation due to skipping of measurement update step, it suffers from the following shortcomings:

1. OLE diverges swiftly in the presence of adequate data-loss [33],
2. Spikes and/or oscillations (particularly when the output data is recovered),
3. It is harder to attain steady state values completely when data-loss is resumed [19].

4.3 Closed-loop Estimation Scheme

Due to the aforementioned disadvantages of OLE approach, an improved estimation scheme, based on linear prediction concept, presented in [18, 19], is utilized. This scheme is known as "Compensated Closed-loop Kalman Filtering (CCLKF)" wherein the lost observation signal is reconstructed through linear prediction subsystem. There are various methods to predict a lost sample or signal such as Particle Swarm Optimization [2], Maximum-a-posteriori or MAP [34], Linear Prediction Coefficients/Coding (LPC) [5], etc. For several advantages such as close resemblance to FFT, based on source-filter model, easy calculations, LPC technique is adopted to reconstruct missing data signal.

For efficient linear prediction, as commonly adopted, it is speculated that the measurement signal has correlation of some extend. In addition, the statistical properties of the plant model vary slowly with time. According to this scheme, the missing signal is assumed to be

$$\bar{z}_k = \sum_{i=1}^{p} \alpha_i z_{k-i} \tag{20}$$

where the Linear Prediction Coefficients (LPCs) $'\alpha_i'$ represent weights assigned to the previous observations which is decided according to their correlation-degrees and $'p'$ is the order of the linear prediction filter (LPFO). The optimal value(s) of LPC and order of the Linear Prediction Filter (LPFO) in Equation (20) are important elements in achieving an efficient filter [18] in term of optimal results. For this reason, special attention has been considered to compute these two critical elements. For this reason, a simple strategy is adopted using Algorithms 1, which can assure the optimal value of LPFO leading to the optimal values of LPCs.

Algorithm 1: For LPFO

1: Initialization $j = 1$, Compute $R\gamma$ *and* $r\gamma$ {Equations (24-26)}.

2: Recursion j =2, 3, . . . M (LPFO)

Calculate LPC {Equation (27)}

- Calculate compensated observation signal $\bar{z}_k \,|_{R_\gamma^J}$ {Equation (28)}
- Calculate compensated state estimation ${}_c\hat{x}_k^j$ based on this signal {Equation (29)}
- Calculate compensated state estimation error $\bar{e}_j := \| x_k - {}_c\hat{x}_k^j \|_2$

3: Trace $\epsilon_{th} = \min(\bar{e}_j)$, *whereas* $\bar{e}_j \in \{\bar{e}_2, \bar{e}_3, \ldots \bar{e}_M\}$,

4: Select $\bar{z}_j$ which results in ϵ_{th}

5: Decide $p \leftarrow j$ i.e. LPFO.

The detailed implementation of compensating scheme which will help in the understanding of the above algorithm can be described as follows. Consider the discrete LTI plant dynamics are described by the following equations:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + \xi_k \\ y_k &= \gamma_k (Cx_k) + v_k \end{aligned} \tag{21}$$

where $k \in \mathbb{R} = \{0,1,2,....\}$, $x, \xi \in \mathbb{R}^{\mathrm{n}}$, $u \in \mathbb{R}^{\mathrm{t}}$, $z \in \mathbb{R}^{\mathrm{m}}$, $A \in \mathbb{R}^{\mathrm{n\times n}}$, $B \in \mathbb{R}^{\mathrm{n\times l}}$ *and* $\mathrm{C} \in \mathbb{R}^{\mathrm{m\times n}}$ are state transition matrix, the input matrix, and output matrix with $((x_0, \xi_k, v_k) \sim N((\bar{x}_0, 0, 0), (P_0, Q_k, R_k))$. The random variable γ_k is characterized as follows:

$$\gamma_k = \begin{cases} 0; if\ data{-}loss\ is\ \text{det}ected \\ 1; otherwise \end{cases} \tag{22}$$

Assume data-loss is observed at time instant $'k'$, the employed Kalman filtering technique (CCLKF) is outlined as follows:

- **Prediction cycle**
  At time step $(k\text{-}1)$,

$$\begin{aligned} {}_c x_{k|k-1} &= A_c x_{k-1|k-1} + Bu_{k-1} \\ {}_c P_{k|k-1} &= A_c P_{k-1|k-1} A^T + Q_K \end{aligned} \tag{23}$$

where $\mathrm{E}[\xi_k \xi_k^{\mathrm{T}}] = Q_k$ is the process error covariance matrix, with $'\mathrm{E}'$ denoting expectation.

- **Check** loss of observation:

If $\gamma_k = 1$ i.e. no loss has occurred.

⇒Run conventional or normal Kalman filter (see e.g. [7], [3], [32]).

if $\gamma_k = 0$, it means a data-loss condition has been detected

⇒No measurement (data) is available, hence at *a posteriori* step, accommodating-measurement
update procedure is carried out as follows:

- **Chose** a nominal frame size of the previously stored measurement data (say M) modelled through the constraint $M \le f_s * t_k$ [17], where f_s is the sampling frequency and t_k is starting instant of data-loss.

- **Compute** the autocorrelation matrix R_γ as

$$R\gamma = \begin{bmatrix} R[0] & R[1] & R[2] & \cdots & R[p-1] \\ R[1] & R[0] & R[1] & \cdots & R[p-2] \\ R[2] & R[1] & R[0] & \cdots & R[p-3] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R[p-1] & R[p-2] & R[p-3] & \cdots & R[0] \end{bmatrix} \tag{24}$$

and the auxiliary autocorrelation array $r\gamma$ is

$$r\gamma = [\ r[1]\ \ r[2]\ \ r[3]\ \cdots\ r[p]\ ]^T \tag{25}$$

where

$$\begin{aligned} \mathrm{E}[z_{k-i}^T z_{k-j}] &= \begin{cases} R[0], & if\ \mathrm{i} = \mathrm{j} \\ R[|i-j|], & if\ \mathrm{i} \ne \mathrm{j} \end{cases} \\ \mathrm{E}[z_k^T z_{k-j}] &= r[j] \end{aligned} \tag{26}$$

- **Compute** the Linear Prediction Coefficients (LPC) as

$$A_\alpha = [\alpha_j]^T = R_\gamma^{-1}.r\gamma \quad (27)$$

- **Compute** the accommodating observation vector as

$$\bar{z}_k = \sum_{j=1}^{p} \alpha_j z_{k-j} \equiv C\bar{x}_k + \bar{v}_k \quad (28)$$

- **Compute** the compensated residual vector as $\bar{z}_k - \hat{z}_k$.
- **Compute** compensated Kalman gain as ${}_cK_k = {}_{2c}P_kC^T(C_{3c}P_kC^T + R_k)^{-1}$
- **Perform** *a posterior* step as

$${}_cx_{k|k} = {}_cx_{k|k-1} + {}_cK_k(\bar{z}_k - C_cx_{k|k-1})$$

$${}_cP_{k|k} = {}_{1c}P_k - {}_{2c}P_kC^T(C_{3c}P_kC^T + R_k)^{-1}C_{2c}P_k \quad (29)$$

- **Return** to step 1 (prediction cycle).

Three covariance matrices that appear in above Equation are defined as:

$$\begin{aligned}
{}_{1c}P_k &= {}_{1c}P_{k|k-1} \overset{def}{=} \mathrm{E}[(e_{k|k-1})(e_{k|k-1})^T] = \mathrm{E}[(x_k - {}_cx_{k|k-1})(x_k - {}_cx_{k|k-1})^T] \\
{}_{2c}P_k &= {}_{2c}P_{k|k-1} \overset{def}{=} \mathrm{E}[(e_{k|k-1})(\bar{e}_{k|k-1})^T] = \mathrm{E}[(x_k - {}_cx_{k|k-1})(\bar{x}_k - {}_cx_{k|k-1})^T] \\
{}_{3c}P_k &= {}_{3c}P_{k|k-1} \overset{def}{=} \mathrm{E}[(\bar{e}_{k|k-1})(\bar{e}_{k|k-1})^T] = \mathrm{E}[(\bar{x}_k - {}_cx_{k|k-1})(\bar{x}_k - {}_cx_{k|k-1})^T]
\end{aligned} \quad (30)$$

where ${}_cP_{1k} = P_{k|k-1}$ is the normal predicted error covariance matrix. The compensated closed-loop scheme along with the controller strategy is shown in the block diagram fashion in Fig (2). The switching mechanism between conventional Kalman filter (when there is no data-loss) and compensated closed-loop scheme (when there is a data-loss) is shown in Fig (3).

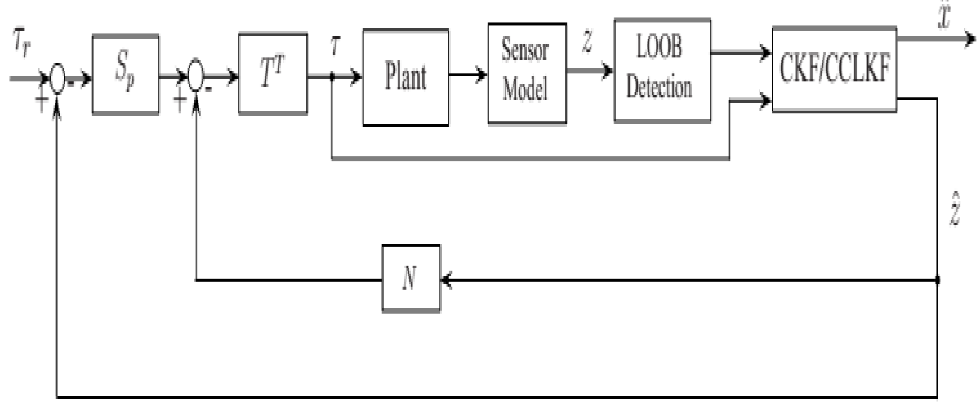

Figure 2. Complete Diagram of Compensated scheme and spacecraft system

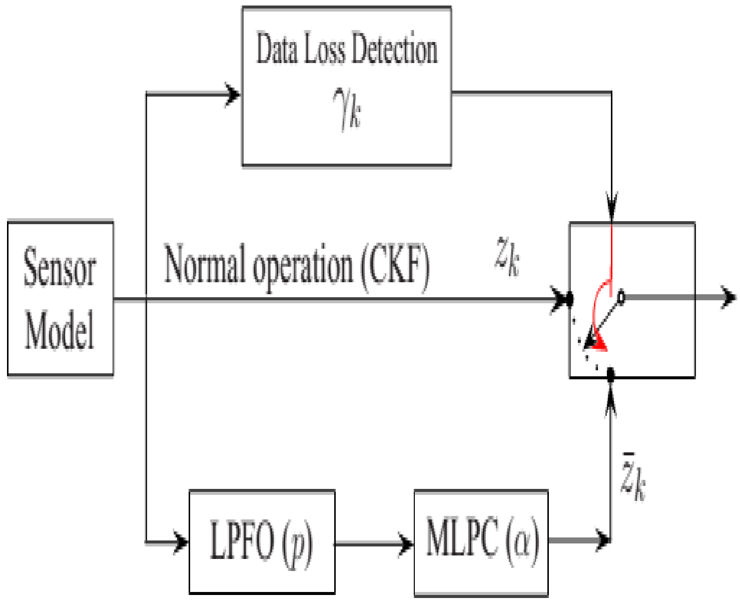

Figure 3. Switching mechanism between loss of information and fault free systems

5. NUMERICAL SIMULATION

As mentioned before, a rigid spacecraft model, subjected to intermittent output data is employed here to test the performance of two schemes. These two schemes are thoroughly discussed in the simulation results with their respective advantages and disadvantages. Emphasis has been made on simulation results based on a data-loss at a specific location (30 - 45 sec). However, to show the flexibility and efficiency of the compensated scheme, a few simulation results based on a data-loss at another location (15 - 25 sec) are also shown.

5.1 Spacecraft model

The non-zero equilibrium points in Equations (1) and (7) are computed off-line through Levenberg- Marguardt iterative least-square scheme in Matlab-Simulink environment. The plant model is lin- earized using Jacobian linearization at these operating points to conclude the state-space model. The mathematical description of the linearized spacecraft attitude model is given by

$$\begin{aligned} \dot{x}_t &= Ax_t + Bu_t + G\xi_t \\ z_t &= Cx_t + Du_t + \theta_t \end{aligned} \tag{31}$$

where x, u, ξ, z are state vectors, deterministic system input, plant disturbance and measured output. The linear time invariant system matrices $A, B, C\ and\ D$ computed through Jacobian linearization at non-zero operating points are as follows:

$$A = \begin{bmatrix} -0.0160 & 0.0621 & 0.3567 & 0.2151 & 0.2087 & -0.0133 \\ -0.0621 & -0.0160 & 0.1462 & -0.2010 & 0.2010 & -0.0962 \\ -0.3567 & -0.1462 & -0.0160 & -0.0580 & 0.0779 & 0.2247 \\ 0 & 0 & 0 & 0.0465 & 0.0661 & 0.0234 \\ 0 & 0 & 0 & -0.0435 & 0.0163 & -0.1275 \\ 0 & 0 & 0 & -0.1921 & 0.0274 & -0.0628 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0.0503 & -0.0033 & -0.0027 \\ 0 & 0 & 0 & -0.0033 & 0.0595 & -0.0054 \\ 0 & 0 & 0 & -0.0027 & -0.0054 & 0.0673 \end{bmatrix}$$

$$G^T = \begin{bmatrix} -0.2151 & 0.2010 & 0.0580 & -0.0465 & 0.0435 & 0.1921 \\ -0.2087 & -0.2010 & -0.0779 & -0.0661 & -0.0163 & -0.0274 \\ 0.0133 & 0.0962 & -0.2247 & -0.0234 & 0.1275 & 0.0628 \end{bmatrix}$$

$$C = \begin{bmatrix} 2.7069 & -2.7731 & 0.3026 & 0 & 0 & 0 \\ 2.6196 & 2.5571 & -1.1132 & 0 & 0 & 0 \\ 1.1156 & -0.2864 & 2.6252 & 0 & 0 & 0 \end{bmatrix}$$

and

$$D = 0_{3\times 3}$$

For simulation purpose, the initial state vector is selected as $x_0 = [0.2 \;\; 0.2 \;\; 0.2 \;\; -0.3 \;\; -0.4 \;\; 0.2]^T$,the operating points computed through Levenberg-Marguardt methods are $\tau = u = [0.1284 \;\; 0.5767 \;\; 0.9365]^T$; $\sigma = [0.1741 \;\; 0.0447 \;\; -0.4097]^T$ and $\omega = [0.4779 \;\; -0.4779 \;\; 0.228]^T$. The output and process noise covariance matrices are chosen as $R = 0.05 * I_{3\times 3}$ and $Q = 0.01 * I_{6\times 6}$ where I is an identity matrix.

5.2 Spacecraft model

Certain typical simulation results are shown in this section for the system discussed above, subjected to an induced data-loss. The simulation results obtained for CCLKF are compared with that of OLKF and loss-free (Normal) Kalman filter results. This data-loss is assumed to commence at time instant $t = 30$ *sec* and remains for 15 *sec*. Various studies are investigated and addressed in the subsection below.

5.2.1 MRP Attitude

Figures 4 and 5 show the estimation results of MRP-σ_1 for the said two techniques. Although all the three attitude parameters σ_1, σ_2 and σ_3 could be shown, however to avoid repetitions, they are not included in this paper. It is obvious that the open-loop estimation diverges heavily and instantly. Contrary to this, in the accommodating CCLKF approach, the estimation during data-loss time is significantly stable as the highlighted picture shows in Figure 5. No doubt, the deviation is directly related to the duration of loss of observation. From Figure 5, it can be observed that less deviation caused by CCLKF scheme results in minor moment of inertia (juggling) just after the data-loss is resumed, and hence the steady state is captured sooner than OLKF approach. This is a significant achievement compared to OLKF scheme.

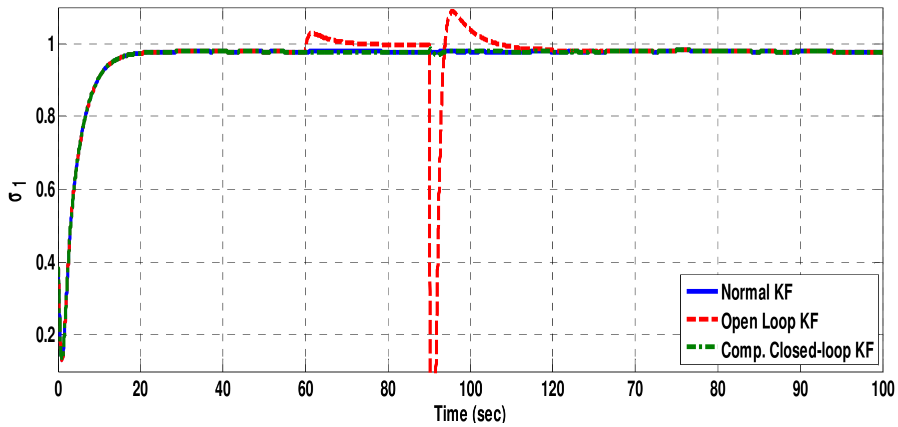

Figure 4: Estimated result of First MRP (σ_1)

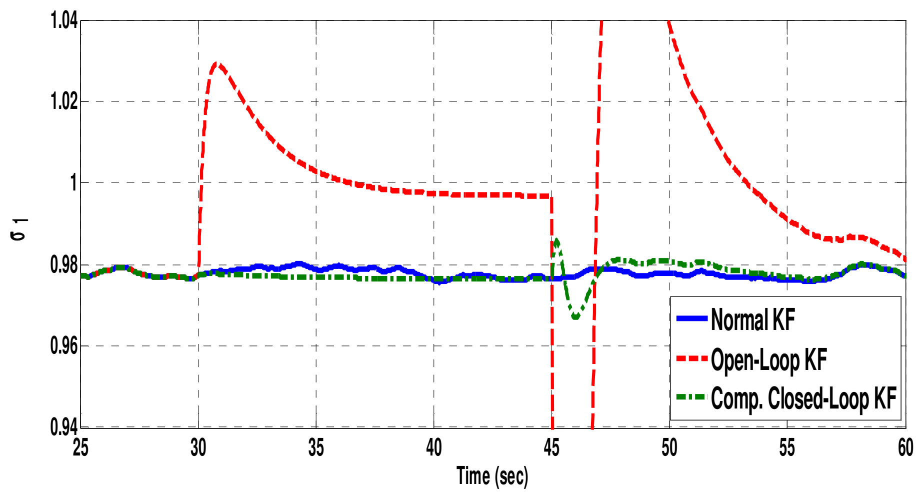

Figure 5: Data-loss period is highlighted

5.2.2 Angular Velocity

Similar to MRPs, three angular velocities related to the spacecraft model are also analyzed. Figures 6 – 8 show the comparison of the two schemes (OLKF and CCLKF) during the data-loss time for the angular velocities ω_1, ω_2 and ω_3 along with the base comparison of normal Kalman filtering. These figures illustrate, that the unavailability of observation has made OLKF a poor estimating tool for nominal data-loss. Abrupt changes ("spikes and oscillations") can be frequently realized in the estimation of angular velocities through OLKF scheme. On the other side, the accommodating CCLKF scheme provides smaller chattering compared to OLKF and hence outperforms.

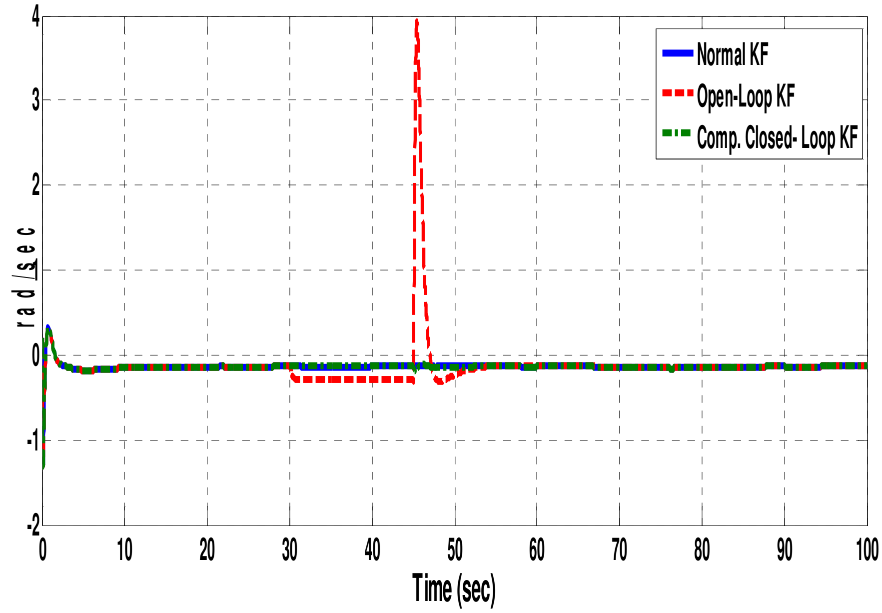

Figure 6: Estimated result of (ω_1)

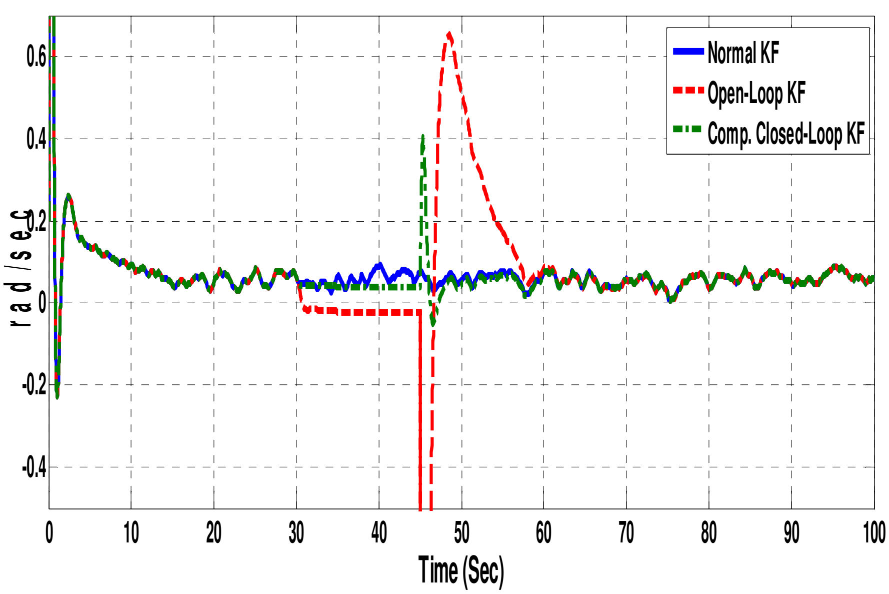

Figure 7: Estimated result of (ω_2)

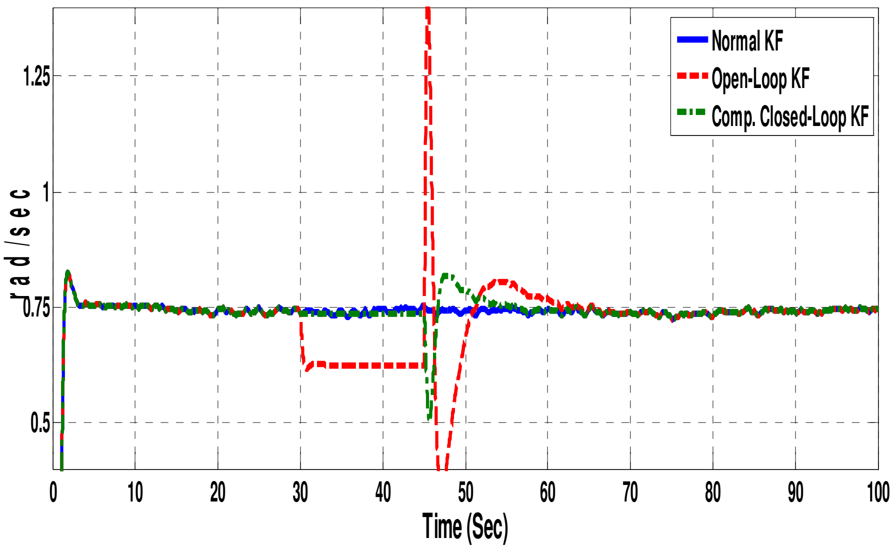

Figure 8: Estimated result of ($\omega 3$)

5.2.3 Control Effort

MRPs another important aspect which delivers significant performance impact is the control input signal. The data-loss at output terminal transverses its influence to the input parameters as well due the recursive behaviour of Kalman filter and feedback control system. During the data-loss time, significant overshoots can always be observed using the OLKF scheme than the accommodating CCLKF scheme as shown in Figures 9-11. To observe the effectiveness and flexibility of compensated scheme, Figure 13 shows the performance of OLKF and CCLKF when data-loss has occurred at another location (i.e. from 15 – 25 sec). This figure too shows the efficiency of CCLKF over the OLKF scheme.

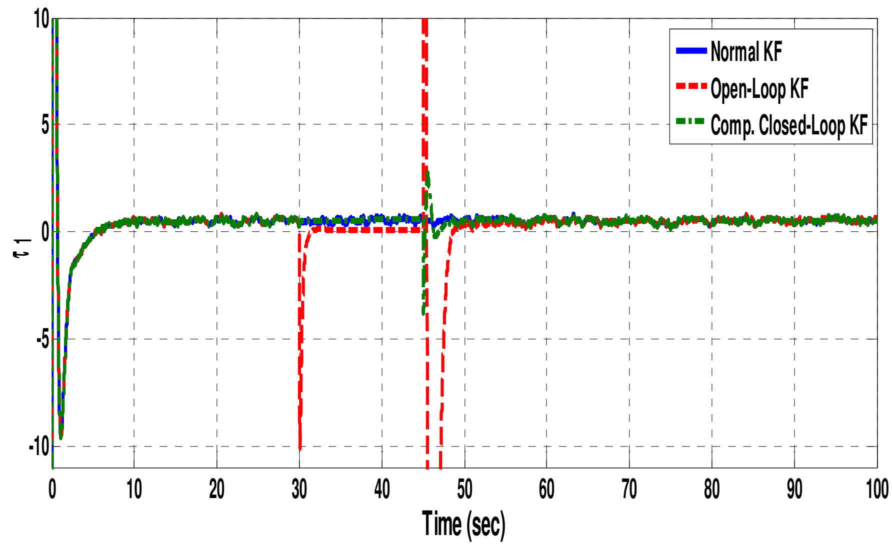

Figure 9: Input Signal τ_1

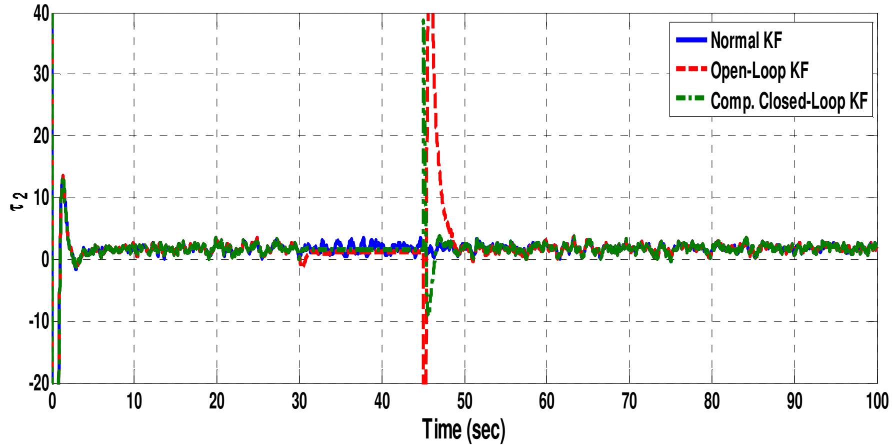

Figure 10: Input Signal τ_2

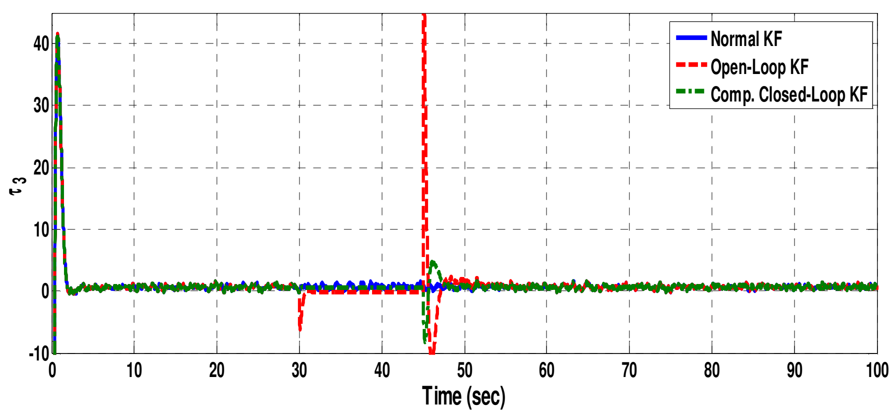

Figure 11: Input Signal τ_3

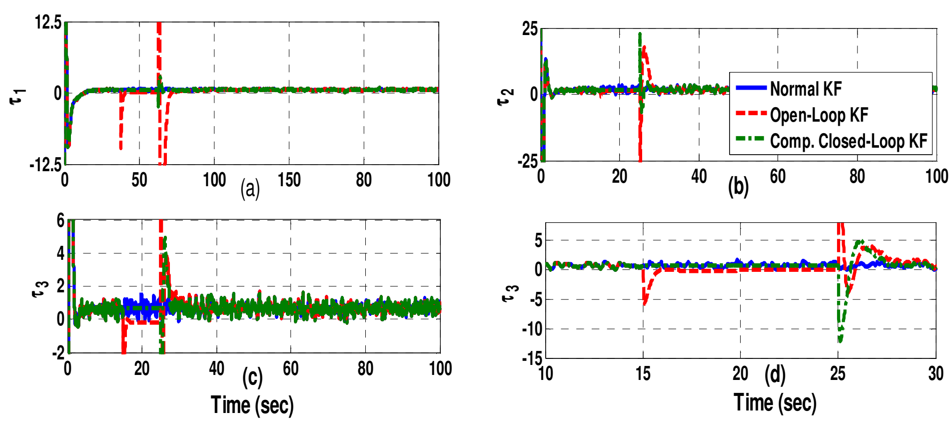

Figure 12: Simulation results of Control inputs (a) τ_1 (b) τ_2 (c) τ_3 and (d) Enlarge view of τ_3 when data loss occurs from 15-25 sec.

5.2.4 Error Analysis

Error analysis is the another investigated characteristic which reveals the efficient performance of the accommodating CCLKF scheme i.e. $e_k(i) = \sigma_r(i) - \hat{\sigma}_k(i)\ \forall\ .\ \ i = \{1,2,3\}$. Similar to the other parameters, less disruption can be seen utilizing CCLKF scheme than that of OLKF approach as shown in Figures 13-15. In other words, state estimation using CCLKF approach is less influenced by data-loss than the OLKF scheme.

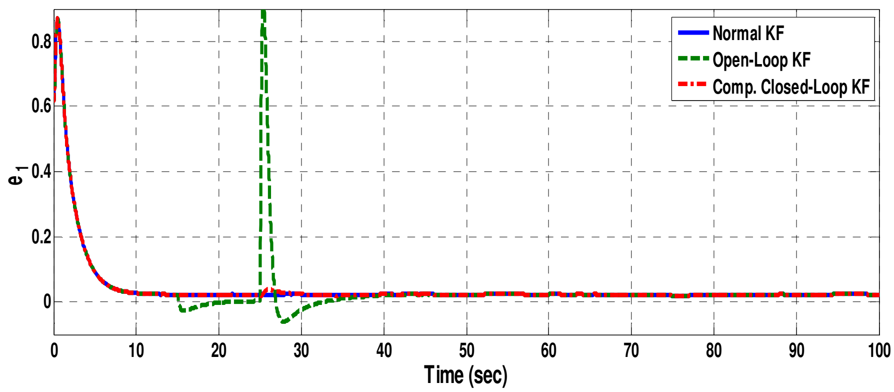

Figure 13: Estimated error in $\sigma 1$ by the three schemes

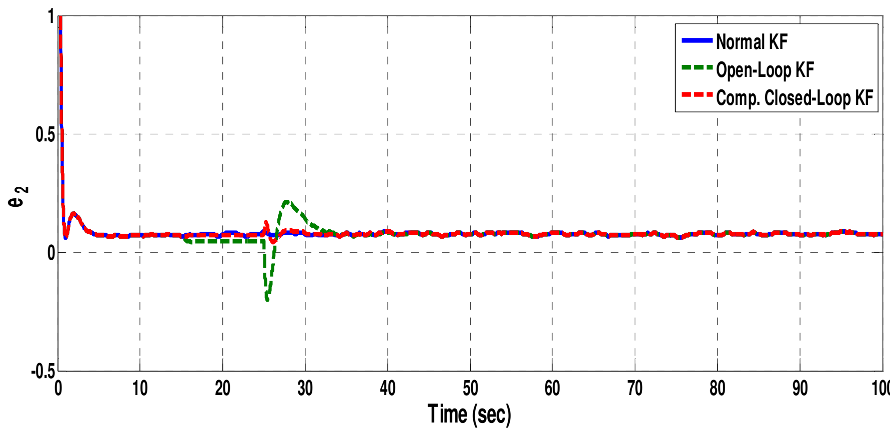

Figure 14: Estimated error in $\sigma 2$ by the three schemes

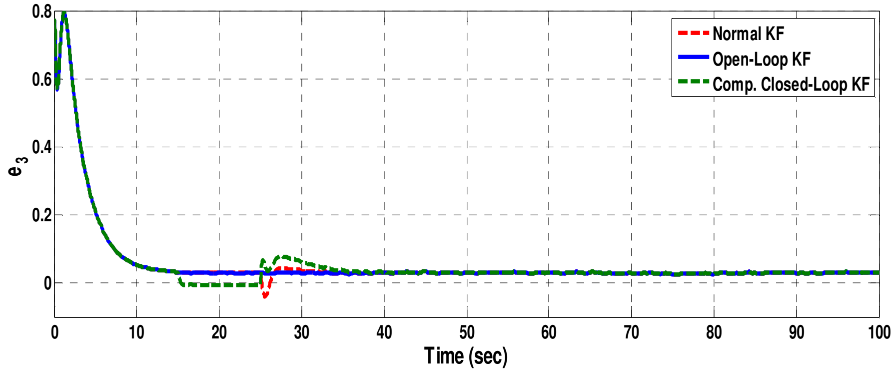

Figure 15: Estimated error in $\sigma 3$ by the three schemes

6. CONCLUSIONS AND FUTURE WORKS

6.1 Conclusion

In this work, theoretical study of two accommodating estimation schemes are discussed for a lin- earized rigid body spacecraft model subjected to loss of observations. Linearized output dynamics of the spacecraft model are derived by examining the two Direction Cosine Matrices of the same sequences for the Euler angels and MRP. The conventional Kalman filter fails to provide bounded er- ror estimation during loss of measurement, open-loop Kalman filtering (OLKF) is frequently utilized to overcome loss of output data. However, the compensated closed-loop Kalman filtering (CCLKF) scheme has been found impressive compared to OLKF scheme to the linearized spacecraft model. Simulation results of the two approaches are compared to normal Kalman filter estimation approach under no loss of output data. A comprehensive analysis of OLKF and CCLKF approaches is presented by demonstrating various characteristics through a numerical example.

6.2 Future Work

In this paper a stationary process of spacecraft dynamics has been considered. However, non-stationary processes are intended to be tested in future. Stability and convergence issues related to accommodating CCLKF approach also need to be explored. In this paper, usual linear prediction technique (*Normal Equation*) is employed to compute linear prediction coefficients. In future, faster techniques including Levinson-Durbin and Leroux-Gueguen algorithms, to handle computational burdensome of the CCLKF approaches are intended to entertain for the discussed case study.

REFERENCES

[1] R. Ahmed, D. Gu, and I. Postelwaite, “A Case Study on Spacecraft Attitude Control”. In 48th IEEE Control and Decision Conference, pages 7345 – 7350. December 2009.
[2] H. N. Al-Duwaish, “Identification of Hammerstein Models with Known Nonlinearity Structure Using Particle Swarm Optimization”. Arabian Journal of Science and Engineering, 36:1269–1276, 2011
[3] B. O. Anderson andJ. B. Moore, Optimal Filtering. Prentice Hall, Inc. 1979.
[4] A. Antoniads, H. J. Kim, and S. Sastry, “Pursuit-Evasion Strategies for Teams of Multiple Agents with Incomplete Information”. In 42nd IEEE Conference on Decision and Control, December 2003. USA.

[5] W. C. Chu. Speech Coding Algorithms; Foundation and Evolution of Standardized Coders. John Wiley & Sons Inc. 2003

[6] J. L. Crassidis and F. L. Markley, “Attitude Estimation Using Modified Rodriques Parameters” Proceeding of the Flight Mechanics/Estimation theory Symposium, pages 71 – 83. May 1999.

[7] J. F., K. Jr., and A. Gelb, Applied Optimal Estimations. 65. The Massachusetts Institute of Technology, Press, TASC. 1974

[8] H. Fang and Y. Shi, “Genetic Adaptive State Estimation with Missing Input/Output Data”. Proceeding IMechE System and Control Engineering, 224(Part I: J.): 611 – 617. March 2010

[9] M . Fu, C. E. de Souza, and Z.-Q. T. Luo, Finite-Horizon Robust Kalman Filter Design. IEEE Transactions On Signal Processing, 49(9):2103 – 2112. September 2001.

[10] T. Gao, Z. Gong, J. Luo, W. Ding, and W. Feng, “An Attitude Determination System For A small Unmanned Helicopter Using Low-Cost Sensors”. In Proceedings of the IEEE, International Conference on Robotics and Biomimetics, pages 1203–1208, December 2006, China.

[11] N. Gollu and L. Rodrigues, USA, “Control of Large Angle Attitude Maneuvers for Rigid Bodies Using Sum of Squares”. In American Control Conference, pages 3156 – 3161. July 2007.

[12] M. S. Grewal and A. P. Andrews, Kalman Filtering: Theory and Practice using MATLAB. John Wiley & Sons, Inc., third edition. 2008

[13] G. Heredia and A. Ollero, “Sensor Fault Detection in Small Autonomous Helicopter using Ob- server Kalman Filter Identification”. Proceeding of the 2009 IEEE International Conference on Mechatronics, April 2009, Spain.

[14] H. W. Jie Ding, YangShi and F. Ding, “A modified stochastic gradient based parameter estimation algorithm for dual-rate sampled-data systems”. Digital Signal Processing, 20: 1238–1247. 2010

[15] M. B. Kadri, “Disturbance Rejection in Nonlinear Uncertain Systems Using Feedforward Control”. Arabian Journal of Science and Engineering, 38:2439–2450. 2013.

[16] F. Khan, M. I. Khattak, M. Shafi, M. Saleem, F. Khan, and N. Khan, “A case study of variable window size in linear prediction techniques”. World Applied Sciences Journal, 23(12):71–80. 2013.

[17] N. Khan ,. Linear Prediction Approaches for Compensation of Missing Measurements in Kalman Filtering. PhD thesis, University of Leicester, December 2011.

[18] N. Khan, S. Fekri, and D. Gu, “A Sub-Optimal Kalman Filtering for Discrete-Time LTI Systems with Loss of Data”. In The 7th IFAC Conference on Intelligent Control Automation and Robotics. June 2010, Portugal.

[19] N. Khan, S. Fekri, and D. Gu, “Improvement on state estimation for discrete time LTI systems with measurement loss”. Measurement, Vol-43(2010):1609 – 1622.

[20] N. Khan, M. I. Khattak, and D. Gu, “Robust State Estimation and its Application to Spacecraft Control”. Automatica, 48(12):3142 – 3150. December 2012

[21] N. Khan, M. I. Khattak, M. N. Khan, F. Khan, L. Khan, S. A. Salam, and D. Gu, “Implementation of Linear Prediction Techniques in State Estimation”. In The 10th IEEE International Bhurban Conference on Applied Sciences & Technology, IBCAST. January 2013 Pakistan.

[22] E. J. Lefferts, F. L. Markley and M. D. Shuster, “Kalman Filtering for Spacecraft Attitude Estimation”. AIAA Journal of Guidance, Control and Dynamics, 5(5): pages 417 – 429. September-October 1982.

[23] X. Liu and A. Goldsmith, “Kalman Filtering with Partial Observation Loss”. In 43rd IEEE Conference on Decision and Control, pages 4180 – 4186, December 2004

[24] R. Patton and J. Chen, “Observer-based Fault Detection and Isolation: Robustness and Applications”. Control Engineering Practice, 5(5):671 – 682. 1997.

[25] R. J. Patton, F. J. Uppal, S. Simani, and B. Polle, “Robust FDI Applied to Thruster Faults of a Satellite system”. Control Engineering Practice, 2010(18):1093 – 1109. July 2010

[26] F. N. Pirmoradi, F. Sassani, and C. W. de Silva, “Fault Detection and Diagnosis in a Spacecraft Attitude Determination System”. ACTA ASTRONAUTICA, 65:710 – 729. April 2009

[27] L. Schenato, July 2006. “Kalman Filter for Network Control Systems with Random Delay and Packet Loss. In Conf. of Mathematical Theory of Networks and Systems, Japan.

[28] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, January 2007. “Foundation of Control and Estimation Over Lossy Network”. Proceeding of The IEEE, 95(1): 163 – 187.

[29] L. Shi, M. Epstein, A. Tiwari, and R. M. Murray, December 2005. “Estimation with Information Loss: Asymptotic Analysis and Error Bounds”. In 44th IEEE Conference on Decision and Control, pages 1215 – 1221, Spain.

[30] Y. Shi, H. Fang, and M. Yan,, November 2009. "Kalman filter-based adaptive control for networked systems with unknown parameters and randomly missing outputs". International Journal of Robust and Nonlinear Control, 1976-1992.

[31] S. Simani, C. Fantuzzi, and R. J. Patton, 2003. Model Based Fault Diagnosis in Dynamic System Using Identification Techniques. Advances in Industrial Control, Springer.

[32] D. Simon., 2006. Optimal State Estimation Kalman, H∞ and Nonlinear Approaches. John Wiley & Sons, Inc.

[33] B. Sinopoli, M. Franceschetti, L. Schenato, K. Poolla, M. I. Jordon, and S. S. Sastry, September 2004. "Kalman Filtering with Intermittent Observations". IEEE Transections on Automatic Control, 49(9):1453– 1464.

[34] F. Q. Thomas, 2002. Discrete time Speech Signal Processing; Principles and Practice. Prentice Hall Signal Processing Series. Prentice Hall, Massachusetts Institute of Technology, Lincoln Lab.