GDQ SIMULATION FOR FLOW AND HEAT TRANSFER OF A NANOFLUID OVER A NONLINEARLY STRETCHING SHEET

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ABSTRACT

This paper presents the generalized differential quadrature (GDQ) simulation for analysis of a nanofluid over a nonlinearly stretching sheet. The obtained governing equations of flow and heat transfer are discretized by GDQ method and then are solved by Newton-Raphson method. The effects of stretching parameter, Brownian motion number (Nb), Thermophoresis number (Nt) and Lewis number (Le), on the concentration distribution and temperature distribution are evaluated. The obtained results exhibit that the heat transfer rate can be controlled by choosing different nanoparticles and stretching parameter.

KEYWORDS

GDQ, Nonlinear stretching sheet, Nanofluid, Brownian motion, Thermophoresis

1. INTRODUCTION

Boundary layer behaviour over a stretching surface has important industrial applications and considerable role on many technological processes. Sakiadis [1] presented his work as a pioneering study about boundary layer flow over a continuous solid surface moving with constant velocity, then many researchers attention to this, and published many papers about that. The flow created by the stretching of a sheet is obtained by Crane [2]. Many researchers such as Gupta and Gupta [3], Dutta et al. [4], Chen and Char [5], Andersson [6] developed the work of Crane [2] by adding the effects of heat and mass transfer analysis under different physical conditions. Wang [7] found the closed form similarity solution of a full Navier–Stoke's equations for the flow due to a stretching sheet with partial slip. Furthermore, Wang [8] investigated stagnation slip flow and heat transfer on a moving plate. Kelson and Desseaux [9, 10] investigated the effect of surface conditions on the micropolar flow driven by a porous stretching sheet and the flow of a micropolar fluid bounded by a linearly stretching sheet, and then Bhargava et al. [11] developed that work by using nonlinear stretching sheet.

Meanwhile, Choi [12] proposed nanofluid for the first time and after that studies related to the nanofluid dynamics have increased greatly due to its wide applications in industrial and engineering systems [13-17]. Nanofluid is a suspension of a nanometer size solid particles and fibres in convectional base fluids. Commonly used base fluids are water, toluene, oil and ethylene glycol mixture and etc. Usually, the nanoparticles are constructed of metals such as Aluminum and Copper, metal oxides (Al_2O_3), carbides (SiC), nitrides (AlN, SiN) or nonmetals such as Graphite and Carbon nanotubes. Khan and Pop [18] obtained the flow of a nanofluid caused by stretching sheet. After that, Khan and Aziz [19] used the same model to earn the boundary layer flow of a nanofluid with a heat flux. Kuznetsov and Nield [20] studied the influence of nanoparticles on natural convection boundary layer flow past a vertical plate by taking Brownian motion and thermophoresis into account. Rana and Bhargava [21] extended the work of Khan and Pop [18] by nonlinearly stretching sheet with finite element method (FEM).

The most numerical methods such as finite difference method (FDM) and finite element method (FEM) should be applied to large number of grid points for having a good accuracy. Therefore, these methods require to high calculations volume. But some new methods such as the generalized differential quadrature method can have an exact response with a few chosen grid point. The technique of differential quadrature (DQ) was proposed by Bellman et al [22]. The DQ method is based on determination of weighting coefficients for any order derivative discretization. Bellman et al. [22] suggested two approaches to determine the weighting coefficients of the first order derivative. Shu [23] proposed the generalized differential quadrature (GDQ), which can calculates the weighting coefficients of the first order derivative by a simple algebraic formulation without any limitation in choice of nodes, and the weighting coefficients of the second and higher order derivatives by a recurrence relation [24]. Now, this method is applied to solve many engineering problems with many researchers [25-28].

In this paper, GDQ method is used to discrete the governing equations of flow and heat transfer of nanofluid over nonlinearly stretching sheet. The discretized equations are solved by Newton-Raphson method. The effects of stretching parameter (κ), Brownian motion number (*Nb*), thermophoresis number (*Nt*) and Lewis number (*Le*), on the concentration distribution and temperature distribution are evaluated.

2. FORMULATION OF PROBLEM

Consider laminar, steady, two dimensional boundary layer flow of an incompressible viscous nanofluid over a nonlinearly stretching sheet. This geometry is shown in Fig.1, the sheet stretching non-linearly that caused by applying two equal and opposite forces along x-axis and this force makes the flow. Velocity of stretching is $u = ax^{\kappa}$, where, *a* is constant and κ is non-linearly parameter. The pressure gradient and external force are neglected.

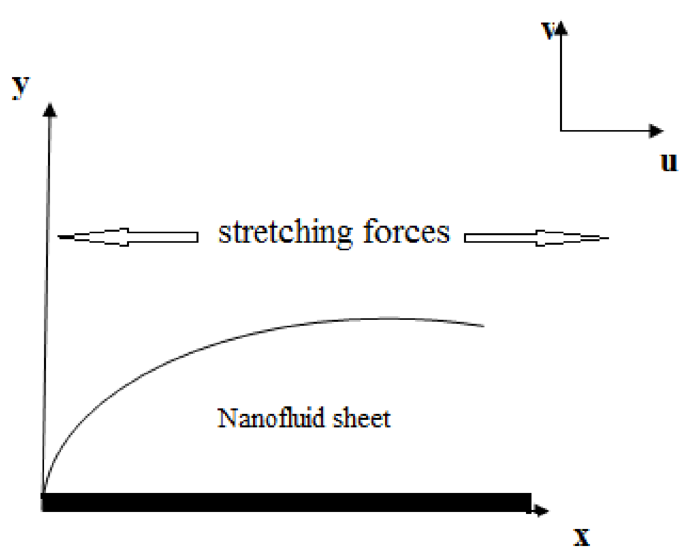


Figure 1. Physical model

The basic equations of nanoparticles and conservation of mass, momentum, thermal energy for this geometry and flow can be expressed as [18,21];

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\nabla^2 T+\tau[D_B\frac{\partial C}{\partial y}.\frac{\partial T}{\partial y}+(D_T/T_\infty)(\frac{\partial T}{\partial y})^2] \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+(D_T/T_\infty)\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where

$$\alpha_m=\frac{k_m}{(\rho c)_f}\ ,\ \tau=\frac{(\rho c)_p}{(\rho c)_f}$$

Here, α_m is the thermal diffusivity and τ is the proportion of the effective heat capacity of the nanoparticle to heat capacity of the fluid. Also, D_B and D_T are the Brownian diffusion and the thermophoretic diffusion coefficients, respectively.

The boundary condition for this problem are given by;

$$v=0,\ u_w=ax^{\kappa}, T=T_w, C=C_w \quad \text{at } y=0 \tag{5a}$$

$$u=v=0, T=T_\infty, C=C_\infty \quad as \quad y\to\infty \tag{5b}$$

Introducing the suitable transformations as;

$$\eta=y\sqrt{\frac{a(\kappa+1)}{2\nu}}x^{\frac{\kappa-1}{2}},\quad u=ax^{\kappa}f'(\eta),\quad v=-\sqrt{\frac{a\nu(\kappa+1)}{2}}x^{\frac{\kappa-1}{2}}(f+(\frac{\kappa-1}{\kappa+1})\eta f')$$

$$\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

By substituting Eq. (6) into Eqs. (1-4) and Eqs. (5a, 5b), the governing equations and boundary condition transform to;

$$f'''+ff''-(\frac{2\kappa}{\kappa+1})f'^2=0 \tag{7}$$

$$\frac{1}{\Pr}\theta'' + f\,\theta' + Nb\,\theta'\phi' + Nt\,(\theta')^2 = 0 \tag{8}$$

$$\phi'' + \frac{1}{2}Lef\,\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{9}$$

And

$$\begin{aligned} &f = 0,\ f' = 1,\ \theta = 1,\ \phi = 1 \quad at\ \eta = 0 \\ &f' = 0,\ \theta = 0,\ \phi = 0 \qquad\qquad \eta \to \infty \end{aligned} \tag{10}$$

where $\Pr = \dfrac{\nu}{\alpha}$ and $Le = \dfrac{\nu}{D_B}$ are the Prandtl number and the Lewis number, respectively. Also the Brownian motion and the thermophoresis parameters are introduced as follow:

$$\begin{aligned} Nb &= \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f\,\nu} \\ Nt &= \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f\,\nu T_\infty} \end{aligned} \tag{11}$$

To solve the high nonlinear Eqs. (7-9), a powerful method is used that describes in the next section.

3. METHOD OF SOLUTION

3.1. Generalized differential quadrature

The GDQ method is based on the finding of weight coefficients and discretization derivation of equations. The weighting coefficients for the first-order and higher-order derivatives are calculated by the simple algebraic formulation and the recurrence relation, repectively. The details of GDQM can be found in [24].

n-th order derivative with respect to *x*, at the grid point x_i as;

$$f_x^{(n)}(x_i) = \sum_{k=1}^{N} C_{ik}^{(n)} f(x_k), \quad n = 1,\ldots,N-1 \tag{12}$$

Weighting coefficients for the first order derivative;

$$C_{ij}^{(1)} = \frac{A^{(1)}(x_i)}{(x_i - x_j)A^{(1)}(x_j)}, \quad i, j = 1,\ldots,N, i \neq j \tag{13}$$

Where

$$A^{(1)}(x_i) = \prod_{\substack{j=1 \\ j\neq i}}^{N} (x_i - x_j) \tag{14}$$

Weighting coefficients for the second and higher order derivatives;

$$C_{ij}^{(n)} = n\left(C_{ii}^{(n-1)} C_{ij}^{(1)} - \frac{C_{ij}^{(n-1)}}{x_i - x_j} \right), \quad i, j = 1, 2, ..., N;\ n = 2, 3, ..., N-1 \tag{15}$$

When i=j, the weighting coefficients given by;

$$C_{ii}^{(n)} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} C_{ij}^{(n)}, \quad i = 1, ..., N-1;\ n = 1, 2, ..., N-1 \tag{16}$$

The sample points are obtained from the Chebyshev-Gauss-Lobatto as follows:

$$X_i = \frac{1}{2}(1 - \cos(\frac{(i-1)\pi}{N-1})) \tag{17}$$

3.2. Discretization of the governing equations

Now Eqs. (7-9) are discreted with GDQ method:

$$\sum_{k=1}^{N} C_{ik}^{(3)} f_k + f_i \sum_{k=1}^{N} C_{ik}^{(2)} f_k - \frac{2n}{n+1} (\sum_{k=1}^{N} C_{ik}^{(1)} f_k)^2 = 0 \tag{18}$$

$$\frac{1}{\Pr} \sum_{k=1}^{N} C_{ik}^{(2)} \theta_k + f_i \sum_{k=1}^{N} C_{ik}^{(1)} \theta_k + Nt (\sum_{k=1}^{N} C_{ik}^{(1)} \theta_k)^2 + Nb (\sum_{k=1}^{N} C_{ik}^{(1)} \theta_k)(\sum_{k=1}^{N} C_{ik}^{(1)} \phi_k) = 0 \tag{19}$$

$$\sum_{k=1}^{N} C_{ik}^{(1)} \phi_k + \frac{1}{2} Le f_i \sum_{k=1}^{N} C_{ik}^{(1)} \phi_k + \frac{Nt}{Nb} \sum_{k=1}^{N} C_{ik}^{(2)} \theta_k = 0 \tag{20}$$

Also, discreted boundary conditions are

$$\begin{gathered} f_1 = 0, \quad \theta_1 = \phi_1 = 1, \quad \theta_N = \phi_N = 0 \\ \sum_{k=1}^{N} C_{1k}^{(1)} f_k = 1, \quad \sum_{k=1}^{N} C_{Nk}^{(1)} f_k = 0 \end{gathered} \tag{21}$$

After that, the set of equations are solved by Newton-Raphson method.

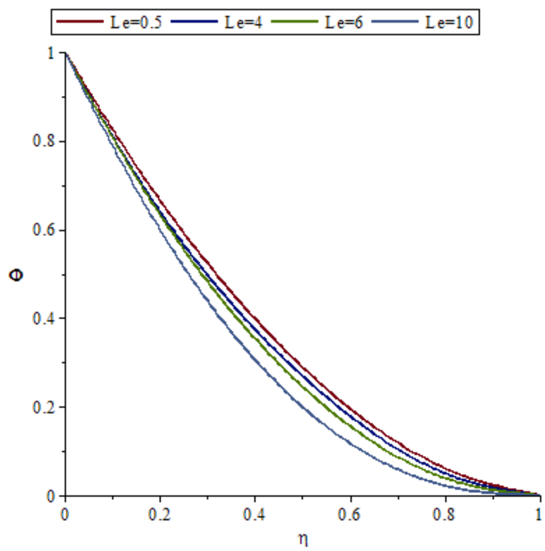
4. RESULTS AND DISCUSSION

Numerical results are demonstrated for different values of Lewis number (Le), Brownian motion parameter (Nb), thermophoresis parameter (Nt) and non-linearly parameter (κ). Also, Prandtl number is assumed equal to Pr=2.

The obtained results by GDQ method are compared with presented results in Ref [29], where the present problem is solved without Brownian motion parameter (Nb) and thermophoresis parameter (Nt). This comparison is shown in Table 1 for reduced Nusselt number $\left|-\theta'(0)\right|$.

Table 1. Comparison of results for reduced Nusselt number $\left|-\theta'(0)\right|$ and Pr=1.

κ	**Ref [29]**	**Present results**
0.2	0.610262	0.6112
0.5	0.595277	0.5953
1.5	0.574537	0.5755
3.0	0.564472	0.5643
10	0.554960	0.55510



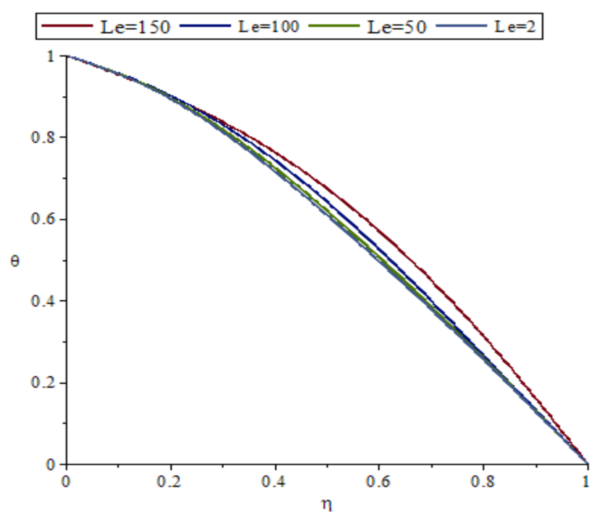


Figure 2. Effect of Lewis number (Le) on concentration distribution and temperature distribution for Nb = 0.5, Nt = 0.5, Pr= 2.0, κ = 2.

Effects of Lewis number (Le) on concentration distribution and temperature distribution are shown in Figures 2. These figures show that by increasing the Lewis number increases the temperature distribution and decreases the concentration distribution. Figures 3 show variations of concentration and temperature distribution by stretching parameter.

Effects of Brownian motion parameter (Nb) on concentration distribution and temperature distributions are displayed in Figures 4. These figures prove that by increasing the Brownian motion increases the temperature distribution and decreases the concentration distribution.

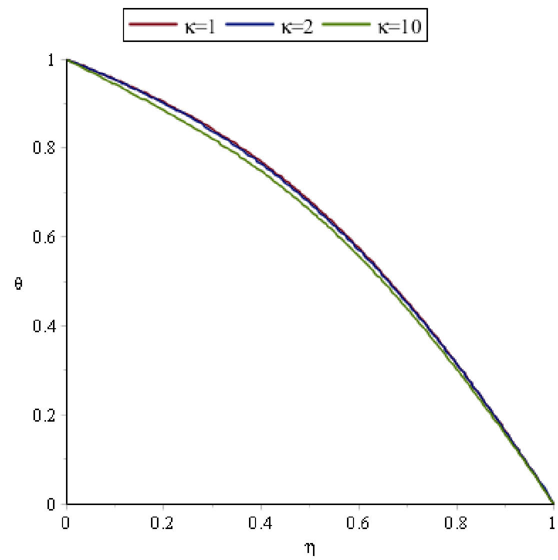


Figure 3. Effect of stretching parameter (κ) on concentration distribution and and temperature distribution for Nb = 0.5, Nt = 0.5, Pr = 2.0, Le = 10.

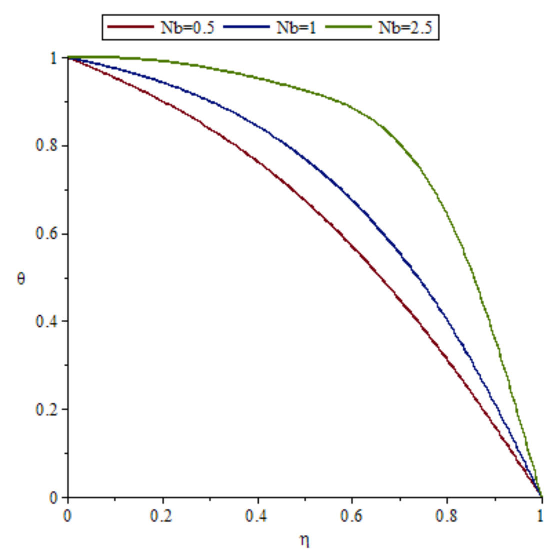


Figure 4. Effect of Brownian motion parameter (Nb) on concentration distribution and temperature distribution for Nt = 0.5, Pr = 2.0, Le = 2.0, $\kappa = 2$.

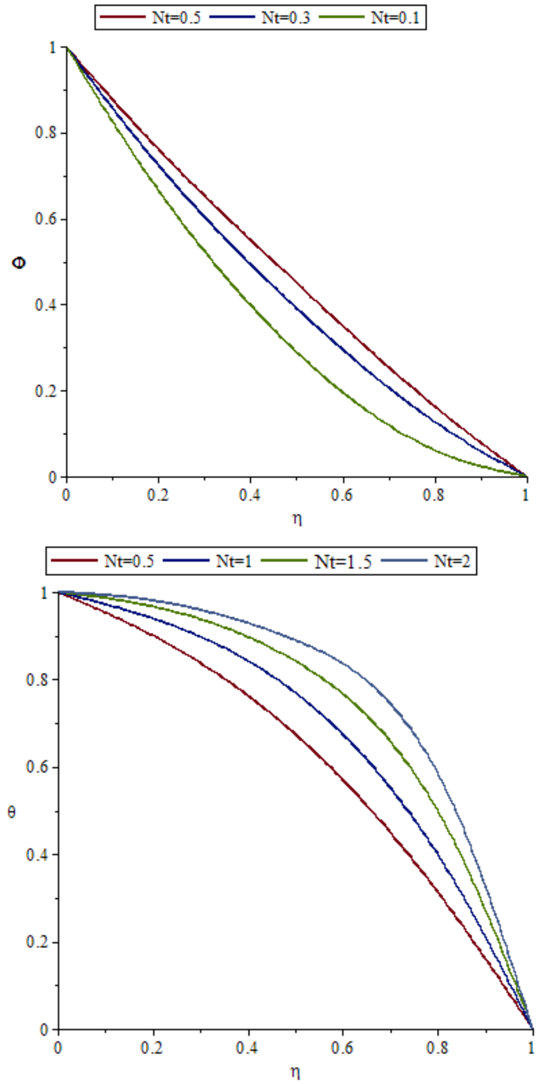


Figure 5. Effect of thermophoresis parameter (Nt) on concentration distribution and temperature distribution for Nb = 0.5, Pr = 2.0, Le = 2.0, $\kappa = 2$

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Also, curves of concentration and temperature distributions versus variations of thermophoresis parameter (Nt) are presented in Figures 5. The results indicate that, increasing the thermophoresis parameter increases the temperature distribution and decreases the concentration distribution.

5. CONCLUSIONS

In this paper, a powerful method (GDQ) was used to solve set of nonlinear equations for nanofluid flow which are created by nonlinear stretching sheet. The GDQ method can be solved this problem with few grid points and its results are accurate with minimum volume of calculations. This study shows effects of different parameters on concentration distribution and temperature distribution. As can be seen, increasing the Lewis number (*Le*) decrease both temperature and concentration profiles and increasing the Brownian motion parameter (*Nb*) increases temperature profile and decreases concentration profile. Therefore, the heat transfer rate can be controlled by choosing different nano-particles for nanofluid. Also, increasing thermophoresis parameter (*Nt*) increases concentration profile and decreases temperature profile. The heat transfer rate and the concentration profile can be changed by decreasing or increasing stretching parameter (*n*).

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