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ANALYSIS AND CONTROL OF THE DYNAMICS OF ECONOMIC MODELS

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ABSTRACT

The complex dynamics of various nations' economies have led to extensive research into economic models to prevent market collapse and mitigate fluctuations, while developing strategies to control the gross domestic product (GDP). Bifurcation analysis is a powerful mathematical tool for examining the nonlinear dynamics of any process. Several factors must be considered, and multiple objectives need to be met simultaneously. Bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC) calculations are applied to two economic models. The MATLAB program MATCONT was used for the bifurcation analysis. The MNLMPC calculations were carried out employing the optimization language PYOMO in conjunction with advanced global optimization solvers IPOPT and BARON. The bifurcation analysis identified Hopf bifurcation points that lead to unwanted limit cycles, which are mitigated using an activation factor involving the tanh function. The MNLMPC calculations produced optimal control and variable profiles. However, the control profiles exhibited a significant amount of noise, which was addressed using the Savitzky-Golay Filter.

KEYWORDS

financial system, bifurcation, optimization, control

1. BACKGROUND

Lavoie and Godley(2001) [1] discussed Kaleckian models of growth in a coherent stock-flow monetary framework. Szydłowsk et al (2001) [2] demonstrated the existence of nonlinear oscillations in business cycle model with time lags. Gallegati et al (2003)[3] showed that the Hicks' trade cycle exhibited cycles and bifurcations. Torres and Vela(2003)[4] , researched trade integration and synchronization between the business cycles of Mexico and the United States. Puu and Sushko(2004)[5] developed a business cycle model with cubic nonlinearity. Dubois (2004)[6] extended the Kaldor-Kalecki model of business cycle with a computationally anticipated capital stock. Kyrtsou and Vorlow(2005)[7] demonstrated the existence of complex dynamics in macroeconomics.

Rana(2007)[8] discussed the economic integration and synchronization of business cycles in East Asia. De Paoli(2009)[9] investigated the monetary policy and welfare in an open economy. Matsumoto A., and Szidarovszky(2010)[10] developed a continuous Hicksian trade cycle model with consumption and investment time delays. Asada et al (2012)[11] , developed an analytical expressions of periodic disequilibrium fluctuations generated by Hopf bifurcations in economic dynamics. Volos et al (2012)[12] studied the synchronization phenomena in coupled nonlinear systems applied in economic cycles. Bouali et al (2012)[13] emulated complex business cycles by using an electronic analogue. He and Liao(2012)[14] , studied the asian business cycle synchronization.

Calderon and Fuentes(2014)[15] conducted an empirical investigation of changes in business cycles over the last two decades. Chu et al (2015)[16] discussed the inflation, R&D, and growth in an open economy. Zhaoet al (2016)[17] demonstrated the existence of Hopf Bifurcations in a nonlinear Business Cycle Model. Kufenko, and Geiger (2016)[18] provided an econometric analysis of economic business cycles. Sella et al (2016)[19] investigated the economic cycles and their synchronization in three European countries. Groth and Ghil(2017) [20] studied the synchronization of world economic activity. Gardini et al (2018) [21] investigated the nonlinear behavior of economic dynamics. Campos et al (2019)[22] reviewed the research on business cycle synchronisation and currency unions. Hanias et al (2019)[23] describe the reverse engineering in econo-physics. Camargo et al (2022)[24] demonstrated the Synchronization and bifurcation in an economic model. Amaral et al (2022)[25] studied the interaction between economies in a business cycle model.

Some of this work involved bifurcation analysis and single-objective optimal control calculation. The main objective of this paper is to perform multiobjective nonlinear model predictive control(MNLMPC) in conjunction with bifurcation analysis for two dynamic economic models. The two models that will be used are those described in (Camargo et al (2022)[24]) and Zhao et al (2016)[17]. The paper is organized as follows. First, the model equations are presented. The numerical procedures (bifurcation analysis and multiobjective nonlinear model predictive control(MNLMPC) are then described. The results, discussion, and conclusions are then presented.

2. ECONOMY MODELS

The two models described here are those of (Camargo et al (2022)[24]);and Zhao et al (2016)[17]

Model 1(Camargo et al (2022)[24])

The model equations are

$$
\begin{aligned}
\frac{d(xval)}{dt} &= m(yval) + pval(xval)(d-(yval^2)) \\
\frac{d(yval)}{dt} &= v(yval) + \mathrm{w}(xval) + \mathrm{c}(zval) \qquad (1) \\
\frac{d(zval)}{dt} &=) = s(xval) - r(yval) + u1\left(u2 - zval\right)
\end{aligned}
$$

$xval, yval, zval$ represent savings , gross domestic product (GDP) *y*, and foreign capital inflow. The parameter values are

$m = 0.02; pval = 0.4; d = 1.0; c = 50.0; s = 10; r = 0.1; v = 0.05; w = 0.1;$

$u1$ is the bifurcation and control parameter, while $u2$ is taken as 0.5 for the bifurcation analysis and treated as a variable for the MNLMPC calculations. *m* is the marginal propensity to save, *pval* represents the fraction of capitalized profit, *d* represents the GDP potential, and *c* is the output-capital ratio. *s* is the inflow-saving ratio, *r* is the indebtedness factor, *v* is the marginal propensity to consume, and *w* represents the proportion of savings. $u1$ measures the effect of the commercial trade and $u2 - zval$ represents the commercial balance of the country.

Model 2 Zhao et al (2016)[17]

The model equations are

$$\frac{d(yval)}{dt} = \alpha * (-0.66yval + 15\sqrt{yval} - 0.3kval + mval + 108)$$
$$\frac{d(kval)}{dt} = 15\sqrt{yval} - 0.3kval + mval \quad (2)$$
$$\frac{d(mval)}{dt} = -0.3yval + 10\beta\sqrt{yval} - \beta mval - 6\beta + 50$$

$yval, kval, mval$ represent the actual gross domestic product, the physical capital stock and the nominal currency supply, α is the market adjustment coefficient, and β is the degree of capital movement. Both are used as bifurcation and control parameters.

3. NUMERICAL PROCEDURES

This section describes the numerical procedures for bifurcation analysis and multi-objective nonlinear model predictive control.

Bifurcation analysis

The MATLAB software MATCONT is used to perform the bifurcation calculations. Bifurcation analysis deals with multiple steady-states and limit cycles. Multiple steady states occur because of the existence of branch and limit points. Hopf bifurcation points cause limit cycles . A commonly used MATLAB program that locates limit points, branch points, and Hopf bifurcation points is MATCONT(Dhooge Govearts, and Kuznetsov, 2003[26]; Dhooge Govearts, Kuznetsov, Mestrom and Riet, 2004[27]). This program detects Limit points(LP), branch points(BP), and Hopf bifurcation points(H) for an ODE system

$$\frac{dx}{dt} = f(x, \alpha) \quad (3)$$

$x \in R^n$ Let the bifurcation parameter be α Since the gradient is orthogonal to the tangent vector,

The tangent plane at any point $w = [w_1, w_2, w_3, w_4, ...w_{n+1}]$ must satisfy

$$Aw = 0 \; (4)$$

Where A is

$$A = [\partial f / \partial x \quad | \partial f / \partial \alpha] \qquad (5)$$

where $\partial f / \partial x$ is the Jacobian matrix. For both limit and branch points, the matrix $[\partial f / \partial x]$ must be singular. The n+1 th component of the tangent vector w_{n+1} = 0 for a limit point (LP)and for a branch point (BP) the matrix $\begin{bmatrix} A \\ w^T \end{bmatrix}$ must be singular. At a Hopf bifurcation point,

$$\det(2 f_x(x,\alpha) @ I_n) = 0 \qquad (6)$$

@ indicates the bialternate product while I_n is the n-square identity matrix. Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov (1998[28]; 2009[29]) and Govaerts [2000] [30]

Hopf bifurcations cause unwanted oscillatory behavior and limit cycles. The tanh activation function (where a control value u is replaced by) $(u \tanh u/\varepsilon)$ is commonly used in neural nets (Dubey et al 2022[31]; Kamalov et al, 2021[32] and Szandała, 2020[33])and optimal control problems(Sridhar 2023[34]) to eliminate spikes in the optimal control profile. Hopf bifurcation points cause oscillatory behavior. Oscillations are similar to spikes, and the results in Sridhar(2024)[35] demonstrate that the tanh factor also eliminates the Hopf bifurcation by preventing the occurrence of oscillations. Sridhar (2024)[35] explained with several examples how the activation factor involving the tanh function successfully eliminates the limit cycle causing Hopf bifurcation points. This was because the tanh function increases the time period of the oscillatory behavior, which occurs in the form of a limit cycle caused by Hopf bifurcations.

Multi-Objective Nonlinear Model Predictive Control (MNLMPC)

Flores Tlacuahuaz et al (2012)[36] developed a multiobjective nonlinear model predictive control (MNLMPC) method that is rigorous and does not involve weighting functions or additional constraints. This procedure is used for performing the MNLMPC calculations Here $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ (j=12..n) represents the variables that need to be minimized/maximized simultaneously for a problem involving a set of ODE

$$\frac{dx}{dt} = F(x,u) \qquad (7)$$

t_f being the final time value, and n the total number of objective variables and . u the control parameter. This MNLMPC procedure first solves the single objective optimal control problem independently optimizing each of the variables $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ individually. The minimization/maximization of $\sum_{t_i=0}^{t_i=t_f} q_j(t_i)$ will lead to the values q_j^* . Then the optimization problem that will be solved is

$$\begin{aligned} &\min\left(\sum_{j=1}^{n}\left(\sum_{t_i=0}^{t_i=t_f} q_j(t_i) - q_j^*\right)\right)^2 \\ &\textit{subject to } \frac{dx}{dt} = F(x,u); \end{aligned} \qquad (8)$$

This will provide the values of u at various times. The first obtained control value of u is implemented and the rest are discarded. This procedure is repeated until the implemented and the first obtained control values are the same or if the Utopia point where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j) is obtained. Pyomo (Hart et al, 2017)[37] is used for these calculations. Here, the differential equations are converted to a Nonlinear Program (NLP) using the orthogonal collocation method The NLP is solved using IPOPT (Wächter And Biegler, 2006)[38]and confirmed as a global solution with BARON (Tawarmalani, M. and N. V. Sahinidis 2005)[39].

The steps of the algorithm are as follows

1. Optimize $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ and obtain q_j^* at various time intervals t_i. The subscript i is the index for each time step.
2. Minimize $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$ and get the control values for various times.
3. Implement the first obtained control values
4. Repeat steps 1 to 3 until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) = q_j^*$ for all j.

4. RESULTS AND DISCUSSION

For model 1, the bifurcation analysis revealed a Hopf bifurcation point of $(xval, yval, zval, u1)$ of (0.014863 3.638778 -0.003669 0.427351). This is shown in Fig. 1a. The limit cycle produced by this Hopf bifurcation point is shown in Fig. 1b. When u1 is modified to u1tanh(u1) the Hopf bifurcation point and the limit cycle disappear (confirming the analysis of Sridhar(2024).

In model 2, when α is the bifurcation parameter, the bifurcation analysis revealed a Hopf bifurcation point of $(yval, kval, mval, \alpha)$ = (163.636364 1047.013683 122.223460 1.433291)(Fig. 2a). The limit cycle produced by this Hopf bifurcation point is shown in Fig. 2b . When α is modified to $\alpha\tanh(\alpha)/0.2$ the Hopf bifurcation point disappears. (Fig. 2c).When β is the bifurcation parameter, the bifurcation analysis revealed a Hopf bifurcation point of $(yval, kval, mval, \beta)$ = ((163.636364 1047.009483 122.222200 3.012525)(Fig. 3a). The limit cycle produced by this Hopf bifurcation point is shown in Fig. 3b . When β is modified to $\beta\tanh(\beta)/0.2$ the Hopf bifurcation point disappears. (Fig. 3c). Both models show the presence of limit cycles causing Hopf bifurcations, which can be eliminated using the activation factor involving the tanh function, confirming the analysis of Sridhar(2024)[35].

When the MNLMPC calculations were performed for model 1, $\sum_{t_{i=0}}^{t_i=t_f} xval_j(t_i), \sum_{t_{i=0}}^{t_i=t_f} yval_j(t_i), \sum_{t_{i=0}}^{t_i=t_f} zval_j(t_i)$ were maximized individually, resulting in values of 7.1377e-03, 1000, and 1000, respectively. The overall optimal control problem will involve the minimization of

$$(\sum_{t_i=0}^{t_i=t_f} xval_j(t_i) - 7.1377e-03)^2 + (\sum_{t_i=0}^{t_i=t_f} yval_j(t_i) - 1000)^2 + (\sum_{t_i=0}^{t_i=t_f} zval_j(t_i) - 1000)^2$$

was minimized subject to the equations governing the model. This led to a value of 5.02e-05. The first of the control variables is implemented, and the rest are discarded. The process is repeated until the difference between the first and second values of the control variables is the same. This MNLMPC control value of u_1 obtained was 0.01063. The MNLMPC profiles of xval, yval, and zval are shown in Figs 4a, 4b and 4c. . The obtained control profile of u1 exhibited noise (Fig. 4d). This was remedied using the Savitzky-Golay Filter. The smoothed-out version of this profile is shown in Fig.4e.

When the MNLMPC calculations were performed for model 2, $\sum_{t_i=0}^{t_i=t_f} yval_j(t_i), \sum_{t_i=0}^{t_i=t_f} kval_j(t_i), \sum_{t_i=0}^{t_i=t_f} mval_j(t_i)$ are maximized individually producing values of 11018.477, 20000, and 13221.553, respectively. The overall optimal control problem will involve the minimization of

$$(\sum_{t_i=0}^{t_i=t_f} yval_j(t_i) - 11018.477)^2 + (\sum_{t_i=0}^{t_i=t_f} kval_j(t_i) - 20000)^2 + (\sum_{t_i=0}^{t_i=t_f} mval_j(t_i) - 13221.553)^2)1.e-08$$

was minimized subject to the equations governing the model. A scaling factor of 1.e-08 was used. This minimization led to a value of 0.5679. The first of the control variables is implemented, and the rest are discarded. The process is repeated until the difference between the first and second values of the control variables is the same. The MNLMPC control values of α, β obtained were 0.1394 and 0. The MNLMPC profiles of yval, kval, and mval are shown in Figs 5a, 5b and 5c. . The obtained control profiles of α, β exhibited noise (Fig. 5d). This was remedied using the Savitzky-Golay Filter. The smoothed-out version of this profile is shown in Fig.5e.

5. CONCLUSIONS

Multiobjective nonlinear model predictive control(MNLMPC) calculations were performed along with bifurcation analysis on two dynamic financial engineering models. The bifurcation analysis revealed the existence of Hopf bifurcation points. The Hopf bifurcation points cause unwanted limit cycles and were eliminated using an activation factor involving the tanh function.The MNLMPC calculations generated the optimal control and variable profiles. The control profiles exhibited a considerable amount of noise. This was remedied using the Savitzky-Golay Filter.

Data Availability Statement

All data used is presented in the paper

Conflict of interest

The author, Dr. Lakshmi N Sridhar has no conflict of interest.

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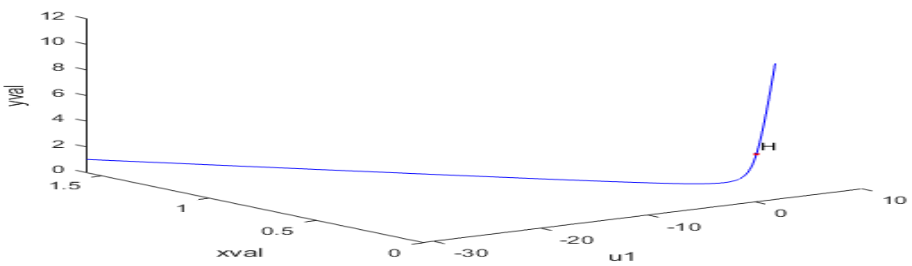

Fig. 1a (bifurcation analysis model 1 showing Hopf bifurcation

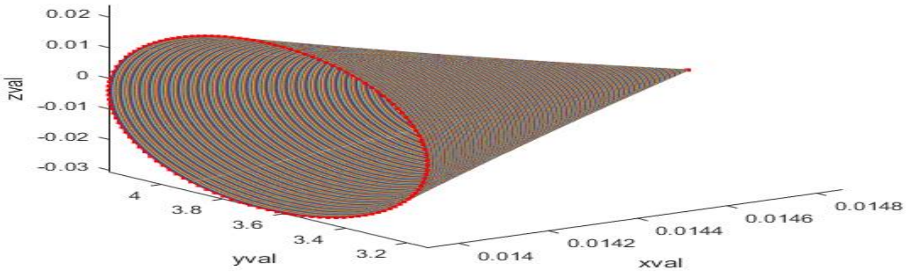

Fig. 1 b(limit cycle for Model 1)

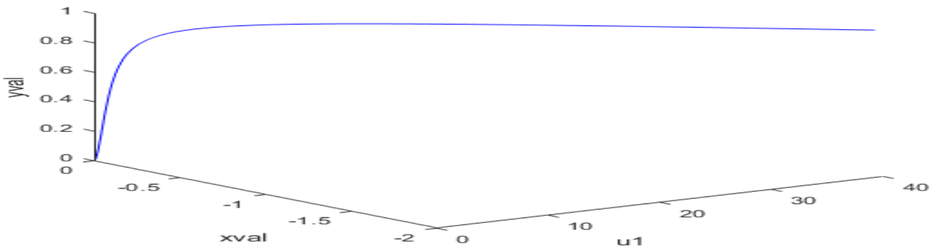

Fig. 1c(Hopf bifurcation disappears with tanh activation factor in model 1)

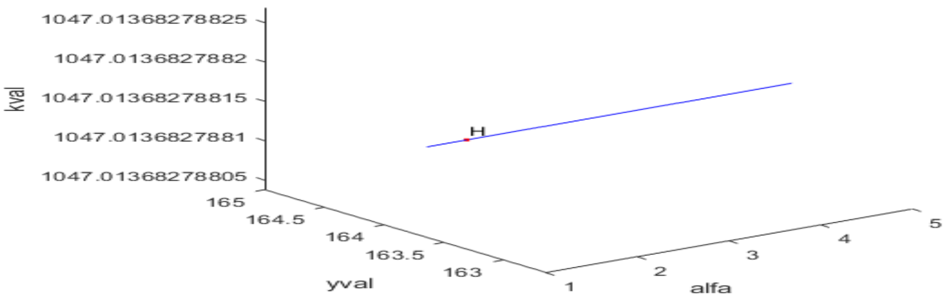

Fig. 2a (hopf bifurcation model 2 alfa is bifurcation parameter)

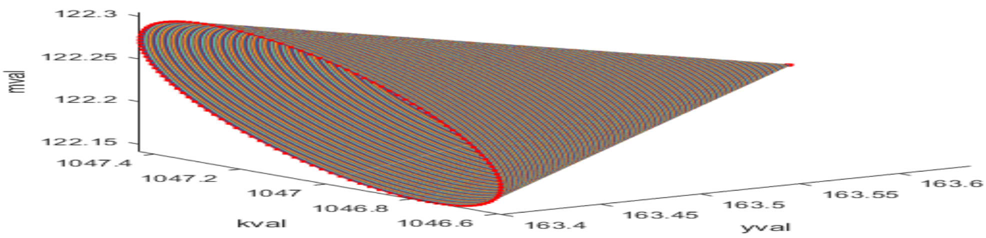

Fig. 2b (limit cycle alfa is bifurcation parameter)

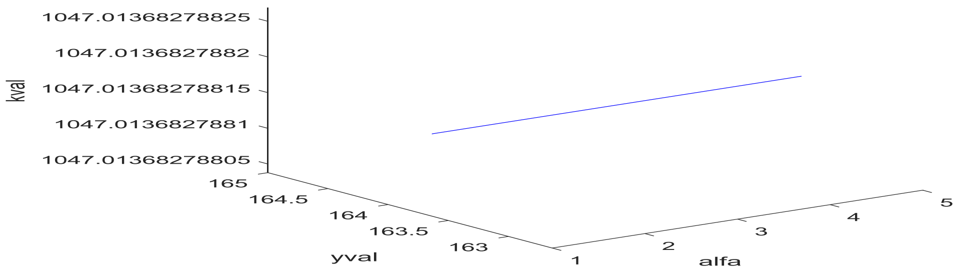

Fig. 2c Hopf bifurcation disappears with tanh activation factor alfa is bifurcation parameter

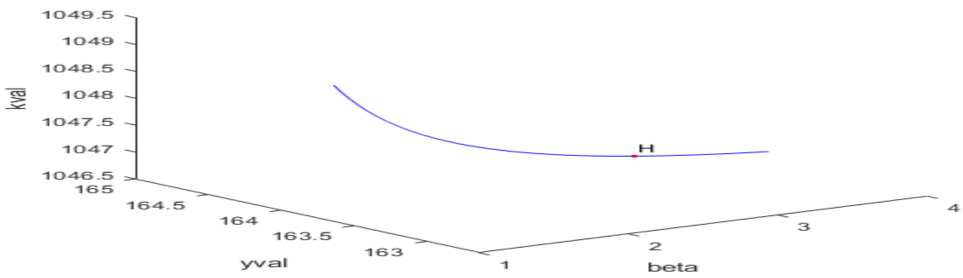

Fig. 3a(Hopf bifurcation model 2 beta is bifurcation parameter)

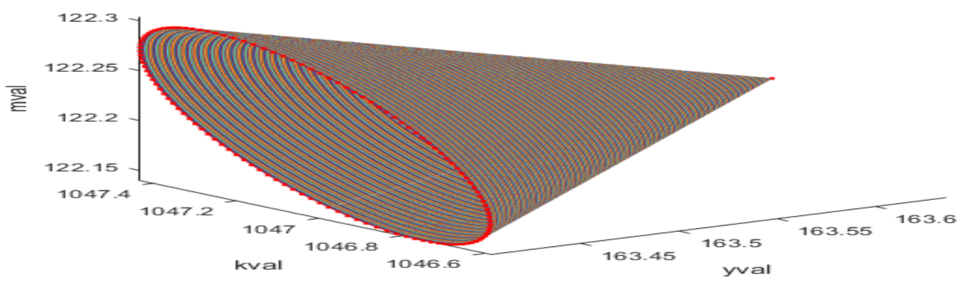

Fig. 3b(limit cycle beta is bifurcation parameter)

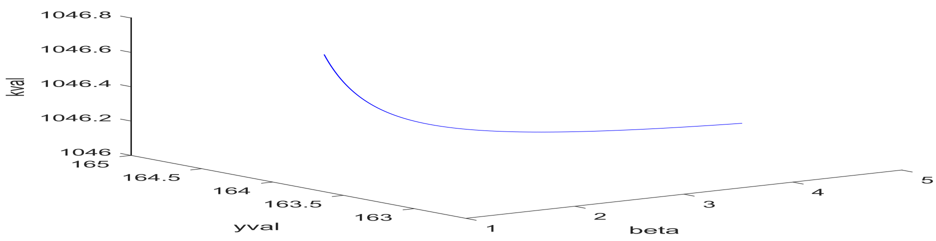

Fig. 3c Hopf bifurcation disappears with tanh activation factor alfa is bifurcation parameter

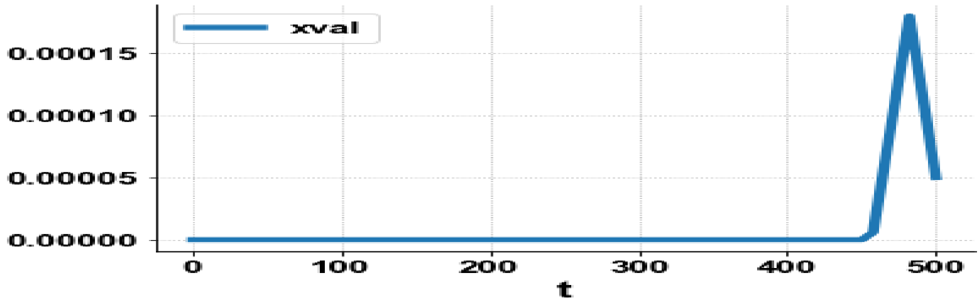

Fig. 4a (xval vs t MNLMPC model 1)

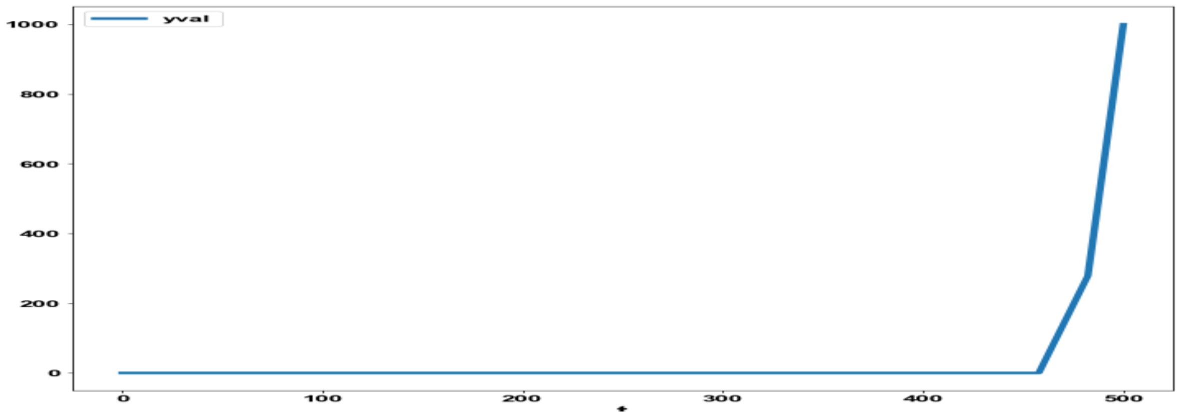

Fig. 4b (yval vs t MNLMPC model 1)

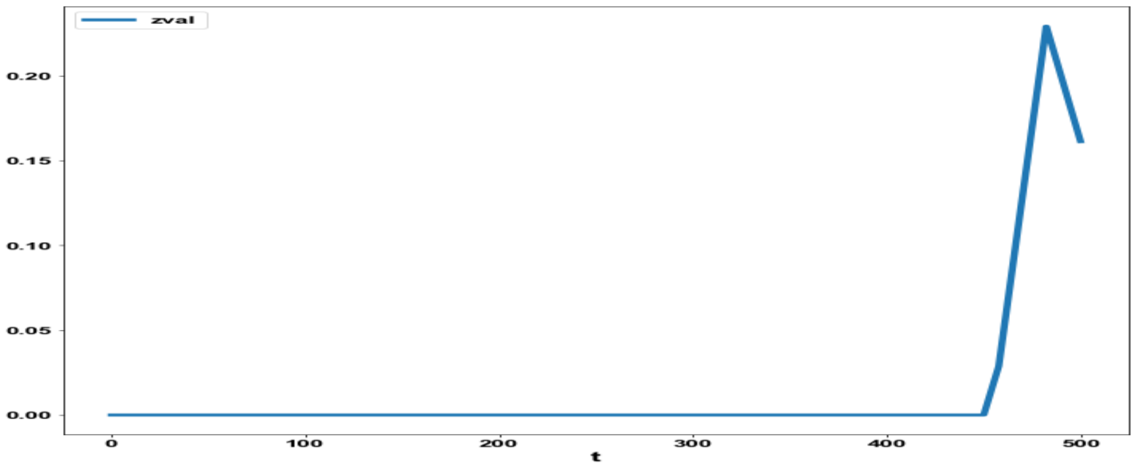

Fig. 4c (zval vs t MNLMPC model 1)

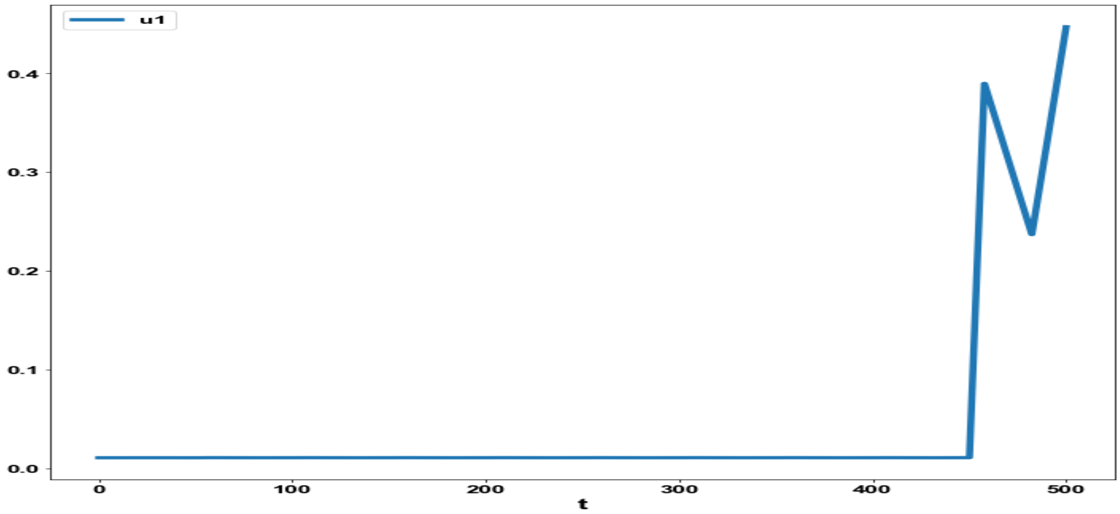

Fig. 4d (u1 vs t MNLMPC model 1)

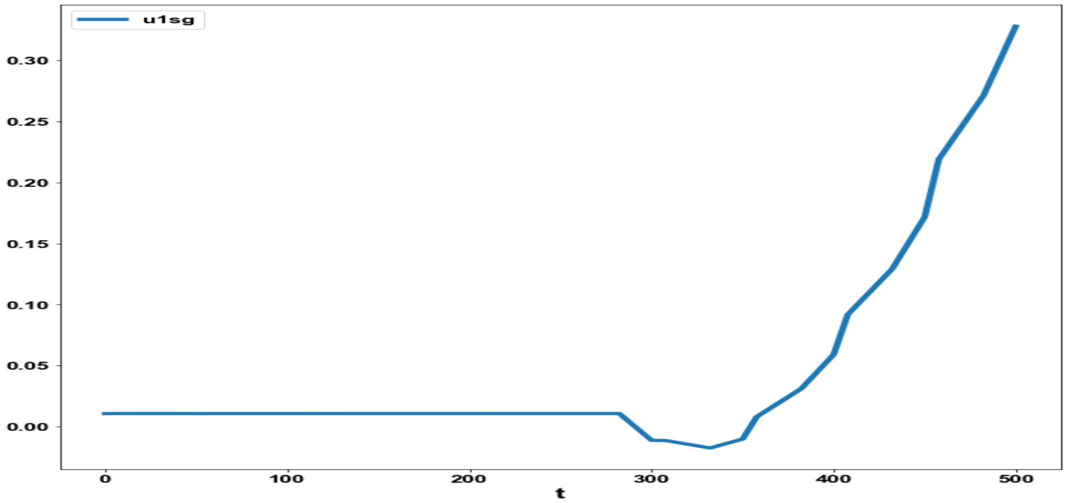

Fig. 4e (u1 with Savitzky Golay filter MNLMPC model 1)

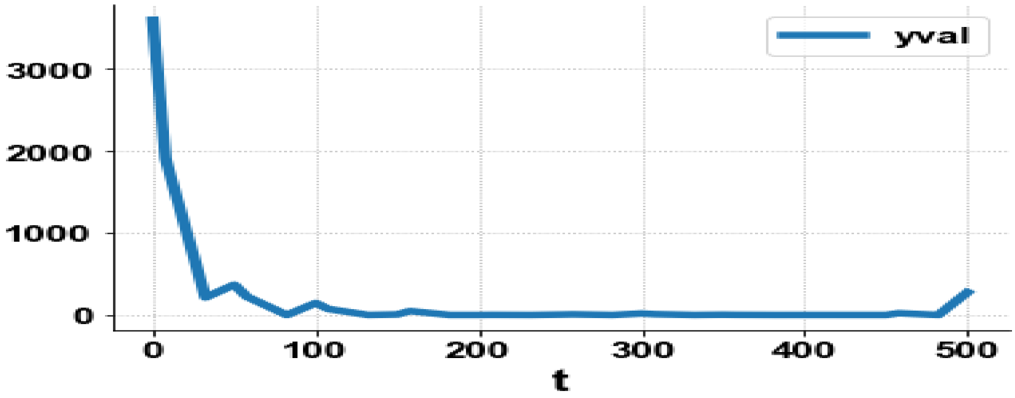

Fig. 5a (yval vs t MNLMPC model 2)

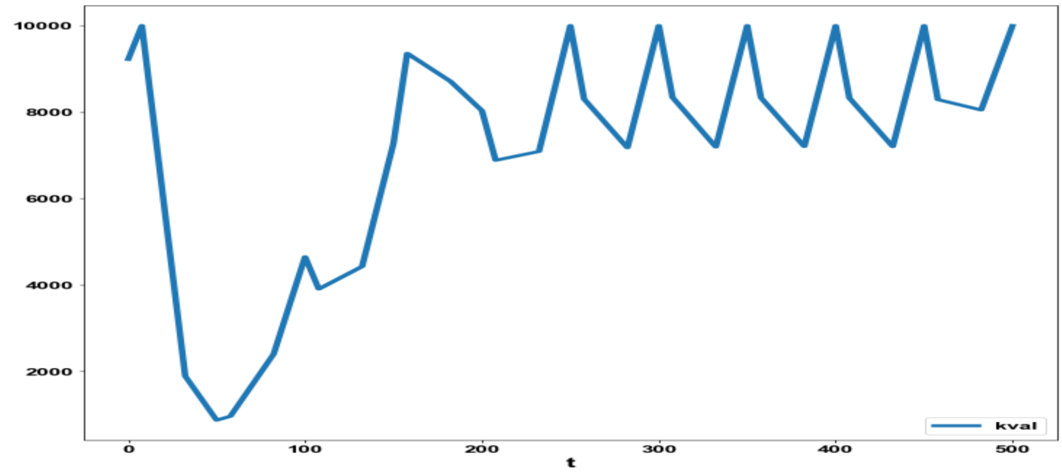

Fig. 5b (kval vs t MNLMPC model 2)

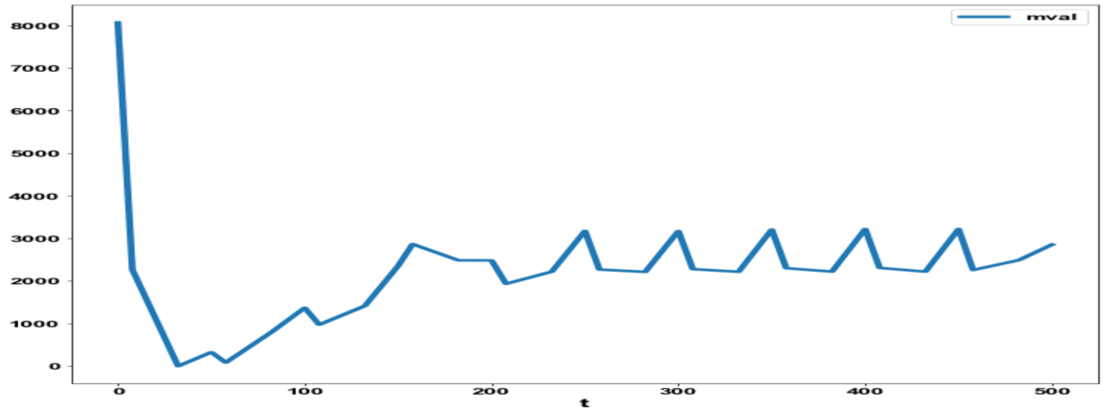

Fig. 5c (mval vs t MNLMPC model 2)

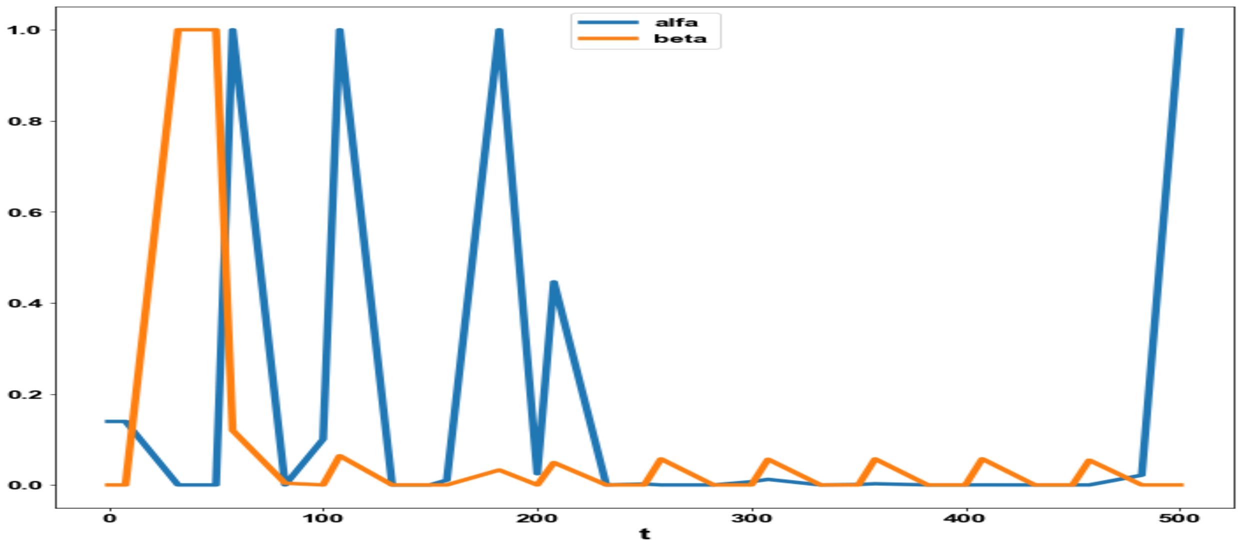

Fig. 5d (alfa beta vs t MNLMPC model 2)

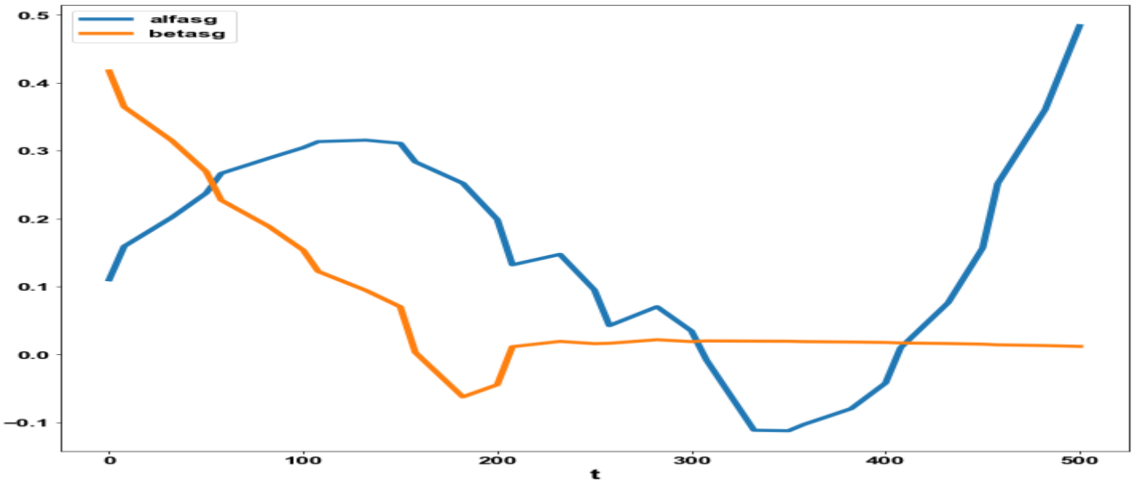

Fig. 5e (alfa beta (with Savitzky Golay) vs t MNLMPC model 2)