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FREE CONVECTION HEAT TRANSFER OF NANOFLUIDS FROM A HORIZONTAL PLATE EMBEDDED IN A POROUS MEDIUM

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ABSTRACT

In this paper the natural convection heat transfer from a horizontal plate embedded in a porous medium saturated with a nanofluid is numerically analyzed. By a similarity approach the partial differential equations are reduced to a set of two ordinary differential equations. In order to evaluate the influence of nanoparticles on the heat transfer, Ag and Cuo as the nanoparticles were selected. Results show that heat transfer rate (Nur) is a decreasing function of volume fraction of nanoparticles.

KEYWORDS

Free convection, Nanofluids, Porous medium, Horizontal plate, Darcy model

1. INTRODUCTION

Heat transfer in porous media has numerous engineering applications such as ground water pollution, geothermal energy recovery, flow filtering media, thermal energy storage and crude oil extraction (Gorla and Ghamkha 2011).

A nanofluid is a fluid which contains nanometer-sized solid particles. Nanometer-size solid particles have unique chemical and physical properties. Since suspending nanometer-size particles to the conventional heat transfer fluids leads a better heat transfer, nanofluids are proposed to be employed in several applications such as transportation, nuclear reactors and electronics (Saidura et al. 2011).

Evaluating natural convection heat transfer from a horizontal plate embedded in a saturated porous medium has been conducted in some literatures. Cheng and Chang [3] investigated buoyancy induced flows in a saturated porous medium adjacent to impermeable horizontal surfaces. Nield and Bejan [4] investigated natural convection heat transfer from a horizontal plate in porous medium.

Gorla and Chamkha [1] studied natural convective boundary layer flow over a horizontal plate embedded in a porous medium saturated with a nanofluid. In the work of Gorla and Chamkha [1] the lower surface of the plate maintains at constant temperature and they considered dynamic model of nanofluids and focused on the variations of dynamic forces in heat transfer of nanofluids.

In the present analyze, the temperature of the lower surface is differs with longitudinal distance and static model of nanofluids is considered. The influence of variations of volume fraction of Ag and CuO nanoparticles and porosity of the porous medium on natural convection heat transfer of nanofluids above a horizontal plate embedded in a porous medium is studied.

2. MATHEMATICAL ANALYSIS

Consider the steady free convection heat transfer of a nanofluid from a horizontal flat plate embedded in a saturated porous medium as depicted in Fig. 1. It is assumed that the nanofluid is homogeneous, working fluid and porous medium are in local thermodynamic equilibrium and the properties of working fluid and porous medium are constant. Recently, Noghrehabadi et al. [5] have studied the influence of thermophoresis forces and Brownian motion on the heat transfer of nanofluids. They reported that on the heat transfer of nanofluids the effect of nanoparticles migration because of thermophoresis effects and Brownian motion is negligible. Therefore, in the present paper, the heat transfer of nanofluids because of migration of nanoparticles in the base fluid is ignored.

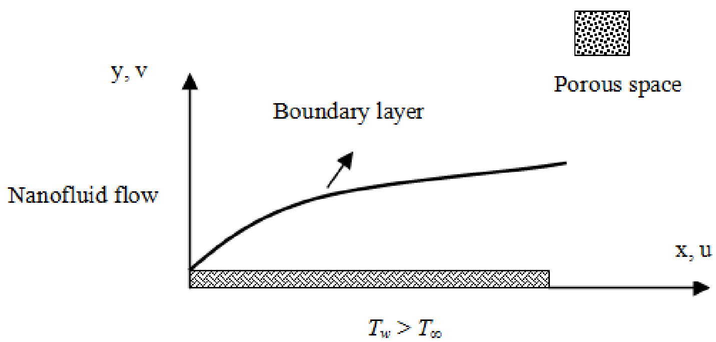

Figure 1. Schematic diagram and coordinate system

Assuming an isotropic Darcy porous medium and employing Boussinsq approximation, the governing equations in the Cartesian coordinate system of *x* and *y* are written as follow (Nield and Bejan 2013 and Rosca et al. 2012):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u=-\frac{K}{\mu_{nf}}\frac{\partial P}{\partial x} \quad (2)$$

$$v=-\frac{K}{\mu_{nf}}\left(\frac{\partial P}{\partial y}+\rho_{nf}\,g\right) \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{m,nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

$$\rho_{nf}=\rho_{nf,\infty}[1-\beta_{nf}(T-T_\infty)] \quad (5)$$

The boundary conditions for these equations are

$$y = 0: \qquad T_w = T_\infty + Ax^\lambda \qquad v = 0 \tag{6}$$
$$y \to \infty: \qquad T = T_\infty \qquad u = 0 \tag{7}$$

The above symbols are introduced in nomenclature.
By introducing stream function ψ, the continuity equation is satisfied automatically

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{8}$$

By invoking boundary layer approximations and using definition of stream function, the pressure gradient is omitted and governing equations become:

$$\frac{\partial^2 \psi}{\partial y^2} = -\frac{\rho_{nf,\infty}\beta_{nf} gK}{\mu_{nf}}\frac{\partial T}{\partial x} \tag{9}$$
$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha_{m,nf}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

In terms of ψ, boundary conditions become:

$$y = 0: \qquad T_w = T_\infty + Ax^\lambda \qquad \frac{\partial \psi}{\partial x} = 0 \tag{11}$$
$$y \to \infty: \qquad T = T_\infty \qquad \frac{\partial \psi}{\partial y} = 0 \tag{12}$$

In the following denoting $\rho_{nf.\infty}$ and $\rho_{bf,\infty}$ by ρ_{nf} and ρ_{bf} respectively. Using the following equations, the thermo-physical properties of the porous medium and working fluid can be evaluated [4, 7, 8]:

$$\alpha_{m,f} = \frac{k_{m,f}}{(\rho c_p)_f} \tag{13}$$
$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_{bf} + \varphi(\rho c_p)_p \tag{14}$$
$$k_{eff} = k_m^{1-\varepsilon} k_f^{\varepsilon} \tag{15}$$
$$\frac{k_{nf}}{k_{bf}} = \frac{k_p + 2k_{bf} + 2\varphi(k_p - k_{bf})}{k_p + 2k_{bf} - \varphi(k_p - k_{bf})} \tag{16}$$
$$\beta_{nf} = \frac{(1-\varphi)\rho_{bf}\beta_{bf} + \varphi\rho_p\beta_p}{\rho_{nf}} \tag{17}$$
$$\mu_{nf} = \frac{\mu_{bf}}{(1-\varphi)^{2.5}} \tag{18}$$

The above symbols are introduced in the nomenclature. Eq. (15) computes the effective thermal conductivity of the porous medium and the base fluid when k_f=k_{bf}, and the effective thermal conductivity of the porous medium and the nanofluid is obtained when k_f=k_{nf}. When k_f and k_m are not much different, Eq. (15) provides a proper estimate [4]. It should be noted that subscript f denotes the working fluid (either the base fluid or the nanofluid).
In order to obtain a similarity solution, the dimensionless variables are introduced as follows:

$$\eta = (\frac{\rho_{bf} g \beta_{bf} KA}{\mu_{bf} \alpha_{m,bf}})^{\frac{1}{3}} y x^{\frac{\lambda-2}{3}} = \frac{y}{x}(Ra_x)^{\frac{1}{3}} \tag{19}$$

$$\psi = \alpha_{m,bf}[\frac{\rho_{bf} g \beta_{bf} KA}{\mu_{bf} \alpha_{m,bf}}]^{\frac{1}{3}} x^{\frac{\lambda+1}{3}} f(\eta) = \alpha_{m,f}(Ra_x)^{\frac{1}{3}} f(\eta) \tag{20}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{21}$$

the local Rayleigh number is introduced as

$$Ra_x = \frac{\rho_{bf} \beta_{bf} g K (T_w - T_\infty) x}{\mu_{bf} \alpha_{m,bf}} \tag{22}$$

Applying similarity variables, on the Pdes, Eqs. (9) and (10), lead to the following Odes:

$$f'' + \{(1-\varphi)^{2.5}[1-\varphi+\varphi(\frac{\rho_p}{\rho_{bf}})(\frac{\beta_p}{\beta_{bf}})]\}[\lambda\theta + (\frac{\lambda-2}{3})\eta\theta'] = 0 \tag{23}$$

$$\frac{\alpha_{m,nf}}{\alpha_{m,bf}}\theta'' - \lambda\theta f' + (\frac{\lambda+1}{3}) f\theta' = 0 \tag{24}$$

The transformed boundary conditions are

$$\eta = 0: \qquad \theta(0) = 1 \qquad f(0) = 0 \tag{25}$$

$$\eta \to \infty \qquad \theta(\infty) = 0 \qquad f'(\infty) = 0 \tag{26}$$

Here, α_D is an effective parameter in heat transfer rate and defined as follow:

$$\alpha_D = \frac{\alpha_{m,nf}}{\alpha_{m,bf}} \tag{27}$$

The local surface heat flux can be evaluated as follow:

$$q = -k_{m,nf}(\frac{\partial T}{\partial y})_{y=0} \tag{28}$$

Using the similarity variables on Eq. (28) leads to:

$$q(x) = -k_{m,nf} A[\frac{\rho_{bf} g \beta_{bf} KA}{\mu_{bf} \alpha_{m,bf}}]^{\frac{1}{3}} x^{\frac{4\lambda-2}{3}} \theta'(0) \tag{29}$$

The quantity of practical interest, the local Nusselt number (Nu_x) is defined as follow:

$$Nu_x = \frac{xq}{k_{m,bf}(T_w - T_\infty)} \tag{30}$$

Using Eq. (30) with the aid of Eq. (29), lead to

$$Nur = Nu_x Ra_x^{-\frac{1}{3}} = -\frac{k_{m,nf}}{k_{m,bf}}\theta'(0) \tag{31}$$

It should be noted that when 1/2 ≤ λ ≥ 2, the problem is physically realistic. When λ=1/2 constant heat flux boundary condition is dominant [3].

3. RESULTS AND DISCUSSION

in the present paper, the thermo-physical properties of the selected nanoparticles and water are given in Table 1 and soil is adopted as the porous medium with the porosity (ε) 0.5 and thermal conductivity (k) 1.5 ($W/m.K$) [9].

The system of the ordinary differential equations, Eqs. (23) and (24) subject to the boundary conditions, Eqs. (25) and (26), are solved numerically by means of Runge-Kutta-Fehelberg method with shooting technique. The asymptotic value of infinity is assumed as η_∞=8.

Assuming φ=0 (no nanoparticles suspend in the base fluid), present work reduces to the work of Cheng and Chang [3]. In order to check the accuracy of the numerical method, values to
–θ'(0) reported by Cheng and Chang [3] are compared with the slope of the diagram of dimensionless temperature at η=0 obtained in the present paper in Table 2. Further more Fig. 2 show a comparison for the $\theta(\eta)$ diagram of the present paper for pure fluid (φ=0) when λ=1 with the work of Cheng and Chang [3]. These comparisons show excellent agreement between the results. Moreover, Fig. 2 present the $\theta(\eta)$ diagram for Ag and CuO nanoparticles and for various values of φ (φ=0.1, 0.2) when λ=1. It can be seen that the $\theta(\eta)$ diagram for Ag and CuO nanoparticles are nearly equal. However, for lower values of φ (φ=0.1), the non-dimensional temperature is lower.

Fig. 3 shows the variations of reduced Nusselt number against φ for chooses values of λ (λ=0.5, 1, 2). Heat transfer rate (Nur) is a decreasing function of φ for different values of λ. The results shows that increase of φ significantly decrease the Nur.

Variations of Nur against ε for various values of λ (λ=0.5, 1, 2) when (φ=0.1) and k_m=1.5 ($W/m.K$) is depicted in Fig. 4. For all values of λ, Nur is an increasing function of porosity. It is obviously clear that using nanofluids in higher porosity leads a better heat transfer because when ε is higher the basic role in heat transfer devoted to the working fluid not porous medium.

Variations of Nur against ε for different values of φ (φ=0.05, 0.1, 0.2) when λ=1 and k_m=1.5 ($W/m.K$) is illustrated in Fig. 5. For all values of φ, Nur is an increasing function of ε. When ε is low, the basic role in heat transfer devoted to the solid part (porous medium); in contrast, when ε is high the basic role in heat transfer devoted to the working fluid. Therefore when ε is higher, Nur is higher for higher values of φ respect to lower values of φ.

Table 1. Properties of silver and copper oxide and water [10].

Physical properties	Fluid phase (Water)	Ag	CuO
c_p (J/kg k)	4179	235	531.8
ρ (kg/m^3)	997.1	10500	3620
k (W/m k)	0.613	429	76.5
$\beta \times 10^5$ (K^{-1})	21	1.89	1.80

Table 2. comparison between the results of present paper when φ=0 with the results of Cheng and Chang [3].

	Present work for pure fluid	Cheng and Chang [3]
λ	$-\theta'(0)$	$-\theta'(0)$
0.5	0.81565	0.8164
1	1.09894	1.099
1.5	1.34554	1.351
2	1.57094	1.571

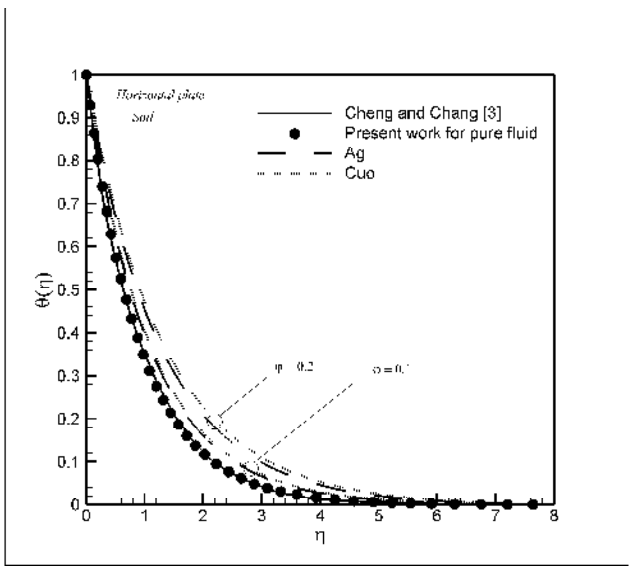

Figure 2. Comparison between results and variations of $\theta(\eta)$ against η.

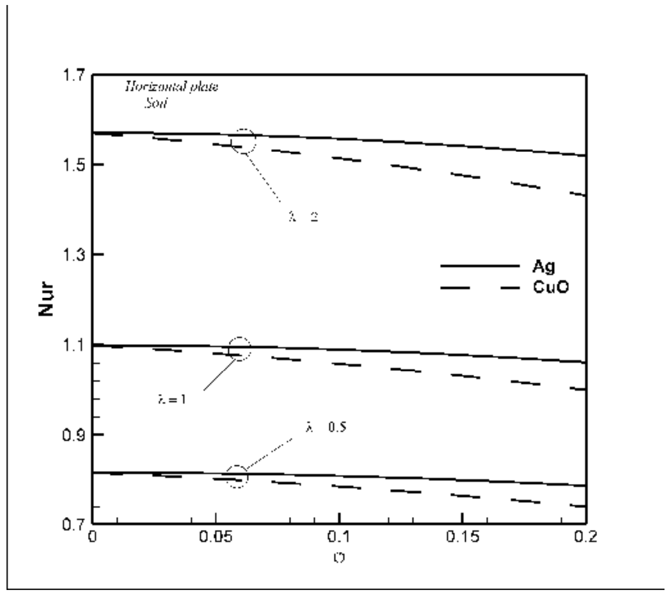

Figure 3. Variations of Nur against φ for Ag and CuO for λ=0.5, 1, 2.

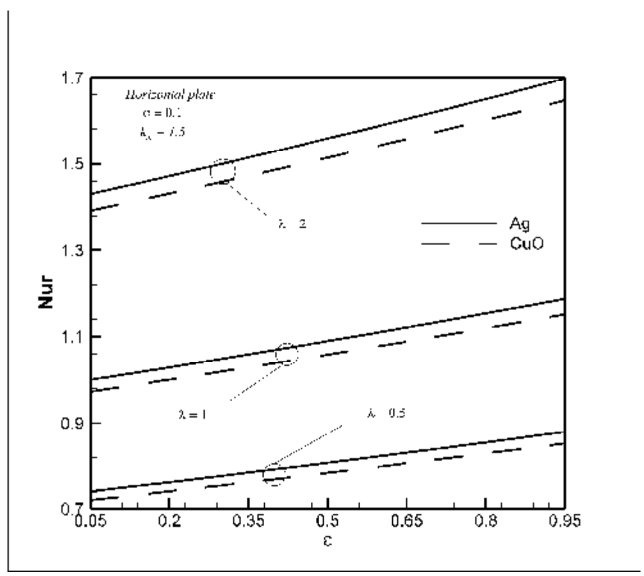

Figure 4. Variations of Nur against ε for Ag and CuO for λ=0.5, 1, 2.

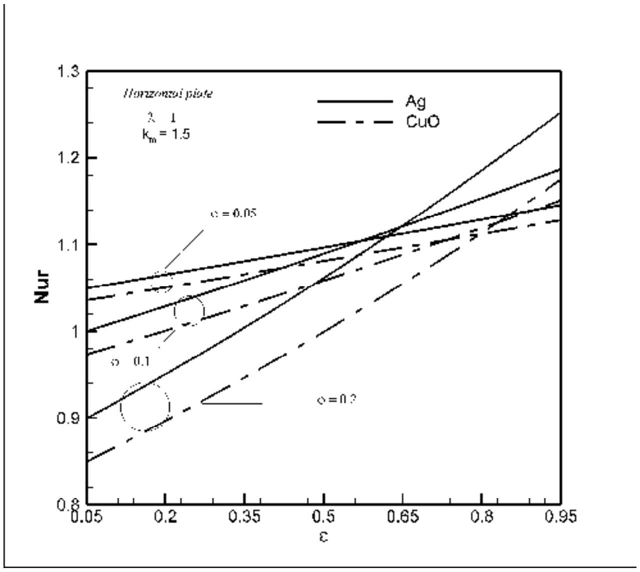

Figure 5. Variations of Nur against ε for Ag and CuO for φ=0.05, 0.1, 0.2.

4. CONCLUSIONS

In the present paper, the influence of nanoparticles on natural convection heat transfer from a horizontal plate in a porous medium is numerically analyzed. The Pdes are transformed to Odes. In summary, the results have shown the following:

1) For all studied cases, Nur is an increasing function of ε.
2) For all studied cases, Nur is a decreasing function of φ.
3) For higher values of λ, the value of Nur is higher.
4) For higher values of ε, Nur is higher for higher values of φ respect to lower values of φ.
5) For lower values of ε, Nur is higher for lower values of φ respect to higher values of φ.

Nomenclature	
K	permeability (m^2)
k	thermal conductivity (W/m K)
c_p	specific heat at constant pressure (J/kg K)
f	dimensionless stream function
Nu_x	local Nusselt number
A	constant defined by Eq. (6)
Ra_x	Rayleigh number
g	gravitational acceleration (m/s^2)

T	fluid temperature (K)
U_∞	velocity at infinity (m/s)
T_∞	ambient temperature (K)
u, v	Darcy velocities (m/s)
x, y	Cartesian coordinates (m)
q	local surface heat flux (W/m^2)
Greek symbol	
α	thermal diffusivity (m^2/s)
β	thermal expansion (K^{-1})
φ	volume fraction of nanoparticles
θ	dimensionless temperature
μ	dynamic viscosity (kg/m s)
υ	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
(ρc_p)	heat capacity (kg/m^3 K)
η	similarity variable
ε	Porosity
ψ	stream function (m^2/s)
λ	constant defined by Eq. (6)
Subscripts	
bf	base fluid
p	particle
nf	nanofluid

m	porous media
f	fluid (pure fluid or nanofluid)
m,f	effective value for base fluid and porous medium
m,nf	effective value for porous medium and nanofluid
∞	ambient condition
Superscript	
$'$	differentiation with respect to η

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References

[1] Gorla, R. S. R. & Chamkha, A. (2011) "Natural Convective Boundary Layer Flow over a Horizontal Plate Embedded in a Porous Medium Saturated with a Nanofluid", *Journal of Modern Physics*, Vol. 2, pp62-71.

[2] Saidura, R. Leong, K. Y. & Mohammad, H. A. (2011) "A review on applications and challenges of nanofluids", *Renewable and Sustainable Energy Reviews*, Vol. 15, pp1646-1668.

[3] Cheng, P. & Chang, I-D. (1976) "Buoyancy induced flows in a saturated porous medium adjacent to impermeable horizontal surfaces", *Int. J. Heat Mass Transfer*, Vol. 19, pp1267-1276.

[4] Nield, D. A. & Bejan, A. (2013) Convection in Porous Media, *Springer*.

[5] Noghrehabadi, A. Ghalambaz, M. & Behseresht, A. (2013) "Natural-Convection Heat and Mass Transfer from a Vertical Cone in Porous Media Filled with Nanofluids Using the Practical Ranges of Nanofluids", *Chemical Engineering Research and Design*.

[6] Rosca, A. V. Rosca, N. C. Grosan, T. & Pop, I. (2012) "Non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media", *International Communications in Heat and Mass Transfer*, Vol. 39, pp1080-1085.

[7] Khanafer, K. & Vafai, K. (2011) "A Critical Synthesis of Thermophysical Characteristics of Nanofluids", *international Journal of Heat and Mass Transfer*, Vol. 54, Nos. 19-20, pp4410-4428.

[8] Das, S.K. Choi, S.U.S. Yu, W. & Pradeep, T. (2007) "Nanofluids Science and Technology", *Wiley*.

[9] Rabeti, M. Noghrehabadi, A. & Ghanbarzadeh, A. (2013) "Forced Convection Heat Transfer over a Horizontal Plate Embedded in a Porous Medium Saturated with a Nanofluid in the Presence of Heat Sources", *Journal of Advances in Energy Engineering*, Vol. 1, No. 2, pp46-51.

[10] Mahdy, A. (2012) "Unsteady mixed convection boundary layer flow and heat transfer of nanofluids due to stretching sheet", *Nuclear Engineering and Design*, Vol. 249, pp248-255.

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Masoud Rabeti is currently lecture in the department of mechanical engineering at Islamic Azad University. His research interests are on the field of heat transfer of Nanofluids in the porous media.