

# ON OPTIMIZATION OF MANUFACTURING OF FIELD EFFECT HETEROTRANSISTORS FRAMEWORK A SINGLE STAGE MULTIPATH OPERATIONAL AMPLIFIER, TO INCREASE THEIR DENSITY

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## **ABSTRACT**

*We consider an approach for increasing density of field-effect heterotransistors in a single-stage multi-path operational amplifier. At the same time one can obtain decreasing of dimensions of the above transistors. Dimensions of the elements could be decreased by manufacturing of these elements in a heterostructure with specific structure. The manufacturing is doing by doping of required areas of the heterostructure by diffusion or ion implantation with future optimization of annealing of dopant and/or radiation defects.*

## **KEYWORDS**

*field-effect heterotransistors; single-stage multi-path operational amplifier; optimization of manufacturing.*

## **1. INTRODUCTION**

In the present time it is attracted an interest increasing of performance of elements integrated circuits and increasing of their density. At the same time dimensions of these elements decreases with decreasing of their density. Dimensions of elements of integrated circuit could be decreases by manufacturing of them in thin-film heterostructures [1-4]. As an alternative approach for the decreasing one can use laser and microwave types annealing [5-7]. Using these types of annealing leads to generation inhomogeneous distribution of temperature. Temperature dependence of dopant diffusion coefficient and other parameters of process leads to their inhomogeneity due to inhomogeneity of temperature. The inhomogeneity could leads to decreasing of dimensions of elements of integrated circuits. Properties of electronic materials could be also changes by using radiation processing of these materials [8,9].

In this paper we consider a single-stage multi-path operational amplifier described in Ref. [10] (see Fig.1). We assume, that element of the considered circuit has been manufactured in heterostructure from Fig. 1. The heterostructure includes into itself a substrate and an epitaxial layer. The epitaxial layer consist of a main material and several sections manufactured by using another materials in the main material. The sections should be doped for generation into them

required type of conductivity ( $n$  or  $p$ ). Most simple types of doping are diffusion or ion implantation in the case. Main aim of the paper we analyzing of redistribution of dopant and radiation defects during the doping to formulate conditions to decrease of dimensions of the considered circuit.

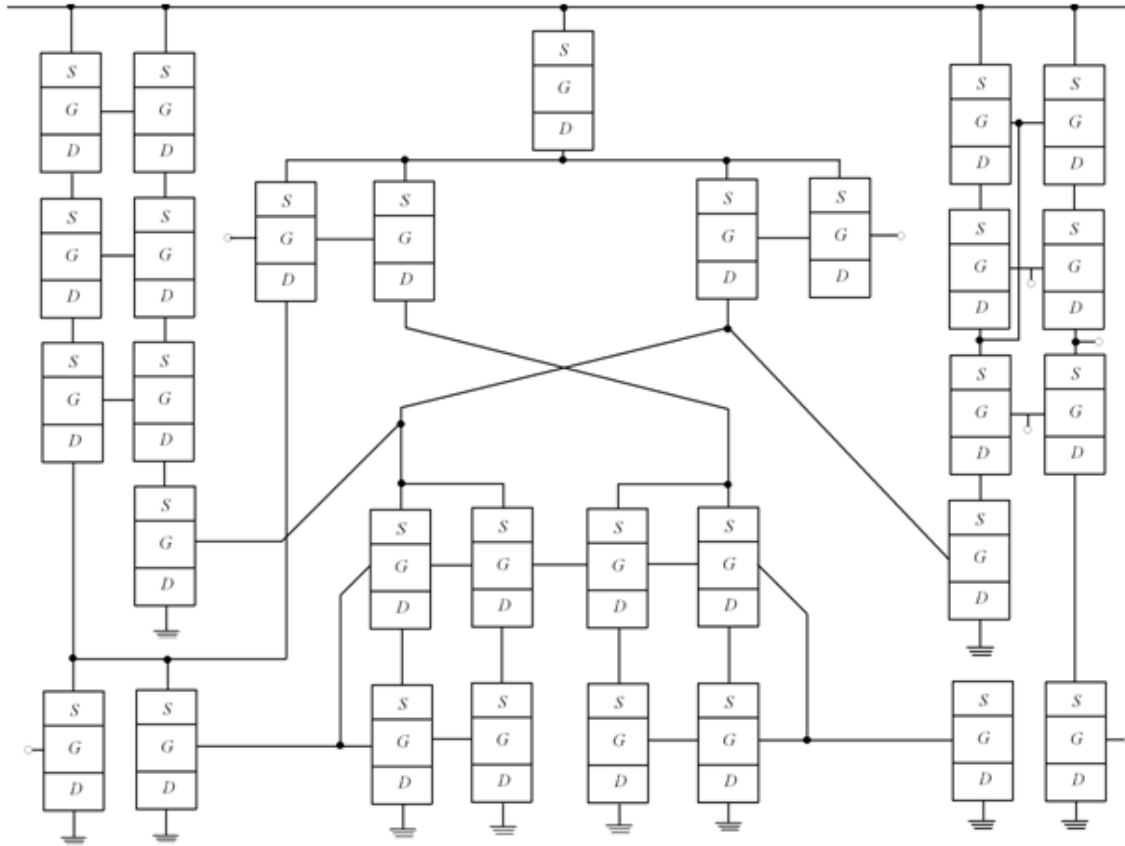


Fig. 1a. Circuit of a pass transistor single-stage multi-path operational amplifier [10]

Source	Channel	Drain		Source	Channel	Drain

Fig. 1b. Heterostructure with substrate and epitaxial layer. The epitaxial layer in-cludes into itself several sections

## 2. METHOD OF SOLUTION

We calculate distribution of concentration of dopant in space and time by solving the following equation

$$\frac{\partial C(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_C \frac{\partial C(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_C \frac{\partial C(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_C \frac{\partial C(x, y, z, t)}{\partial z} \right] \quad (1)$$

Boundary and initial conditions for the equation could be written as

$$\begin{aligned} \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \quad (2) \\ \left. \frac{\partial C(x, y, z, t)}{\partial y} \right|_{x=L_y} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial C(x, y, z, t)}{\partial z} \right|_{x=L_z} = 0, \\ C(x, y, z, 0) = f(x, y, z). \end{aligned}$$

Function  $C(x, y, z, t)$  describes distribution of concentration of dopant in space and time;  $T$  is the temperature of annealing;  $D_C$  is the dopant diffusion coefficient. Dopant diffusion coefficient could be varying with changing of materials, speed of heating and cooling of heterostructure. Approximation of dopant diffusion coefficient could be written as [9,11,12]

$$D_C = D_L(x, y, z, T) \left[ 1 + \xi \frac{C^\gamma(x, y, z, t)}{P^\gamma(x, y, z, T)} \right] \left[ 1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right]. \quad (3)$$

Here  $D_L(x, y, z, T)$  is the dependence of dopant diffusion coefficient on coordinate and temperature;  $P(x, y, z, T)$  is the dependence of limit of solubility of dopant diffusion coefficient on coordinate and temperature; parameter  $\gamma$  should be integer in the interval  $\gamma \in [1, 3]$  [9];  $V(x, y, z, t)$  is the dependence of distribution of concentration of radiation vacancies on coordinate and time with the equilibrium distribution  $V^*$ . Dependence of dopant diffusion coefficient on dopant concentration has been discussed in details in [9]. It should be noted, that using infusion of dopant did not leads to generation radiation defects, i.e.  $\zeta_1 = \zeta_2 = 0$ . We determine distributions of concentrations of point defects on space and time by solving the following system of equations [11,12]

$$\begin{aligned} \frac{\partial I(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) - \\ & - k_{I,I}(x, y, z, T) I^2(x, y, z, t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial V(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[ D_V(x, y, z, T) \frac{\partial V(x, y, z, t)}{\partial z} \right] - k_{I,V}(x, y, z, T) I(x, y, z, t) V(x, y, z, t) - \\ & - k_{V,V}(x, y, z, T) V^2(x, y, z, t) \end{aligned}$$

Boundary and initial conditions for these equations could be written as

$$\begin{aligned} \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \\ \left. \frac{\partial \rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \\ \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \quad \rho(x, y, z, 0) = f_\rho(x, y, z). \end{aligned} \quad (5)$$

Here  $\rho = I, V$ ; function  $I(x, y, z, t)$  describe distribution of concentration of radiation interstitials in space and time; function  $D_\rho(x, y, z, T)$  describe distribution of diffusion coefficients of radiation interstitials and vacancies on coordinate and temperature; squared terms on concentration of defects (i.e.  $V^2(x, y, z, t)$  and  $I^2(x, y, z, t)$ ) describe dependences of parameter of generation of divacancies and diinterstitials on coordinate and temperature, respectively; function  $k_{I,V}(x, y, z, T)$  describe distribution of the parameter of recombination of point radiation defects on coordinate and temperature; function  $k_{\rho,\rho}(x, y, z, T)$  describe distribution of the parameters of generation of simplest complexes of point radiation defects on coordinate and temperature.

We calculate dependences of concentrations of divacancies  $\Phi_V(x, y, z, t)$  and diinterstitials  $\Phi_I(x, y, z, t)$  on coordinate and temperature by solving the following system of equations [11,12]

$$\begin{aligned} \frac{\partial \Phi_I(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_I(x, y, z, t)}{\partial z} \right] + k_{I,I}(x, y, z, T) I^2(x, y, z, t) - k_I(x, y, z, T) I(x, y, z, t) \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial \Phi_V(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial y} \right] + \\ & + \frac{\partial}{\partial z} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_V(x, y, z, t)}{\partial z} \right] + k_{V,V}(x, y, z, T) V^2(x, y, z, t) - k_V(x, y, z, T) V(x, y, z, t) \end{aligned}$$

Boundary and initial conditions for these equations could be written as

$$\begin{aligned} \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial x} \right|_{x=L_x} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=0} = 0, \\ \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial y} \right|_{y=L_y} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=0} = 0, \quad \left. \frac{\partial \Phi_\rho(x, y, z, t)}{\partial z} \right|_{z=L_z} = 0, \end{aligned} \quad (7)$$

$$\Phi_I(x, y, z, 0) = f_{\Phi I}(x, y, z), \quad \Phi_V(x, y, z, 0) = f_{\Phi V}(x, y, z).$$

Functions  $D_{\Phi\rho}(x, y, z, T)$  describe dependences diffusion coefficients of complexes of radiation defects on coordinate and temperature; functions  $k_\rho(x, y, z, T)$  describe dependences of parameters of decay of complexes of radiation defects on coordinate and temperature.

We calculate distributions of concentrations of dopant and radiation defects in space and time by using method of averaging of function corrections [13] with decreased quantity of iteration steps [14]. First of all we used solutions of linear Eqs. (1), (4) and (6) and with averaged values of diffusion coefficients  $D_{0L}, D_{0I}, D_{0V}, D_{0\Phi I}, D_{0\Phi V}$  as initial-order approximations of the considered concentrations. The solutions could be written as

$$C_1(x, y, z, t) = \frac{F_{0C}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nC}(t),$$

$$I_1(x, y, z, t) = \frac{F_{0I}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nI} c_n(x) c_n(y) c_n(z) e_{nI}(t),$$

$$V_1(x, y, z, t) = \frac{F_{0C}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{nC} c_n(x) c_n(y) c_n(z) e_{nV}(t),$$

$$\Phi_{I1}(x, y, z, t) = \frac{F_{0\Phi_I}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_I} c_n(x) c_n(y) c_n(z) e_{n\Phi_I}(t),$$

$$\Phi_{V1}(x, y, z, t) = \frac{F_{0\Phi_V}}{L_x L_y L_z} + \frac{2}{L_x L_y L_z} \sum_{n=1}^{\infty} F_{n\Phi_V} c_n(x) c_n(y) c_n(z) e_{n\Phi_V}(t).$$

Here  $e_{n\rho}(t) = \exp\left[-\pi^2 n^2 D_{0\rho} t \left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)\right]$ ,

$$F_{n\rho} = \int_0^{L_x} c_n(u) \int_0^{L_y} c_n(v) \int_0^{L_z} c_n(w) f_\rho(u, v, w) dw dv du, \quad c_n(\chi) = \cos(\pi n \chi / L_\chi).$$

Approximations of the considered concentrations with higher orders (second, third, ...) have been calculated framework standard iterative procedure [13,14]. To use the procedure we shall replace the functions  $C(x,y,z,t)$ ,  $I(x,y,z,t)$ ,  $V(x,y,z,t)$ ,  $\Phi_I(x,y,z,t)$ ,  $\Phi_V(x,y,z,t)$  in the right sides of the Eqs. (1), (4) and (6) on the following sums  $\alpha_{n\rho} + \rho_{n-1}(x,y,z,t)$ . Equations for concentrations for considered concentrations for the second-order could be written as

$$\begin{aligned} \frac{\partial C_2(x, y, z, t)}{\partial t} = & \frac{\partial}{\partial x} \left( \left[ 1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, t)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \times \right. \\ & \times D_L(x, y, z, T) \frac{\partial C_1(x, y, z, t)}{\partial x} \Big) + \frac{\partial}{\partial y} \left( D_L(x, y, z, T) \left[ 1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \times \right. \\ & \times \left. \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, t)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, t)}{\partial y} \right) + \frac{\partial}{\partial z} \left( D_L(x, y, z, T) \frac{\partial C_1(x, y, z, t)}{\partial z} \times \right. \\ & \times \left. \left[ 1 + \zeta_1 \frac{V(x, y, z, t)}{V^*} + \zeta_2 \frac{V^2(x, y, z, t)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, t)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \right) \end{aligned} \quad (8)$$

$$\frac{\partial I_2(x, y, z, t)}{\partial t} = \frac{\partial}{\partial x} \left[ D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial y} \right] +$$

$$\begin{aligned}
 & + \frac{\partial}{\partial z} \left[ D_I(x, y, z, T) \frac{\partial I_1(x, y, z, t)}{\partial z} \right] - [\alpha_{2I} + I_1(x, y, z, t)] [\alpha_{2V} + V_1(x, y, z, t)] \times \\
 & \quad \times k_{I,V}(x, y, z, T) - k_{I,I}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, t)]^2 \tag{9}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial V_2(x, y, z, t)}{\partial t} & = \frac{\partial}{\partial x} \left[ D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial y} \right] + \\
 & + \frac{\partial}{\partial z} \left[ D_V(x, y, z, T) \frac{\partial V_1(x, y, z, t)}{\partial z} \right] - [\alpha_{2I} + I_1(x, y, z, t)] [\alpha_{2V} + V_1(x, y, z, t)] \times \\
 & \quad \times k_{I,V}(x, y, z, T) - k_{V,V}(x, y, z, T) [\alpha_{2V} + V_1(x, y, z, t)]^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \Phi_{I2}(x, y, z, t)}{\partial t} & = \frac{\partial}{\partial x} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{I1}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_I}(x, y, z, T) \times \right. \\
 & \quad \times \left. \frac{\partial \Phi_{I1}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{I1}(x, y, z, t)}{\partial z} \right] + k_{I,I}(x, y, z, T) \times \\
 & \quad \times I^2(x, y, z, t) - k_I(x, y, z, T) I(x, y, z, t) \tag{10}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial \Phi_{V2}(x, y, z, t)}{\partial t} & = \frac{\partial}{\partial x} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ D_{\Phi_V}(x, y, z, T) \times \right. \\
 & \quad \times \left. \frac{\partial \Phi_{V1}(x, y, z, t)}{\partial y} \right] + \frac{\partial}{\partial z} \left[ D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, t)}{\partial z} \right] + k_{V,V}(x, y, z, T) \times \\
 & \quad \times V^2(x, y, z, t) - k_V(x, y, z, T) V(x, y, z, t).
 \end{aligned}$$

The second-order approximations of concentrations of dopant and radiation defects could be obtained in the final form by integration of the left and right sides of Eqs.(8)-(10)

$$C_2(x, y, z, t) = \frac{\partial}{\partial x} \left( \int_0^t \left[ 1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \times \right.$$

$$\begin{aligned}
 & \times D_L(x, y, z, T) \frac{\partial C_1(x, y, z, \tau)}{\partial x} d\tau + \frac{\partial}{\partial y} \left[ \int_0^t \left[ 1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \right] \times \\
 & \times D_L(x, y, z, T) \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, \tau)}{\partial y} d\tau + \frac{\partial}{\partial z} \left[ \int_0^t D_L(x, y, z, T) \times \right. \\
 & \times \left. \left[ 1 + \zeta_1 \frac{V(x, y, z, \tau)}{V^*} + \zeta_2 \frac{V^2(x, y, z, \tau)}{(V^*)^2} \right] \left\{ 1 + \xi \frac{[\alpha_{2C} + C_1(x, y, z, \tau)]^\gamma}{P^\gamma(x, y, z, T)} \right\} \frac{\partial C_1(x, y, z, \tau)}{\partial z} d\tau \right] + \\
 & + f_c(x, y, z) \tag{8a}
 \end{aligned}$$

$$\begin{aligned}
 I_2(x, y, z, t) &= \frac{\partial}{\partial x} \left[ \int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[ \int_0^t D_I(x, y, z, T) \times \right. \\
 & \times \left. \frac{\partial I_1(x, y, z, \tau)}{\partial y} d\tau \right] + \frac{\partial}{\partial z} \left[ \int_0^t D_I(x, y, z, T) \frac{\partial I_1(x, y, z, \tau)}{\partial z} d\tau \right] - \int_0^t k_{I,I}(x, y, z, T) \times \\
 & \times [\alpha_{2I} + I_1(x, y, z, \tau)]^2 d\tau + f_I(x, y, z) - \int_0^t k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)] \times \\
 & \times [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau \tag{9a}
 \end{aligned}$$

$$\begin{aligned}
 V_2(x, y, z, t) &= \frac{\partial}{\partial x} \left[ \int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[ \int_0^t D_V(x, y, z, T) \times \right. \\
 & \times \left. \frac{\partial V_1(x, y, z, \tau)}{\partial y} d\tau \right] + \frac{\partial}{\partial z} \left[ \int_0^t D_V(x, y, z, T) \frac{\partial V_1(x, y, z, \tau)}{\partial z} d\tau \right] - \int_0^t k_{V,V}(x, y, z, T) \times \\
 & \times [\alpha_{2I} + V_1(x, y, z, \tau)]^2 d\tau + f_V(x, y, z) - \int_0^t k_{I,V}(x, y, z, T) [\alpha_{2I} + I_1(x, y, z, \tau)] \times \\
 & \times [\alpha_{2V} + V_1(x, y, z, \tau)] d\tau
 \end{aligned}$$

$$\Phi_{I2}(x, y, z, t) = \frac{\partial}{\partial x} \left[ \int_0^t D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[ \int_0^t D_{\Phi_I}(x, y, z, T) \times \right.$$



$$\times \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial y} d\tau \left] + \frac{\partial}{\partial z} \left[ \int_0^t D_{\Phi_I}(x, y, z, T) \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial z} d\tau \right] + \int_0^t k_{I,I}(x, y, z, T) \times \right. \\ \left. \times I^2(x, y, z, \tau) d\tau - \int_0^t k_I(x, y, z, T) I(x, y, z, \tau) d\tau + f_{\Phi_I}(x, y, z) \right] \quad (10a)$$

$$\Phi_{V2}(x, y, z, t) = \frac{\partial}{\partial x} \left[ \int_0^t D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial x} d\tau \right] + \frac{\partial}{\partial y} \left[ \int_0^t D_{\Phi_V}(x, y, z, T) \times \right. \\ \left. \times \frac{\partial \Phi_{I1}(x, y, z, \tau)}{\partial y} d\tau \right] + \frac{\partial}{\partial z} \left[ \int_0^t D_{\Phi_V}(x, y, z, T) \frac{\partial \Phi_{V1}(x, y, z, \tau)}{\partial z} d\tau \right] + \int_0^t k_{V,V}(x, y, z, T) \times \\ \times V^2(x, y, z, \tau) d\tau - \int_0^t k_V(x, y, z, T) V(x, y, z, \tau) d\tau + f_{\Phi_V}(x, y, z)$$

We calculate average values of the second-orders approximations of concentrations of dopant and radiation defects by using the following standard relations [13,14]

$$\alpha_{2\rho} = \frac{1}{\Theta_{L_x L_y L_z}} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [\rho_2(x, y, z, t) - \rho_1(x, y, z, t)] dz dy dx dt. \quad (11)$$

Using the relation (11) with account relations (8a)-(10a) leads to the following average values

$\alpha_{2\rho}$

$$\alpha_{2C} = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_C(x, y, z) dz dy dx, \quad (12)$$

$$\alpha_{2I} = \frac{1}{2A_{II00}} \left\{ (1 + A_{IV01} + A_{II10} + \alpha_{2V} A_{IV00})^2 - 4A_{II00} [\alpha_{2V} A_{IV10} - A_{II20} + A_{IV11} - \right. \\ \left. - \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx \right\}^{\frac{1}{2}} - \frac{1 + A_{IV01} + A_{II10} + \alpha_{2V} A_{IV00}}{2A_{II00}} \quad (13a)$$

$$\alpha_{2V} = \frac{1}{2B_4} \sqrt{\frac{(B_3 + A)^2}{4} - 4B_4 \left( y + \frac{B_3 y - B_1}{A} \right) - \frac{B_3 + A}{4B_4}}. \quad (13b)$$

Here

$$A_{abij} = \frac{1}{\Theta L_x L_y L_z} \int_0^{\Theta} (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_{a,b}(x, y, z, T) I_1^i(x, y, z, t) V_1^j(x, y, z, t) dz dy dx dt,$$

$$B_4 = A_{IV00}^2 A_{IV00}^2 - 2(A_{IV00}^2 - A_{II00} A_{VV00})^2,$$

$$B_3 = A_{IV00} A_{IV00}^2 + A_{IV01} A_{IV00}^3 + A_{IV00} A_{II10} A_{IV00}^2 - \\ - 4(A_{IV00}^2 - A_{II00} A_{VV00}) [2A_{IV01} A_{IV00} + 2A_{IV00} (1 + A_{IV01} + A_{II10}) - 2A_{II00} (A_{IV10} + A_{VV10} + 1)] - \\ - 4A_{IV10} A_{II00} A_{IV00}^2 + 2A_{IV00} A_{IV01} A_{IV00}^2,$$

$$B_2 = A_{IV00}^2 \left\{ (1 + A_{IV01} + A_{II10})^2 + A_{IV00}^2 A_{IV01}^2 + A_{IV00} \times \right. \\ \times 2A_{IV00} (A_{IV00} + A_{IV00} A_{IV01} + A_{IV00} A_{II10} - 4A_{IV10} A_{II00}) - 4A_{II00} \left[ A_{IV11} - A_{II20} - \frac{1}{L_x L_y L_z} \times \right. \\ \left. \times \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx \right] \left. \right\} \left\{ [2A_{IV01} A_{IV00} + 2A_{IV00} (1 + A_{IV01} + A_{II10}) - 2A_{II00} (A_{IV10} + 1 + \right. \\ \left. + A_{VV10})]^2 + 2 \left[ A_{IV01} (1 + A_{IV01} + A_{II10}) + \frac{2}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_V(x, y, z) dz dy dx - 2A_{II00} (A_{VV20} - \right. \\ \left. - A_{IV11}) + A_{IV01} (1 + A_{IV01} + A_{II10}) \right] [2A_{IV00} (1 + A_{IV01} + A_{II10}) + 2A_{IV01} A_{IV00} - 2A_{II00} (A_{IV10} + 1 + \\ \left. + A_{VV10})] \right\},$$

$$B_1 = 2A_{IV00} A_{IV01} (1 + A_{IV01} + A_{II10})^2 - 8 \left[ A_{IV11} - \frac{1}{L_x} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx \times \right. \\ \left. \times \frac{1}{L_y L_z} - A_{II20} \right] + A_{IV00} A_{IV01} A_{II00} + A_{IV01}^2 (A_{IV00} + A_{IV00} A_{IV01} + A_{IV00} A_{II10} - 4A_{IV10} A_{II00}) -$$

$$\begin{aligned}
 & -2 \left[ \frac{2A_{II00}}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx + A_{IV01}(1 + A_{IV01} + A_{II10}) - 2A_{II00}(A_{VV20} - A_{IV11}) + \right. \\
 & \left. + A_{IV01}(1 + A_{IV01} + A_{II10}) \right] [2A_{IV00}(1 + A_{IV01} + A_{II10}) - 2(A_{IV10} + A_{VV10} + 1)A_{II00} + 2A_{IV01}A_{IV00}] \\
 & , \\
 & B_0 = 4A_{II00}A_{IV01}^2 \left[ A_{II20} + \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_I(x, y, z) dz dy dx - A_{IV11} \right] + A_{IV01}^2 (A_{IV01} + A_{II10} + \\
 & + 1)^2 - \left[ \frac{2A_{II00}}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_V(x, y, z) dz dy dx + A_{IV01}(1 + A_{IV01} + A_{II10}) - 2A_{II00}(A_{VV20} - A_{IV11}) + \right. \\
 & \left. + A_{IV01}(1 + A_{IV01} + A_{II10}) \right]^2, \quad y = \sqrt[3]{\sqrt{q^2 + p^3} - q} - \sqrt[3]{\sqrt{q^2 + p^3} + q} + \frac{B_2}{6}, \\
 & q = (2B_1 B_3 - 8B_0) \times
 \end{aligned}$$

$$\times \frac{B_2}{48} + \frac{B_2^3}{216} + \frac{B_0(4B_2 - B_3^2) - B_1^2}{8}, \quad p = [3(2B_1 B_3 - 8B_0) - 2B_2^2] / 72,$$

$$A = \sqrt{8y + B_3^2 - 4B_2},$$

$$\begin{aligned}
 \alpha_{2\Phi_I} = & A_{II20} - \frac{1}{\Theta L_x L_y L_z} \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_I(x, y, z, T) I(x, y, z, t) dz dy dx dt + \\
 & + \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_{\Phi_I}(x, y, z) dz dy dx \quad (14)
 \end{aligned}$$

$$\begin{aligned}
 \alpha_{2\Phi_V} = & A_{VV20} - \frac{1}{\Theta L_x L_y L_z} \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_V(x, y, z, T) V(x, y, z, t) dz dy dx dt + \\
 & + \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_{\Phi_V}(x, y, z) dz dy dx.
 \end{aligned}$$

Now one can obtain equation for parameter  $\alpha_{2c}$ . Form of the equation depends on of parameter  $\gamma$ . Analysis of distributions of concentrations of dopant and radiation defects in space and time has been done by using the second-order approximations by using of modified method of averaging of function corrections (with decreased quantity of iterative steps) of these concentrations. The considered approximation is usually enough good approximation to analyze qualitatively the considered concentration and obtain some quantitative results. The analytical calculation results have been checked comparison of the analytical results with results of numerical simulation.

### 3. DISCUSSION

Main aim of the section is analysis of distribution of concentration of dopant in space and time in the considered heterostructure during annealing. Figs. 2 shows distributions of concentrations of dopants in space, which where infused or implanted in the considered epitaxial layer (see Fig. 2a and Fig. 2b, respectively) at the same values of annealing time (the same framework each figure). Larger numbers of curves corresponds to larger difference between dopant diffusion coefficients in epitaxial layer and substrate. The figures show that inhomogeneity could leads to increasing of absolute value of gradient of concentration of dopant (i.e. one can obtain increasing sharpness of the end of doped structure). The increasing leads to decreasing dimensions of the considered transistors and increasing of homogeneity of concentration of dopant in doped areas.

We choose optimal value annealing time as compromise between increasing of homogeneity of dopant concentration in doped area and increasing of sharpness of the concentration in neighborhood of interface between substrate and epitaxial layer (see Fig. 3a for diffusion doping of materials and Fig. 3b for ion doping of materials) [15-20]. Framework the criteria one shall approximate real distributions of concentration of dopant by ideal step-wise distribution  $\psi(x,y,z)$ . compromise value of annealing time by minimization of the following mean-squared error.

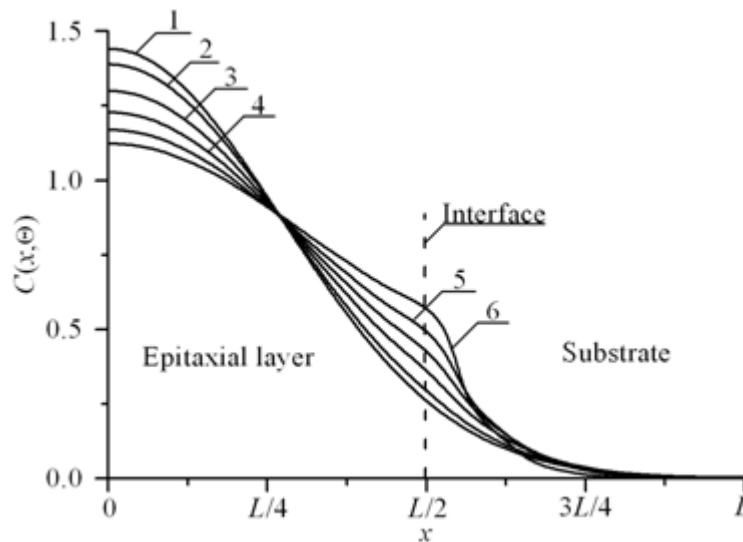


Fig.2a. Distributions of infused dopant concentration in the considered heterostructure. Larger numbers of curves corresponds to larger difference between dopant diffusion coefficients in epitaxial layer and substrate

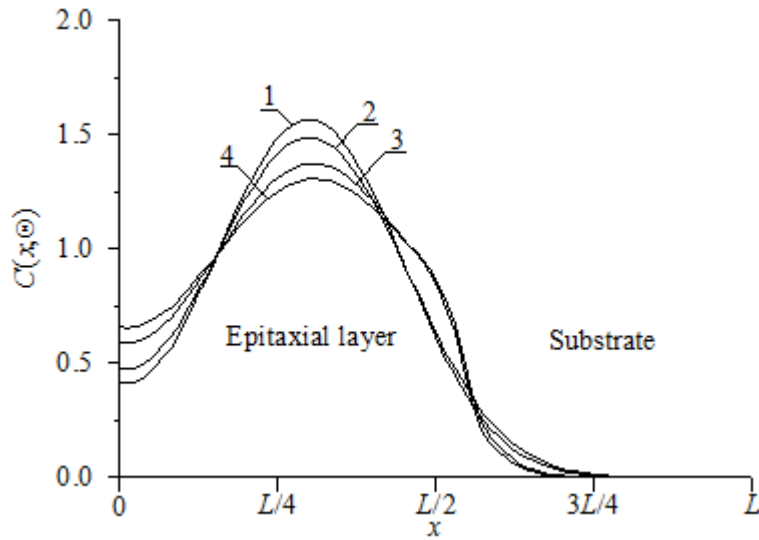


Fig.2b. Distributions of implanted dopant concentration in the considered heterostructure. Larger numbers of curves corresponds to larger difference between dopant diffusion coefficients in epitaxial layer and substrate

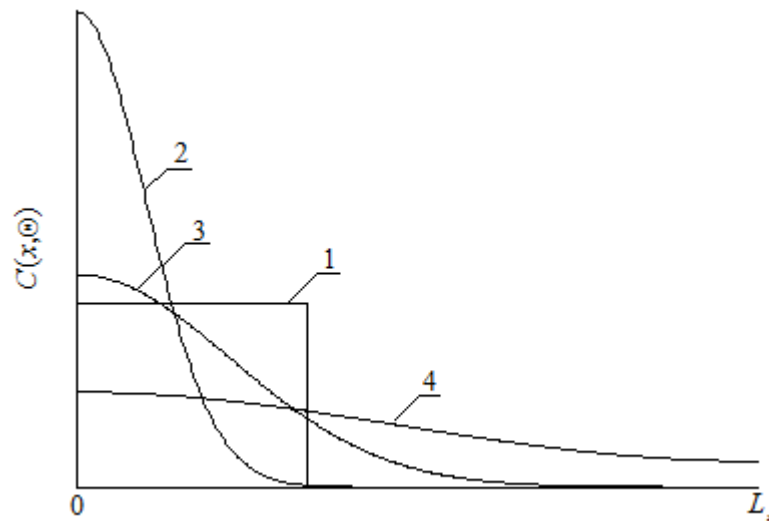


Fig. 3a. Dependences of distributions of concentration of infused dopant on coordinate. Curve 1 is idealized distribution of dopant. Curves 2-4 are real dependences of concentrations dopant on coordinate for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

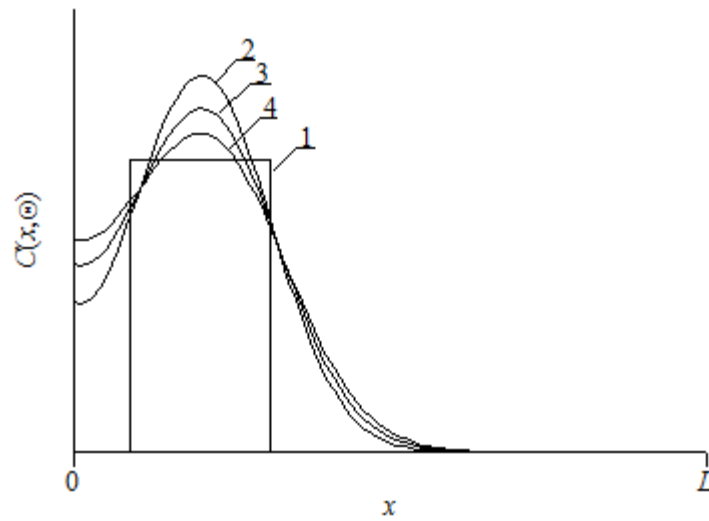


Fig.3b. Dependences of distributions of concentration of implanted dopant on coordinate. Curve 1 is idealized distribution of dopant. Curves 2-4 are real dependences of concentrations dopant on coordinate for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time

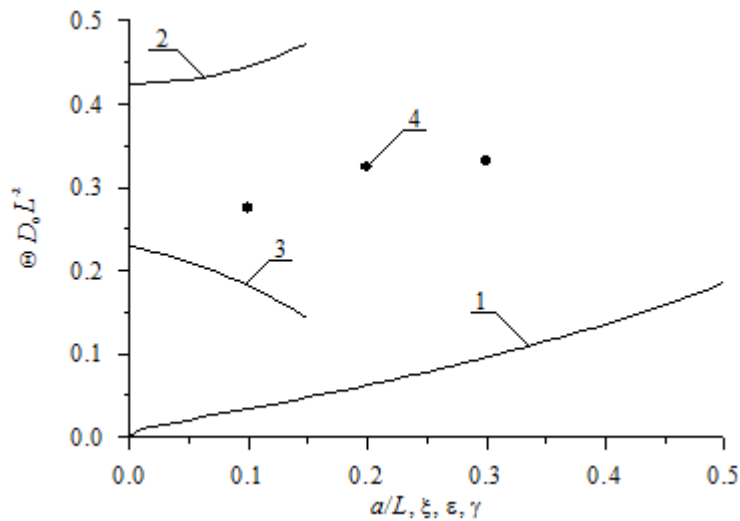


Fig.4a. Dependences of dimensionless optimized annealing time of infused dopant on several parameters. Curve 1 describes dependence of dimensionless optimized annealing time on the value  $a/L$  and  $\xi = \gamma = 0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter  $\epsilon$  for  $a/L=1/2$  and  $\xi = \gamma = 0$ . Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter  $\xi$  for  $a/L=1/2$  and  $\epsilon = \gamma = 0$ . Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter  $\gamma$  for  $a/L=1/2$  and  $\epsilon = \xi = 0$

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} [C(x, y, z, \Theta) - \psi(x, y, z)] dz dy dx. \quad (15)$$

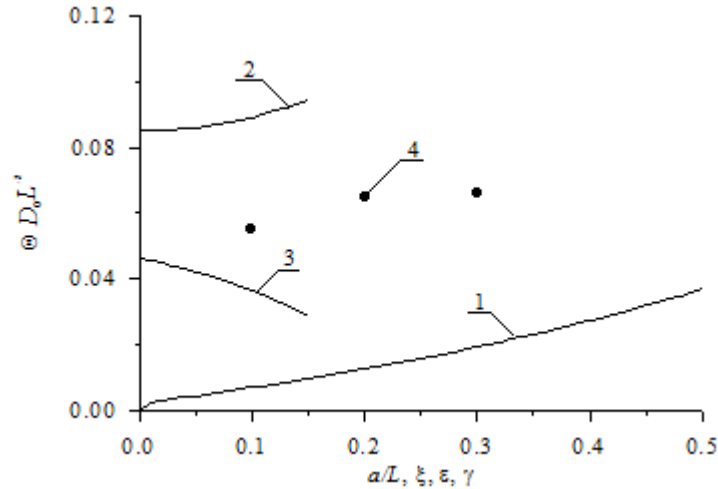


Fig.4b. Dependences of dimensionless optimized annealing time of implanted dopant on several parameters. Curve 1 describes dependence of dimensionless optimized annealing time on the value  $a/L$  and  $\xi = \gamma = 0$  for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter  $\varepsilon$  for  $a/L=1/2$  and  $\xi = \gamma = 0$ . Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter  $\xi$  for  $a/L=1/2$  and  $\varepsilon = \gamma = 0$ . Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter  $\gamma$  for  $a/L=1/2$  and  $\varepsilon = \xi = 0$

We present optimized annealing time on Figs. 4 (Fig. 4a for diffusion type of doping and Fig. 4b for ion type of doping). It is known, that radiation defects should be annealed after ion implantation. The annealing leads to spreading of distribution of concentration of dopant. The ideal spreading leads to achievement of nearest interfaces between materials of heterostructure. If dopant has no time for the achievement during the annealing, it is attracted an interest additional annealing of dopant. The situation leads to smaller value of optimal annealing time of implanted dopant in comparison with diffusion one. At the same time it should be noted, that ion type of doping leads to decreasing of mismatch-induced stress in heterostructure [21].

#### 4. CONCLUSION

We consider an approach for increasing density of field-effect heterotransistors in a single-stage multi-path operational amplifier. At the same time one can obtain decreasing of dimensions of the above transistors. Dimensions of the elements could be decreased by manufacturing of these elements in a heterostructure with specific structure. The manufacturing is doing by doping of required areas of the heterostructure by diffusion or ion implantation with future optimization of annealing of dopant and/or radiation defects.

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