

# THE IMPACT OF ASPECT RATIO ON THE BEHAVIOUR OF RIGID WATER STORAGE TANKS

Samir H. Helou

An-Najah National University, Nablus - Palestine

## ABSTRACT

*Ubiquitous reinforced concrete water storage tanks are quite popular and widely used in Palestine as in elsewhere in the world; they form pivotal components of major bulk-water carrier systems. In essence, they form lifelines to many communities; the water storage concept is as old as civilization itself. Location and land availability often dictate the topology of the tank's structure. They may be either shallow and stubby or deep yet slender or anywhere in between having an aspect ratio dictated by overall site conditions. In all cases adequate structural analysis is mandatory.*

*Modern computer programs demand adequate numerical models and proper loading data. The purpose of the present discourse is to briefly present a general formulation of a three-dimensional hydrodynamic model with the intention of evaluating and comparing three different topologies and to conduct a parametric study to evaluate the impact of the aspect ratio of cylindrical rigid tanks on the general response. This includes, yet it is not limited, to a thorough presentation of the prescribed hydrodynamic pressure, indispensable for accurate evaluation of the induced forces on the tank's shell. The loading part of the seismic analysis procedure forms the principal focus of the present study. The study targets three different geometries of vernacular upright rigid water storage cylindrical tanks built on grade. Such tanks are customarily comprised of a flat roof supported by inner columns. One criterion for the investigation is the magnitude of the base shear and the bending moment at the base knowing that the hydrostatic forces have little or no bearing on such forces. The magnitude of the hydrodynamic force contribution on the overall design in the three selected cases is numerically evaluated and compared. The results of the study point in the direction that from a seismic perspective the short and stubby tanks have the edge over other tank topologies. Furthermore, the present discourse is limited to versatile ground-supported column-free structures. The narration highlights analysis procedures based on the current state of the art practice. However, reinforced concrete section design is beyond the stated objective.*

## KEYWORDS

*Water Storage Tanks, Impulsive Pressure, Convective Pressure, Rigid Water Tanks, Hydrodynamic Forces.*

## 1. INTRODUCTION

Rigorous evaluation of the dynamically induced stresses on the tank's wall is generally a complex undertaking. It involves, inter alia, the interaction between the lateral displacement of the tank's wall and that of the fluid motion. However, in rigid tanks this effect is less pronounced. Hydrodynamic forces in fluid storage tanks are enormous under seismic action while the damping influence is lower than in regular structures. Two widely separated vibration periods govern the structural behavior during earthquake excitations; the sloshing frequency of the contained fluid which is long whereas the coupled vibration modes of the elastic shell and the contained fluid have periods less than 1 second. A water tank's behavior is significantly dictated by its topology including, inter alia, its specific aspect ratio, i.e. height to radius ratio.

Available rigid liquid storage tanks' design codes, are based on the works of G. W. Housner, i.e. the spring mass model that essentially considers the first mode of vibration of the tank and in which the body of water, during seismic events is split into two components. One is the convective mass component while the other is the impulsive mass component. The impulsive mass i.e. the lower portion of the water body moves in unison with the body of the tank. The convective part i.e. the upper portion of the fluid moves relative to the walls of the tank, thus creating a sloshing motion. The two components do not necessarily add up to the total water mass particularly for deeper tanks. In the following narration the standard procedure is followed for evaluating the hydrodynamic loads based on the ACI 350.3-06 on three selected rigid cylindrical tanks built at the same seismic zone and have nearly the same storage capacity. The entire exercise refers to ASCE 7-22 and IBC 2012. One is shallow and stubby, the second is of medium height while the third is rather deep and slender. All assumed to be cast in situ. The customary design assumptions for such an undertaking are as follows.

- All Tanks are cylindrical, rigid, ground supported having flat bottoms.
- All Tanks are subjected to horizontal and vertical excitations.
- Normal environmental exposure is assumed.

Since standard structural analysis packages are not programed to depict seismic hydrodynamic forces resulting from the contained fluid motion. It is imperative that such forces are evaluated a priori and properly applied to the structural model. The following discourse presents hand calculation for water tank analysis and design conducted under the following primary loading conditions:

- Inertia forces resulting from the walls and the roof acceleration.
- Hydrostatic fluid pressure.
- Hydrodynamic forces resulting from induced surface waves.
- Effects of vertical ground acceleration.

## **2. HYDRODYNAMIC PRESSURE DISTRIBUTION**

Stresses in the walls of vertical circular cylindrical liquid storage tanks depend primarily on the distribution of the internal fluid pressure. For the static condition the stress distribution from the fluid at rest is linear and poses a trivial study case; such stresses follow a triangular distribution with the maximum value occurring at the base. However, when lateral ground excitation is of importance complex considerations become indispensable. Hydrodynamic pressure distribution involves two components of pressure; the impulsive component, frequency independent, and the convective component which is frequency dependent. Both occur simultaneously in addition to the hydrostatic pressure distribution; this is clearly manifested by the general Bernoulli's equation. The impulsive component involves the volume of water at the bottom of the tank while the convective component involves the upper volume of the water because it is the region where the surface dynamic effects on the fluid motion are exhibited.

## **3. THE TOPOLOGY OF THE THREE TANKS**

The three tanks selected for the present exercise have a modest capacity of about 1100 m<sup>3</sup>. The tanks are built on an assumed Site Class B location according to ASCE 22, table 20.2-1. All tank roofs are comprised of flat slabs supported on periphery walls. Pertinent details are shown in Table 1. The first two tanks are classified as shallow since the radius to height ratio is greater than 0.6 whereas the third tank is classified as a deep one.

Table 1: Structural Properties of the Three Tanks

|                    | Shallow & Sturdy    | Medium              | Deep & Slender      |
|--------------------|---------------------|---------------------|---------------------|
| Inner Diameter     | 20 m                | 17 m                | 12.5 m              |
| Water Depth        | 3.60 m              | 5.0 m               | 9.25 m              |
| Volume of Water    | 1130 m <sup>3</sup> | 1134 m <sup>3</sup> | 1135 m <sup>3</sup> |
| Free Board         | 1.25 m              | 1.25 m              | 1.25 m              |
| Weight of Water    | 11,085 KN           | 11,125 KN           | 11,134 KN           |
| Depth of Roof Slab | 35 cm               | 35 cm               | 35 cm               |
| Total Tank Height  | 5.2 m               | 6.60 m              | 10.85 m             |
| Wall Height        | 4.85 m              | 6.25 m              | 10.50 m             |
| Wall Thickness     | 40 cm               | 40 cm               | 40 cm               |
| S O G Thickness    | 50 cm               | 50 cm               | 50 cm               |
| Roof Thickness     | 35 cm               | 35 cm               | 35 cm               |
| H <sub>L</sub> /D  | 0.18                | 0.29                | 0.74                |
| D/H <sub>L</sub>   | 5.56                | 3.45                | 1.35                |
| Weight of Walls    | 2979 KN             | 3360 KN             | 4557 KN             |
| Weight of Roof     | 2748 KN             | 1985 KN             | 1073 KN             |
| R/H                | 2                   | 1.31                | 0.54                |

#### 4. SEISMIC DATA FOR THE TANKS

Table 2: Seismic Data for the Three Tanks

|   |                        |
|---|------------------------|
| Occupancy Category  | IV                     |
| Seismic Design Category   | D                      |
| Seismic Importance factor   | 1.25                   |
| Response Modification factor R <sub>i</sub>                             | 2                      |
| Response Modification factor R <sub>c</sub>                             | 1                      |
| PGA   | 0.3g                   |
| S <sub>1</sub>  | 0.20g                  |
| S <sub>s</sub>  | 0.95g                  |
| S <sub>DS</sub>   | 1.05                   |
| S <sub>D1</sub>   | 0.4                    |
| T <sub>S</sub> = S <sub>D1</sub> /S <sub>DS</sub> [Transitional Period] | 0.38                   |
| Site Class  | B                      |
| Vertical Seismic Coefficient  | 0.2 S <sub>DS</sub>    |
| Unit Weight of water  | 9.81 KN/m <sup>3</sup> |
| Unit Weight of Concrete   | 25 KN/m <sup>3</sup>   |

The selected tanks for the present exercise are all of Type 1 in accordance with the classifications of ACI 350.3-06, i.e. they are circular with a fixed base. Being essential facilities, they belong to Occupancy Category IV, [ASCE table 11.5-1]. The relevant design parameters follow:

- ◆ Vertical Seismic Coefficient = 0.2 S<sub>DS</sub>
- ◆ Seismic Design Category D [ASCE 7-22; 11.6-1 & 11.6-2]
- ◆ Seismic Importance Factor = 1.25, (ASCE 7-22; Table 1.5 -2)
- ◆ Response Modification R<sub>i</sub> = 2; R<sub>c</sub> = 1 [ACI 350.3-06, Table 4.1.1 B]

## 5. ANALYSIS PROCEDURE

Structural analysis and design of reinforced concrete water storage tanks involve the following fundamental steps, albeit section design is beyond the present discourse:

1. Computing the Impulsive and the Convective mass components of the contained fluid assuming the tanks are full.
2. Evaluating of the tanks' dynamic properties.
3. Evaluating the lateral seismic forces.
4. Computing the force distribution due to lateral and vertical acceleration.
5. Computing the base shear of the systems.
6. Evaluating the bending moment at the base.
7. Evaluating hoop stresses and forces.

### 5.1. Computing the Convective and the Impulsive Mass Components

The impulsive and the convective masses of all tanks are computed from Equations 9-15 and 9-16 of ACI 350-3-06 or from Figure 9.2.1:

The Shallow Tank

$$W_c = 0.74 \times 11085 = 8203 \text{ KN}$$

$$W_i = 0.21 \times 11085 = 2328 \text{ KN}$$

The Medium Tank

$$W_c = 0.63 \times 11125 = 7009 \text{ KN}$$

$$W_i = 0.33 \times 11125 = 3671 \text{ KN}$$

The Deep Tank

$$W_c = 0.31 \times 11134 = 3452 \text{ KN}$$

$$W_i = 0.70 \times 11134 = 7794 \text{ KN}$$

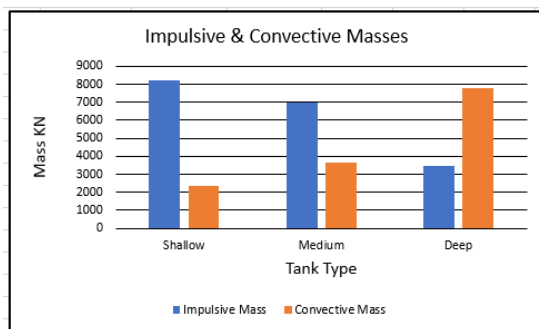


Figure 1: Relative Magnitude of Impulsive and Convective Masses Relative

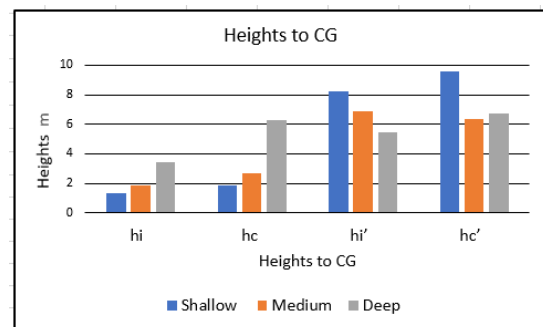


Figure 2: Heights of the Center of Gravity

## 5.2. Heights of the Centers of Gravity above base for the Impulsive and the Convective Components

**For the Shallow Tank, EBP** [excluding base pressure]

The following exercise is based on ACI 350.3-06, Equations 9-17 and 9-19

$$D/H_L = 5.56 \geq 1.333; \quad h_i/H_L = 0.375$$

$$h_i = 0.375 \times 3.6 = 1.35 \text{ m} \quad [\text{Impulsive Component Height}]$$

$$h_c/H_L = 0.52$$

$$h_c = 0.52 \times 3.6 = 1.86 \text{ m} \quad [\text{Convective Component Height}]$$

**IBP** [Including Base Pressure]

Based on equations 9-20 to 9-22 of ACI 350.3-1 the following values are computed

$$h'_i / H_L = 2.28$$

$$h'_i = 2.28 \times 3.6 = 8.21 \text{ m}$$

$$h'_c / H_L = 2.7$$

$$h'_c = 2.7 \times 3.6 = 9.59 \text{ m}$$

In similar manner the heights of the centres of gravity for other topologies are computed. Table 3 presents the results.

Table 3: Height above base to the centres of Gravity

|     | Shallow | Medium | Deep |
|-----|---------|--------|------|
| hi  | 1.35    | 1.875  | 3.47 |
| hc  | 1.86    | 2.72   | 6.29 |
| hi' | 8.21    | 6.85   | 5.44 |
| hc' | 9.59    | 6.38   | 6.72 |

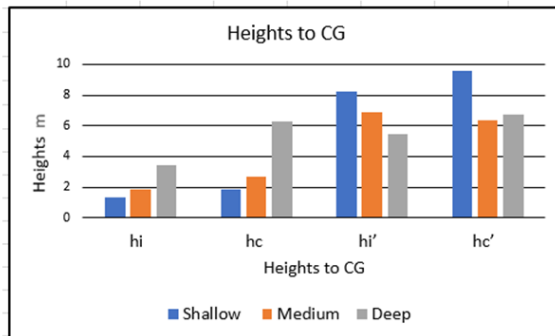


Figure 3: Height to the Center of Gravity

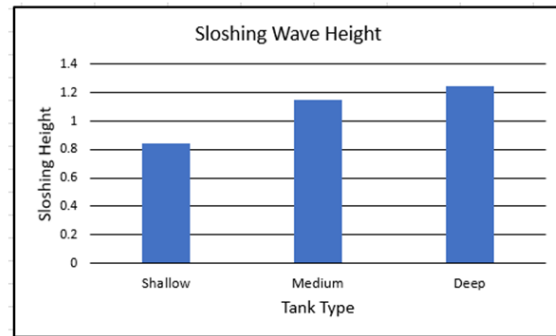


Figure 4: Sloshing Wave Heights above Base

## 6. DYNAMIC PROPERTIES OF THE TANKS-SITE SPECIFIC SEISMIC RESPONSE COEFFICIENT-

Following the procedure of paragraph 9.3.4 of ACI 350.3-06, the following the dynamic properties of the tanks are computed:

**The Shallow and Stubby Tank**

$$D/H_L = 5.56$$

From Figure 9.3.4 ACI 350.3-06 and from Equation 9-24

$$C_w = 0.126$$

$$C_I = 0.126 \times [400 / (10 \times 10)]^{1/2} = 0.252$$

$$\omega_i = (0.252/3.6) \times [1000 \times 24870 \times 9.81/25]^{1/2}$$

$$= 218.7 \text{ rad/sec}$$

$$T_i = 2 \pi / 218.7 = 0.029$$

$$T_s = 0.38$$

Since  $T_i < T_s$ ,  $C_i = S_{DS}$

$$C_i = 1.05$$

$$\Omega_c = \lambda / \sqrt{D}$$

$$\lambda = [3.68 \text{ g} \times \tanh(3.68 H_L/D)]^{1/2} \text{ Equation 9.29 ACI 350.3.}$$

$$= [3.68 \times 9.81 \times \tanh(3.68 \times 3.6/20)]^{1/2}$$

$$= 4.58$$

$$\omega_c = 4.58 / \sqrt{20} = 1.02$$

$$T_c = 2 \pi / \omega_c$$

$$= 6.16 > 1.6/T_s; [1.6/0.38 = 4.21]$$

$$C_c = 2.4 \times S_{DS} / T_c^2 = 2.4 \times 1.05 / 6.16^2$$

$$= 0.067$$

Table 4 shows the results for all tank topologies that are quantified in exactly the same procedures.

Table 4: Seismic Parameters

| Tank    | T <sub>i</sub> | T <sub>c</sub> | C <sub>I</sub> | C <sub>C</sub> |
|---------|----------------|----------------|----------------|----------------|
| Shallow | 0.029          | 6.16           | 1.05           | 0.067          |
| Medium  | 0.032          | 4.83           | 1.05           | 0.108          |
| Deep    | 0.044          | 3.72           | 1.05           | 0.16           |

## 7. CALCULATIONS OF THE FREE BOARD HEIGHT

The sloshing heights are computed using Equation 7.2, ACI 350.3-06

The Shallow Tank

$$d_{\max} = C_c \times I \times D/2 = 0.067 \times 1.25 \times 20/2 = 0.84 \text{ m}$$

The Medium Tank

$$d_{\max} = C_c \times I \times D/2 = 0.108 \times 1.25 \times 17/2 = 1.15 \text{ m}$$

The Deep Tank

$$d_{\max} = C_c \times I \times D/2 = 0.16 \times 1.25 \times 12.5/2 = 1.25 \text{ m}$$

Hence, an assigned freeboard height of 1.25 m for all is adequate.

### 7.1. Estimation of the Minimum Wall Thicknesses

$$t = \frac{\epsilon_{sh} E_s + f_s - n f_{ct} T}{100 f_s f_{ct}}$$

$$= 0.00042 T$$

$$E_s = 200,000 \text{ MPa}$$

$$f_s = 420/3 = 140 \text{ MPa}$$

Permissible stresses according to ACI 350M-06

-Normal Environmental Exposures-

$$f_{ct} = 0.1 \times 350 = 35 \text{ MPa}$$

$$\varepsilon_{sh} = \text{coefficient of shrinkage of plain concrete} \\ = 0.0003$$

$$n = 8$$

For the shallow and sturdy

$$T = \gamma H R$$

$$= 1000 \times 10 \times 3.6 = 36000 \text{ t/m}$$

$$t_{\min} = 0.00042 \times 36000 = 15.1 \text{ cm}$$

For the medium size tank

$$T = \gamma H R$$

$$= 1000 \times 8.5 \times 5 = 42500 \text{ t/m}$$

$$t_{\min} = 0.00042 \times 42500 = 17.9 \text{ cm}$$

For the deep and slender tank

$$T = \gamma H R$$

$$= 1000 \times 6.25 \times 9.25 = 57812.5 \text{ t/m}$$

$$t_{\min} = 0.00042 \times 57812.5 = 24.3 \text{ cm}$$

$\varepsilon$  = effective mass coefficient defined as the ratio of equivalent dynamic mass of the tank shell to its actual mass. [ACI 350.3-06 section 9.6.2]

For the Shallow and Subby Tank

$$\varepsilon = 0.0151 (D/H_L)^2 - 0.1908 (D/H_L) + 1.021$$

$$= 0.0151 (5.56)^2 - 0.1908 (5.56) + 1.021$$

$$= 0.427 < 1 \text{ OK}$$

For the Medium Tank

$$\varepsilon = 0.542 < 1 \text{ OK}$$

For the Deep and Slender Tank

$$\varepsilon = 0.79 < 1 \text{ OK}$$

## 8. LATERAL HYDROSTATIC PRESSURE DISTRIBUTION

$y = 0$  is at the base

Shallow Tank

- Hydrostatic pressure at walls =  $35.3 - 9.81 y$   
Acts outward on the outer surface of the tank wall
- Hydrostatic Pressure on the base slab =  $35.3 \text{ KN/m}^2$

Medium Tank

- Hydrostatic pressure at walls =  $49.05 - 9.81 y$   
Acts outward on the inner surface of the tank wall
- Hydrostatic Pressure on the base slab =  $49.05 \text{ KN/m}^2$

Deep Tank

- Hydrostatic pressure at walls =  $90.74 - 9.81 y$   
Acts outward on the inner surface of the tank wall
- Hydrostatic Pressure on the base slab =  $90.74 \text{ KN/m}^2$

## 9. LATERAL HYDRODYNAMIC FORCES

For the Shallow Tank

Lateral force caused by the impulsive mass

$$P_i = C_i \times I \times W_i / R_i \quad [\text{ACI 350.3 Equation 4-1}]$$

$$= 1.05 \times 1.25 \times 2328 / 2 = 1528 \text{ KN}$$

Impulsive Pressure at Base

$$P_i = [1528 / 2 [4 \times 3.6 - 6 \times 1.35]] / 3.6^2$$

$$= 371.3 \text{ KN/m}^2$$

Impulsive Pressure at Water Surface

$$P_i = [1528 / 2 [4 \times 3.6 - 6 \times 1.35 - 6 \times 3.6 + 12 \times 1.35]] / 3.6^2$$

$$= 53.1 \text{ KN/m}^2$$

$$P_i y = 371.3 - 88.4 y$$

Lateral force caused by the Convective Mass

$$P_c = C_c \times I \times W_c / R_c \quad [\text{ACI 350.3 Equation 4-4}]$$

$$= 0.067 \times 1.25 \times 8203 / 1 = 687 \text{ KN}$$

Compulsive pressure at Base

$$P_c = [(687 / 2) [4 \times 3.6 - 6 \times 1.86]] / 3.6^2$$

$$= 85.9 \text{ KN/m}^2$$

Compulsive Pressure at Water Surface

$$P_c = [687 / 2 [4 \times 3.6 - 6 \times 1.86 - 6 \times 3.6 + 12 \times 1.86]] / 3.6^2$$

$$= 105 \text{ KN/m}^2$$

$$P_c y = 85.9 + 5.3 y$$

Lateral force due to Wall Inertia

$$P_w = C_i \times I \times \epsilon \times W_w / R_i$$

$$= 1.05 \times 1.25 \times 0.427 \times 2979 / 2 = 835 \text{ KN}$$

Lateral force due to Roof Inertia

$$P_r = C_i \times I \times W_r / R_i$$

$$= 1.05 \times 1.25 \times 2748 / 2 = 1503 \text{ KN}$$

### For the Medium Tank

Lateral force caused by the impulsive mass

$$P_i = C_i \times I \times W_i / R_i \quad [\text{ACI 350.3 Equation 4-1}]$$

$$= 1.05 \times 1.25 \times 3671 / 2 = 2409 \text{ KN}$$

Impulsive Pressure at Base

$$P_i = 2409 / 2 [4 \times 5 - 6 \times 1.875] / 5^2$$

$$= 421.6 \text{ KN/m}^2$$

Impulsive Pressure at Water Surface

$$P_i = [(2409 / 2) [4 \times 5 - 6 \times 1.875 - 6 \times 5 + 12 \times 1.875]] / 5^2$$

$$P_i = 60.2 \text{ KN/m}^2$$

$$P_i y = 421.6 - 72.3 y$$

Lateral force caused by the Convective Mass

$$P_c = C_c \times I \times W_c / R_c \quad [\text{ACI 350.3 Equation 4 - 4}]$$

$$= 0.108 \times 1.25 \times 7009 / 1 = 764 \text{ KN}$$



Convective Pressure at Base

$$P_c = 764/2 [4 \times 5 - 6 \times 2.72]/5^2 = 56.2 \text{ KN/m}^2$$

Convective Pressure at Water Surface

$$P_c = [(764/2) [4 \times 5 - 6 \times 2.72 - 6 \times 5 + 12 \times 2.72]]/5^2 = 96.6 \text{ KN/m}^2$$

$$P_c y = 56.2 + 8.08 y$$

Lateral force due to wall inertia

$$P_w = C_i \times I \times \epsilon \times W_w/R_i = 1.05 \times 1.25 \times 0.542 \times 3360/2 = 1195 \text{ KN}$$

Lateral force due to roof inertia

$$P_r = C_i \times I \times W_r/R_i = 1.05 \times 1.25 \times 1985/2 = 1303 \text{ KN}$$

### For the Deep Tank

Lateral force caused by the impulsive mass

$$P_i = C_i \times I \times W_i / R_i \quad [\text{ACI 350.3 Equation 4-1}] = 0.72 \times 1.25 \times 7794/2 = 4038 \text{ KN}$$

Impulsive Pressure at Base,  $y=0$

$$P_i = [4038/2 [4 \times 9.25 - 6 \times 3.47]]/9.25^2 = 381.8 \text{ KN/m}^2$$

Impulsive Pressure at Water Surface

$$y=9.25 = [(4038/2) [4 \times 9.25 - 6 \times 3.47 - 6 \times 9.25 + 12 \times 3.47]]/9.25^2 = 54.7 \text{ KN/m}^2$$

$$P_i(y) = 381.8 - 35.4 y$$

Horizontal Distribution as specified by ACI 350 =  $(2/\pi r) [318.8 - 35.4 y] \cos \Theta$

Lateral force caused by the convective mass

$$P_c = C_c \times I \times W_c / R_c \quad [\text{ACI 350.3 Equation 4-4}] = 0.16 \times 1.25 \times 3452/1 = 629 \text{ KN}$$

Convective Pressure at base

$$P_c = -2.72 \text{ KN/m}^2$$

Convective Pressure at Water Surface

$$= 70.7 \text{ KN/m}^2$$

$$P_c(y) = -2.72 + 7.94 y$$

Horizontal Distribution as specified by ACI 350 =  $(16/9\pi r) [-2.72 + 7.94 y] \cos \Theta$

Lateral force due to Wall Inertia

$$P_w = C_i \times I \times 0.79 \times W_w/R_i = 0.72 \times 1.25 \times 0.79 \times 4557/2 = 1620 \text{ KN}$$

Lateral force due to Roof Inertia

$$P_r = 0.72 \times 1.25 \times 1073/2 = 483 \text{ KN}$$

Table 5: Seismic Induced Forces at the Tanks' Walls

|         | Pi [KN] | Pc [KN] | Pw [KN] | Pr [KN] |
|---------|---------|---------|---------|---------|
| Shallow | 1528    | 687     | 835     | 1503    |
| Medium  | 2408    | 764     | 1195    | 1303    |
| Deep    | 4038    | 629     | 1620    | 483     |

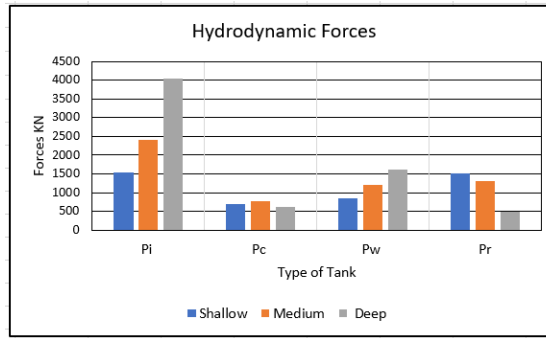


Figure 5: Seismic Induced Forces on the Tanks' Walls

### 9.1. The Total Base Shear

Base Shear in accordance with ACI 350.3-06 and Equation 4-5 the base shear at the base of the tank is

$$\text{Base Shear, } V = [(P_i + P_w + P_r)^2 + P_c^2]^{1/2}$$

$$\begin{aligned} \text{Shallow tank} &= [(1528 + 835 + 1503)^2 + 687^2]^{1/2} \\ &= 3927 \text{ KN} \end{aligned}$$

$$\begin{aligned} \text{Medium tank} &= [(2409 + 1195 + 1303)^2 + 764^2]^{1/2} \\ &= 4966 \text{ KN} \end{aligned}$$

$$\begin{aligned} \text{Deep tank} &= [(4038 + 1620 + 483)^2 + 629^2]^{1/2} \\ &= 6173 \text{ KN} \end{aligned}$$

### 9.2. Bending Moment at Base [EBP]

$$\text{Moment at Base, } M_b = [(M_i + M_w + M_r)^2 + M_c^2]^{1/2}$$

➤ For the Shallow Tank

$$M_w = P_w \times h_w = 835 \times 4.65/2 = 1941.4 \text{ KN-m}$$

$$M_r = P_r \times h_r = 1803 \times 5 = 9015 \text{ KN-m}$$

$$M_i = P_i \times h_i = 1528 \times 1.35 = 2063 \text{ KN-m}$$

$$M_c = P_c \times h_c = 687 \times 1.86 = 1278 \text{ KN-m}$$

$$M_b = 13082 \text{ KN-m}$$

### 9.3. Bending Moment at Base [IBP]

$$\text{Moment at Base, } M_b = [(M_i' + M_w + M_r)^2 + M_c'^2]^{1/2}$$

➤ For the Shallow tank

$$M_i' = P_i \times h_i' = 1528 \times 8.21 = 1254 \text{ KN-m}$$

$$M_c' = P_c \times h_c' = 687 \times 9.59 = 6588 \text{ KN-m}$$

$$M_b = [(1254 + 1941.4 + 9015)^2 + 6588^2]^{1/2} = 13874 \text{ KN m}$$

Similarly values for the other tank topologies are computed; they are shown in Table 6.

Table 6: Forces at the Base

|         | Base Shear<br>KN | Moment at Base<br>-EBP-<br>KN - m | Moment at Base<br>-IBP-<br>KN - m |
|---------|------------------|-----------------------------------|-----------------------------------|
| Shallow | 3927             | 13082                             | 13874                             |
| Medium  | 4926             | 19406                             | 31656                             |
| Deep    | 6173             | 25761                             | 33676                             |

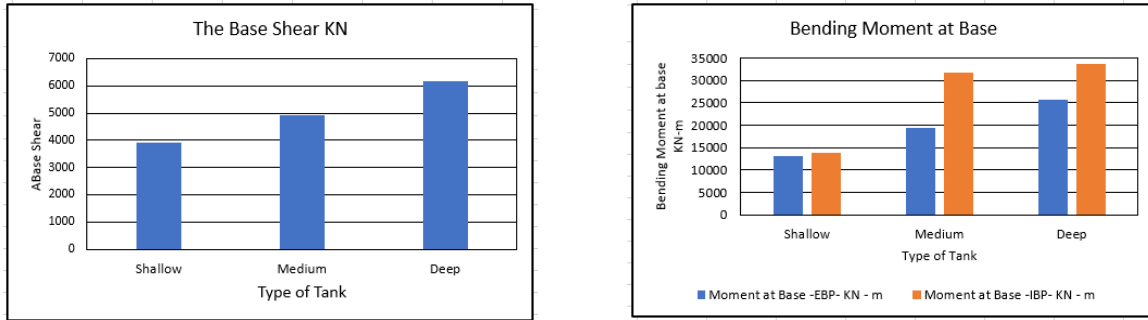


Figure 6: Base Shear & Bending Moment at Base of the Tanks

## 10. PRESSURE AT BASE DUE TO WALL INERTIA

The Shallow and Subby tank

$$835 / (2 \times 3.14 \times 10 \times 4.65) = 2.86 \text{ KN/m}^2$$

The Medium tank

$$1195 / (2 \times 3.14 \times 8.5 \times 6.15) = 3.64 \text{ KN/m}^2$$

The Deep and Slender tank

$$1620 / (2 \times 11.25 \times 3.14 \times 6.25) = 3.67 \text{ KN/m}^2$$

## 11. MAXIMUM HYDRODYNAMIC VERTICAL EXCITATION

From Equation 9-31 and Equation 4-14, ACI 350.3-06 the natural period of vibration of vertical liquid motion and the effective spectral acceleration are computed

$$T_v = 2 \times 3.14 \times [9.81 \times 20 \times 3.6^2 / 2 \times 9.81 \times 400 \times 24870]^{1/2}$$

$$= 0.0287 \text{ sec} \leq 0.38$$

$$\ddot{U} = 1.05 \times 1.25 \times 0.67 / 2 = 0.44g$$

Shallow Tank: Vertical Pressure at base,  $p_v = 3.6 \times 9.81 \times 0.44 = 15.5 \text{ KN/m}^2$

For medium tank

Medium Tank: Vertical Pressure at base,  $p_v = 5 \times 9.81 \times 0.44 = 21.6 \text{ KN/m}^2$

For deep tank

Deep Tank: Vertical Pressure at base,  $p_v = 9.25 \times 9.81 \times 0.44 = 40.0 \text{ KN/m}^2$

$$p_{\max} = [(p_{iw} + p_{ww})^2 + p_{cw}^2 + p_v^2]^{1/2}$$

Maximum Hydrodynamic Pressure

For the shallow tank

$$p_{\max} = [(23.66+2.86)^2 + 4.14^2 + 15.5^2]^{1/2} = 31 \text{ KN/m}^2$$

For the medium tank

$$p_{\max} = [(31.6+3.64)^2 + 3.62^2 + 21.6^2]^{1/2} = 41.5 \text{ KN/m}^2$$

For the deep tank

$$p_{\max} = [(38.9+3.67)^2 + 40^2]^{1/2} = 40.5 \text{ KN/m}^2$$

Table 7: Seismic Induced Pressure at the Tanks' Bases

|         | pi<br>[KN/m <sup>2</sup> ] | pc<br>[KN/m <sup>2</sup> ] | pw<br>[KN/m <sup>2</sup> ] | pv<br>[KN/m <sup>2</sup> ] |
|---------|----------------------------|----------------------------|----------------------------|----------------------------|
| Shallow | 23.66                      | 4.14                       | 2.86                       | 15.5                       |
| Medium  | 31.6                       | 3.62                       | 3.64                       | 21.6                       |
| Deep    | 38.9                       | 0                          | 3.67                       | 40                         |

Table 8: Impact of Hydrodynamic Pressure over Hydrostatic Pressure

|         | Hydrostatic<br>Pressure<br>KN/m <sup>2</sup> | Hydrodynamic<br>Pressure<br>KN/m <sup>2</sup> | Impact |
|---------|--|---|--------|
| Shallow | 35.3   | 31  | 88%    |
| Medium  | 49.03  | 41.5  | 85%    |
| Deep    | 90.74  | 40.5  | 45%    |

## 12. HOOP FORCES DUE TO HYDROSTATIC PRESSURE

Maximum hoop forces due to the generated hydrostatic pressure in all tanks is as follows assuming that the maximum values occur at two thirds of respective depths;

$$HS = (2/3) \gamma_w H R$$

$$\text{For the Shallow Tank} = 10 \times (2 \times 3.6/3) \times 10 = 240 \text{ KN/m}$$

$$\text{For the Medium Tank} = 10 \times (2 \times 5.0/3) \times 8.5 = 283 \text{ KN/m}$$

$$\text{For the Deep Tank} = 10 \times (2 \times 9.25/3) \times 6.25 = 387 \text{ KN/m}$$

## 13. CONCLUSION

1. In the shallow and the medium tanks, the impulsive component of the mass is more than the convective mass. The situation is reversed in the deep tank where the convective component is more pronounced. This is in line with Figure 9.3.1 of ACI 350.3-06.
2. Free board height is substantially larger in magnitude in the deep tank than in the shallow tank.
3. The long and slender tank requires a wall thicker than what the shallow and stubby tank demands.
4. Base Shear due to the hydrostatic pressure distribution is negligible
5. The impulsive component of the total water volume is maximum in the shallow tank while it is minimum in the deep tank
6. The convective pressure at the tank base vanishes with the deep tank case; it is maximum for the shallow tank case
7. Base Shear shown in Figure 6 is about 50% larger in magnitude in the deep tank than it is in the shallow tank.

8. The bending moment at base is substantially larger in the long and slender tank than it is in the short and stubby. The bending moment calculations were based on the resultant force distribution as opposed to distribution across the circumference.
9. Table 8 shows that for the long and slender tank the hydrodynamic pressure is about 45% of the static pressure whereas for the short and stubby tank the hydrodynamic pressure is about 80% of the hydrostatic pressure., meaning the influence is more profound.
10. Hoop stresses are clearly less pronounced in deep and stubby tanks.

In a nutshell, although seismic analysis is mandatory in all cases yet it appears from the present discourse that the short and stubby tank enjoys the edge over the deep and slender due to the smaller design forces and the reduced wall thicknesses.

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## AUTHOR

**Samir H. Helou** is a retired Structural Engineering Professor at An-Najah National University, Nablus, Palestine. He earned his Bachelor of Science Degree from the University of the Bosphorus in 1973 and the Doctorate degree from North Carolina State University in 1980. Helou's main interest lies in Structural Engineering.

